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W WORK FOR UPPER AND MIDDLE-CLASS SCHOOLS.

By E. T. STEVENS, A.K.C., London.

AND

CHARLES HOLE, F.R.G.S.

Editors of 'The Grade Lesson Books,' 'The Advanced Lesson Books,' &c.

It has been suggested by numerous teachers of upper and middle-class schools that the carefully graduated system of teaching to read and spell adopted in the 'Grade Lesson Books,' and highly approved by the teachers of elementary schools under Government inspection, would find equal favour among teachers of schools of a higher class, as the principles of education are the same for the rich as for the poor.

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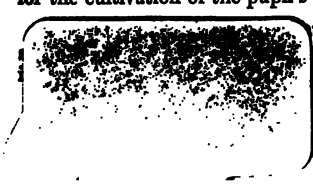

tended to continue this training, pro solid and direct instruction in various

KEN, and CO. Paternoster Row.

MODERN ARITHMETIC.

A TREATISE

ADAPTED FOR

SCHOOL WORK AND PRIVATE STUDY

CONTAINING NUMEROUS

IMPROVEMENTS IN AID OF THE PREPARATION OF CANDIDATES

FOR PUBLIC EXAMINATIONS.

BY THE

REV. JOHN HUNTER, M.A.

Formerly Vice-Principal of the
National Society's Training College, Battersea.

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LONGMANS, GREEN, AND CO.
1866.

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PREFACE.

IT IS HOPED that the title which has been assumed for this publication will be justified by the success of the work in meeting the requirements of the present day. An endeavour has been made to include in one book all modern improvements in their most matured and useful forms, and so to teach and exemplify a general conciseness of method in the solution of problems, as to promote greater facility and power of comprehension in the pupil's mind than the tedious methods that are still so prevalent. The problems themselves, which are very numerous, and nearly all original, are carefully graduated; even those under the head of Miscellaneous Exercises are divided into brief sets, each of which forms a progressively arranged Examination Paper; while several of a kind which has hitherto been found only among miscellaneous questions are, by very simple demonstration, reduced to one Rule.

With regard to the order of the Rules in this Treatise, the author has preferred expedient arrangement to that which according to abstract system may be regarded as more scientific; but as the principal diffi-

culties in Arithmetic arise from the peculiarities of particular examples, an abundant variety of examples are here worked out, with an amount of explanation sufficient to promote in the pupil a proper aptitude for the solution of similar problems.

Those arithmetical students who take pleasure in trying to solve difficult questions will find here, as well as in the author's Appendix to Colenso's Arithmetic, ample scope for the exercise of their [illegible] while those who [illegible]

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Scho [illegible] well-known reputation as a mathematician, and as an experienced and skilful teacher.

LONDON: *October* 1866.

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culties in Arithmetic arise from the peculiarities of particular examples, an abundant variety of examples are here worked out, with an amount of explanation sufficient to promote in the pupil a proper aptitude for the solution of similar problems.

Those arithmetical students who take pleasure in trying to solve difficult questions will find here, as well as in the author's Appendix to Colenso's Arithmetic, ample scope for the exercise of their skill; while those who require or desire nothing beyond a sound and familiar acquaintance with ordinary Arithmetic will observe that the more difficult questions, as being last in order, can be easily avoided.

The Table of the Products of prime factors, at the end of the book, is designed to save trouble in seeking the Greatest Common Measure, and may be used in the construction of Rule of Three questions, and in a variety of other ways which mathematicians will appreciate.

The author has not relied solely on himself in the preparation of this work; and it is his duty gratefully to acknowledge that he has been materially aided by the suggestions of Mr. ISBISTER, of the Stationers' School,—a gentleman of well-known reputation as a mathematician, and as an experienced and skilful teacher.

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MODERN ARITHMETIC.

NOTATION AND NUMERATION.

1. ARITHMETIC teaches the science and use of numbers.

2. The ten figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are made to serve for the expression of all numbers, by having *local* values superadded to their *general* values; their general values being, respectively, *nought* or *nothing*, *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*.

Thus, the figure 6, which always signifies *six*, denotes 6 *units* if it stand alone or is the right-hand figure of a whole number, 6 *tens* if it has one figure on the right of it, 6 *hundreds* if it has two figures on the right, 6 *thousands* if it has three; and so forth. In like manner, each of the other figures increases tenfold with every remove to the left; so that, for example, the value of the number 7265 is, 7 *thousands* 2 *hundreds* 6 *tens* and 5 *units*.

The figure 0, often called *zero* or a *cipher*, expresses no value in itself, but is often wanted to mark position in the scale of local values; thus, 60 means six tens, 605 means six hundreds and five units.

3. NOTATION, which in general means a scheme of marks or characters, is, in Arithmetic, the art of representing by figures any number given in words. The Arabic, or common notation, by means of the ten *digits*—as 0, 1, 2, 3, &c. are often called—was brought into Europe by the Arabs, who learned it from the Hindoos.

4. NUMERATION is the art of expressing in words any number given in figures, and is directed by the following Table of local values :—

NUMERATION TABLE.

8	4	2	3	7	5	9	1	4	8	3	5	3	2	6
&c.	Tens of Billions.	Billions.	Hund. Thous. of Mill.	Ten Thous. of Mill.	Thousands of Mill.	Hundreds of Mill.	Tens of Millions.	Millions.	Hundreds of Thous.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.

Numbers are read from left to right. The name *units* is usually suppressed; thus, 3 units is simply called *three*.

The numbers 10, 11, 12, &c. to 19, are not read 1 *ten*, 1 *ten* and 1, 1 *ten* and 2, &c., but *ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen.*

The numbers 20, 30, 40, &c. to 90, are not read 2 *tens*, 3 *tens*, &c., but *twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety.*

Thus: 718 is read Seven hundred and eighteen, 585 is read Five hundred and eighty-five, 3051 is read Three thousand and fifty-one, 12407090 is read Twelve millions four hundred and seven thousand and ninety.

Ex. 1.

Express in figures, and range in columns, units under units, tens under tens, &c., the following numbers:—

(1) Eighty-seven; Seventy-eight; Sixteen; Thirty-three; Fifty.

(2) Four hundred and sixty-one; Three hundred and sixteen; Ninety-nine; Nine hundred and nine; Nineteen.

(3) Three thousand; Six thousand nine hundred and fifteen; Three hundred and sixty-five; Twenty-eight; Two thousand two hundred and eight.

(4) Four thousand and four; Eight hundred and ninety-two; Seventeen; Nine thousand and sixty; Nine hundred and six.

(5) Seven hundred and eleven; One thousand seven hundred and ten; Ninety-six; Six thousand and fourteen; Eight hundred and one.

(6) Fifteen hundred and fifteen; Twelve hundred and two; Nine hundred and fourteen; One thousand four hundred and nine; Six hundred; Six thousand five hundred and eighteen.

(7) Thirty-eight thousand; Three thousand eight hundred and thirteen; Fifty thousand five hundred; Fifteen thousand and fifteen; One thousand and eleven; Ten thousand three hundred and three.

(8) Four hundred and twenty-six thousand five hundred and twenty-seven; Four hundred and six thousand and six; Nine thousand and ninety-nine; Twelve thousand three hundred and forty-five; One hundred and eight; Nine hundred thousand one hundred and forty-four.

(9) Fifty-two millions two hundred and forty-six thousand seven hundred and five; One hundred and ninety-nine thousand and ninety; Four millions nine hundred and ninety thousand two hundred and fifty-eight; Sixty-seven thousand and forty; Seven hundred thousand seven hundred; Nineteen millions sixteen thousand and six.

Ex. 2.

Express in words the following numbers:—

(1)	(2)	(3)	(4)
20	473	2345	28959
41	112	5432	8099
14	91	77	80609
88	619	1770	354654
12	600	144	880
50	505	4014	80800

(5)	(6)	(7)
3829510	809090009	67890000000
382951	30130550	6079504820
303000	12000020	19029019
7017700	3710011064	207710075
60016	111100	8080900
5010000	7700012	10105540868

SIMPLE ADDITION.

5. SIMPLE ADDITION is the process of finding one number expressing the sum of several numbers taken together.

6. The pupil should first learn to add each of the first nine natural numbers to any number mentally at once; so as to be able, for example, to tell readily that 7 and 8 make 15, that 29 and 7 make 36, that 86 and 9 make 95. In such addition as 29 and 7, or 86 and 9, it will be found a help to recollect, that as 9 and 7 make 16, the sum of 29 and 7 must have 6 as its units figure; and that as 6 and 9 make 15, the sum of 86 and 9 must have 5 as its units figure.

Ex. 3.

Add together mentally—

(1) 3, 2, 7, 4, 8, 9, 7. [40]
(2) 6, 5, 8, 9, 9, 9, 8, 8, 6. [68]
(3) 6, 7, 6, 4, 5, 7, 9, 3, 7, 4, 8, 9, 9, 9. [93]
(4) 9, 8, 1, 4, 5, 6, 9, 6, 0, 8, 8, 8, 8, 6, 9, 9. [104]
(5) 7, 7, 9, 8, 8, 7, 4, 5, 8, 6, 9, 9, 8, 8, 9, 8. [120]
(6) 8, 8, 8, 7, 9, 7, 4, 8, 6, 7, 6, 8, 8, 5, 9, 9, 4. [121]
(7) 7, 9, 7, 9, 7, 5, 5, 6, 6, 6, 8, 8, 6, 5, 7, 8, 8, 9. [126]
(8) 9, 2, 3, 4, 5, 6, 7, 8, 4, 8, 8, 9, 8, 7, 6, 5, 4, 9, 9, 9. [130]

7. The sign of Addition is +, named *plus* (a Latin word meaning *more*); when placed before a number it signifies that that number is to be added to a quantity preceding it. Thus, $7+5+3+2$, which is read 7, *plus* 5, *plus* 3, *plus* 2, means 7, and 5 more, and 3 more, and 2 more, making altogether 17.

The sign = signifies equality, and is read *equal to* or *equals*; thus, $7+5+3+2=17$.

8. When numbers given to be added consist, all or for the most part, of two or more figures, range them one under another, so that units shall stand in one column, tens in another, &c. Add first the units column, and resolving its

sum into tens and units, set down the units and carry the tens to the column of tens. When the sum of the tens column is found, treat it as if the column had been one of units, resolve its sum into tens and units, set down the units and carry the tens to the next column; and so forth.

This procedure is founded on the principle of Numeration, which makes every ten *units*=1 *ten*, every ten *tens*=1 *hundred*, every ten *hundreds*=1 *thousand*, &c.

Ex. Add together 4869,768, 8257, 89, 6075, and 906.

	4869
	768
	8257
	89
	6075
	906
Ans.	20964
Proof	8624
	1234
	20964

Here, begin at the foot of the units column, and add upwards: 6 and 5 is 11, and 9 is 20, and 7 is 27, and 8 is 35, and 9 is 44; set down the right-hand or units figure, 4, and carry the other 4 (which is 4 tens) to the column of tens: 4 and 7 is 11, and 8 is 19, and 5 is 24, and 6 is 30, and 6 is 36; set down 6 and carry 3, &c.

To check, or verify, the operation, we may add the given numbers again in a different order. Thus, begin with the left-hand column, and add downwards, placing 8, the right-hand figure of the sum, under the column, and 1, the left-hand figure, in a lower line, one place to the left; the next column being in like manner added downwards, gives 6 to be set under the column, and 2 to be written one place to the left in next line, &c. Lastly, add the two proof lines thus found; and if the sum is the same as before, you may presume that the ordinary addition was done rightly.

Exercises in Simple Addition.

Ex. 4.

(1)	(2)	(3)	(4)	(5)
84	36	39	65	98
95	70	47	44	74
69	85	58	52	37
76	51	66	40	95
87	23	9	50	63
90	6	84	93	83
88	7	8	96	88
64	87	94	39	76

(6)	(7)	(8)	(9)
463	105	9876	6969
674	594	6387	8958
746	367	5968	6397
899	958	8793	5476
423	670	6500	9869
609	349	7879	3456
906	885	5988	7890

(10) Find the sum of 68549 + 98788 + 766 + 7897 + 959 + 87976 + 679 + 9768.

(11) Find the sum of 7096701 + 55009600 + 87895 + 71787 + 968548 + 5635005 + 98765432 + 126990.

(12) Add together 987654321, 16490058, 69751247, 5318642, 989809, 301002502, 6961990, and 7857760.

(13)	(14)	(15)	(16)
99	704	9812	984
69	2734	8627	3840
87	695	7631	997
698	695	6879	697
76	3586	5635	3869
806	4983	6000	7870
59	599	9482	63
895	87	9593	83
948	889	8986	964
400	3142	9317	4137
765	656	4825	4625
650	6879	7036	369

(17)	(18)	(19)
609760	976178	85400369
478695	69537	23456789
969694	549832	98765432
77857	685869	78541728
6532	56905	93728596
107700	210763	86989869
89878	99096	55367903
9999	768689	13579340
798901	695878	68937958
427546	345678	97531080
208003	59497	68696765
87635	694358	56903008
9969	89639	47568905

SIMPLE SUBTRACTION.

9. SIMPLE SUBTRACTION is the process of finding the difference of two numbers, or the remainder that is left when a number is lessened by any number taken from it.

10. The sign of Subtraction is −, named *minus* (a Latin word meaning *less*); when placed before a number, it signifies that that number is to be subtracted from a quantity preceding it. Thus, $8-3=5$, which is read 8 *minus* 3 equals 5, means that 8 less 3, or 8 lessened by 3, leaves 5; also, $8+9-5=12$, means that the sum of 8 and 9 diminished by 5 gives 12.

11. When numbers, given to find their difference, consist of several figures, range them as in Addition, the less number under the greater; then subtract units from units, tens from tens, &c. successively. If the upper units are less than the lower, suppose the former to be increased by ten units before subtracting, and then set down the remainder; but, on account of that increase of the upper number by *ten units*, increase the lower number by the equivalent value, *one ten*, in the next subtraction, and proceed similarly till the whole of the lower number is subtracted.

This procedure is founded on the principle of local values in Numeration, and on the consideration that two numbers when both increased equally must continue to have the same difference.

Ex. When 760532 is subtracted from 4035478, what remains?

	4035478
	760532
Ans.	3274946

Here, beginning with the units, say 2 from 8 leaves 6; 3 from 7 leaves 4; 5 from 4 cannot be taken, but 5 from 14 leaves 9; and then 1 carried to 0 makes 1, which, subtracted from 5, leaves 4; 6 from 3 cannot be taken, but 6 from 13 leaves 7; then 1 carried to 7 makes 8; and so forth.

The answer may be proved by adding it to the line above it, as the sum ought to be the same as the number that was lessened.

Exercises in Simple Subtraction.

Ex. 5.

(1)	5963189 2681432	(2)	49237656 18306876	(3)	8603419590 7604909260
(4)	40327105 18608506	(5)	11109616 4199625	(6)	3009475210 905697113
(7)	216010542 90039562	(8)	385600942 9010969	(9)	101534730 91887093

(10) From 121690371 subtract 9099389.

(11) By how much does 900036104 exceed 3007605?

(12) Find the difference of 7081736 and 200091345.

(13) Subtract Nine hundred and fourteen thousand nine hundred and twenty, from Four hundred millions and thirteen thousand.

(14) If One hundred and ten millions and three hundred be diminished by Sixteen hundred and sixteen, what will be the remainder?

SIMPLE MULTIPLICATION.

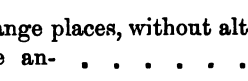
12. Simple Multiplication is a short method of finding what several equal numbers, if added together, would amount to.

This short method is made practicable by committing to memory the results registered in a Table called the Multiplication Table. From that Table, for example, we know at once that $7+7+7+7$, or 4 times 7, is 28. Here, as 7 multiplied by 4 *produces* 28, that amount is called the *product* of 7 and 4, and 7 and 4 are called *factors* of 28.

13. A number that is to be multiplied is called the Multiplicand, and the number of times it is to be repeated is called the Multiplier.

It will readily appear, however, that a simple multipli-

cand and its multiplier may exchange places, without altering the product. Thus, in the annexed representation, we have 4 rows, each containing 7 counters, being in all, 4 times 7; and we have 7 rows, each containing 4 counters, being in all, 7 times 4; hence, 4 times 7=7 times 4.



14. The sign of Multiplication is $\times$; when placed before a number, it signifies that that number is a multiplier of the quantity preceding it. Thus, $7 \times 4 = 28$ is read 7 *multiplied by* 4 equals 28.

15. In the following Table of Products, which should be committed to memory, the second line from the top gives 2 times each of the numbers in the first line; the third gives 3 times each of them; the fourth, 4 times; &c.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

16. When a number given to be multiplied consists of several figures, then, if the multiplier is less than 13, the multiplication can be performed easily in one line, by means of the preceding Table.

Ex. (1) Multiply 72586 by 9.

72586
9
Ans. 653274

Here, having placed the multiplier under the multiplicand, say 9 times 6 is 54, set down 4 and carry 5; 9 times 8 is 72, and 5 carried is 77, set down 7 and carry 7; and so forth.

17. To multiply any number by 10, we need only annex a cipher on the right of the number; for that cipher, then taking the place of units, converts the units of the multiplicand into tens, the tens into hundreds, and so on, thus making a total product ten times as great as the multiplicand. Thus, 10 times 9=9 times 10=9 *tens*=90; 10 times 12=12 times 10=12 *tens*=120; 10 times 365=3650.

18. To multiply by any exact number of tens, as 20, 30, 40, &c. is the same as to multiply successively by 2 and 10, 3 and 10, &c. This may be easily illustrated: for, suppose it were required to multiply 76 by 90, that is, to find the amount of 76 repeated 90 times; we might write down, in the form of an Addition sum, the number 76 ninety times, and then drawing a stroke under every nine lines, we should have 10 groups, each containing 9 times 76, and the whole amount, therefore, would be 9 times 76, taken 10 times, and could be found by first multiplying 76 by 9, which gives 684, and then multiplying 684 by 10, which (by annexing 0) gives 6840.

Hence, to multiply by an exact number of tens, proceed as if the right hand cipher of the multiplier were withdrawn; but annex a cipher on the right of the product. Similarly, to multiply by an exact number of hundreds, thousands, &c., proceed as if the ciphers of the multiplier were withdrawn, but annex them on the right of the product.

Ex. (2)	72586	*Ex.* (3)	72586	*Ex.* (4)	72586
	70		400		3000
	5081020		29034400		217758000

19. When the multiplier is too great to allow the multiplication to be done readily in one line, take partial products by the units figure, the tens figure, &c., in succession, and then add the partial products to obtain the entire product.

Ex. (5) Find the product of 72586 × 3479.

72586
3479

653274	= product by	9,	as in *Ex.*	1.
5081020	= „	70	„	2.
29034400	= „	400	„	3.
217758000	= „	3000	„	4
Ans. 252526694	= „	3479		

Ex. (6) Multiply 20796 by 3095.

20796
3095
103980
187164
62388
Ans. 64363620

Here, according to usual practice, we omit the ciphers that arise from the local values of the digits in the multiplier, and multiply by 5, 9, 3, as if each figure denoted simple units; only taking care, in multiplying the 6 units of the multiplicand, to begin setting down the result directly under the multiplying figure.

The correctness of the results, in Simple Multiplication, may be proved by making the multiplicand and multiplier exchange places, and calculating the product anew.*

* The method of Proof called *Casting out the Nines* is only a presumptive, not an infallible test of accuracy; but it is very convenient, and may in general be trusted. It consists in adding first the digits of the multiplicand, rejecting the value 9 as often as it arises in the addition, and writing the final remainder a little to the right of the multiplicand; then adding the digits of the multiplier, and reserving its final remainder in the same way; then multiplying together these remainders, and casting out 9 from their product, thus obtaining a third remainder, which, if the sum has

Exercises in Simple Multiplication.

Ex. 6.

(1) Multiply 6389470285 by 2, 3, 5, 8, 10, 12, separately.
(2) „ 4099750683 „ 6, 7, 4, 9, 11, 12 „
(3) „ 617493825 „ 40, 60, 900 „
(4) „ 617493825 „ 15, 17, 18 „
(5) „ 709538642 „ 26, 39, 58 „
(6) „ 6859900 „ 810, 9600, 47000 „

(7) 47259638 × 365
(8) 56937488 × 248
(9) 38594076 × 706
(10) 41369570 × 912
(11) 49009856 × 117
(12) 11985275 × 128
(13) 57169483 × 4586
(14) 97946200 × 7932
(15) 29608174 × 5678
(16) 19508469 × 8109
(17) 73850940 × 1320011
(18) 64829000 × 2873500

SIMPLE DIVISION.

20. Simple Division is a short method of finding how often one number is contained in, or can be subtracted from, another. The containing number is called the Dividend, that which it contains is called the Divisor, and the number of times the Divisor is contained in the Dividend is called the Quotient.

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been rightly worked, will be the same as the remainder left when the nines have been cast out from the sum of the digits of the product that is tested.

53674 . . 7
128 . . 2
429392 5
644088
6870272 . . 5

Applying this test to the annexed sum, say, 5 and 3 are 8, and 6 are 14; 5 and 7 are 12; 3 and 4 are 7, remainder; then, 1 and 2 are 3, and 8 are 11, giving 2 remainder; $7 \times 2 = 14$; 1 and 4 are 5, remainder; lastly, 6 and 8 are 14; 5 and 7 are 12; 3 and 2 are 5, remainder, as required.—*See* the Author's Notes to Colenso's Arithmetic.

Thus we can tell at once that 9 is contained in, or can be taken from, 49 *five* times, with a remainder 4, if we remember that 5 times 9 is 45, and that 6 times 9 is more than 49.

21. The sign of Division is $\div$; when placed before a number it signifies that that number is a divisor of the quantity preceding it. Thus $56 \div 8 = 7$ is read 56 *divided by* 8 equals 7.

22. When a given divisor is less than 13, the division can be performed easily in one line.

Ex. (1) Divide 653274 by 9.

9)653274
Ans. 72586

Here we are to employ a process the reverse of that used in the 1st Ex. of Multiplication. Placing the divisor on the left of the dividend, with a bracket curve between them, say 9 in 65 is 7 times, leaving 2, which followed by the next figure 3 makes 23; then 9 in 23 is 2 times, leaving 5, which, followed by 2, makes 52; and 9 in 52 is 5 times, leaving 7; similarly, 9 in 77 is 8 times, leaving 5, and in 54 is 6 times exactly. The quotient, therefore, or the number of times that 9 is contained in, or can be taken from, 653274, is 72586.

It should be observed in the above process that 9 is understood as being contained, first, 7 ten-thousands of times in 65 ten-thousands, secondly, 2 thousands of times in 23 thousand, then, 5 hundreds of times in 52 hundred, &c.

23. To divide by 10, we may simply cut off the right hand figure of the dividend; for hereby the other figures, being each reduced to a tenth of their original value, will be the required quotient, and the figure cut off will be the remainder after division. Thus, $3650 \div 10 = 365$; and $4297 \div 10 = 429$ with a remainder 7. Similarly, to divide any number by 100, 1000, &c., cut off two, three, &c., figures from the right. Thus, $26350 \div 1000 = 26$, with a remainder 350.

To divide by any exact number of tens, hundreds, &c., neglect the ciphers on the right of the divisor, and cut off from the right of the dividend as many figures as the

neglected ciphers; the other figures of the dividend being now divided, will give the required quotient; and if any quantity remains, it will form the first figure or figures of the entire remainder, the rest of that remainder consisting of the figures cut off from the dividend.

Ex. (2) Divide 1096723 by 1200.

1200)10967,23

Ans. 913 *times*; 1123 *rem.*

Here we cut off two figures from the right of the dividend, because we neglect the two ciphers on the right of the divisor and divide only by 12; the remainder 11, prefixed to the figures 23, gives 1123, the full remainder.

24. The form of the preceding examples of Division is that called Short Division. When the divisor is higher than 12, we employ what is called Long Division, as in the two following Examples.

Ex. (3) Find the quotient of 252526694 ÷ 72586.

Ex. (4) „ „ 64363630 ÷ 20796.

72586)252526694(3479
217758
347686
290344
573429
508102
653274
653274

Ans.

20796)64363630(3095
62388
197563
187164
103990
103980

Ans. 3095 times; and 10 *rem.*

In the above Examples is shown the usual practice of placing the dividend between inverted curves of a bracket, with the divisor on the left, and the quotient on the right. In the first of these Examples it is seen that the first six figures of the dividend are the fewest that can be taken to yield a quotient figure, viz. 3; then 3 times the divisor being placed under these six figures, and subtracted from them, we annex to the remainder the seventh figure of the dividend to make the next partial dividend, which is found to contain the divisor 4 times; and so forth. In the next

Example, the first remainder wth the next figure of the dividend annexed to it does not contain the divisor so much as once, and, accordingly, placing 0 in the quotient, we annex another figure of the dividend, and now obtain 9 as a quotient figure; &c.

Note.—When a divisor is large, as in *Ex.* 3 and 4, learners generally find it troublesome to guess rightly the quotient figures. It has been proposed to obviate this difficulty by first writing under the divisor the successive products of it by all the digits from 2 to 9, thus enabling the pupil to perceive at a glance the quotient figure wanted for each partial dividend. It is probably expedient to adopt this plan in a few examples, when the pupil is just entering upon division sums having large divisors; but the sooner he can conveniently dispense with it the better. He should as soon as possible learn to guess readily how often the first two figures of the divisor are contained in the first two or three of the dividend, which will generally give the proper quotient figure.

The accuracy of the result in Simple Division may be tested by multiplying together the divisor and quotient, and then adding the remainder, if any, which should reproduce the dividend.

EXERCISES IN SIMPLE DIVISION.

Ex. 7.

(1) Divide 29653848 and 115960343, each by 2, 3, 5, 8, 10, 12.
(2) „ 101746, 227713, 850915, each by 11, and also each by 12.
(3) „ 23471903007 by 7, 8, 9, 12, separately.
(4) „ 6103950 by 10, 1000, 1200.
(5) „ 107579900 by 1100, 7000, 120.
(6) „ 77047724 „ 13, 17, 19.
(7) „ 11866806 „ 14, 15, 18.
(8) „ 16696576 „ 23, 29, 37.
(9) „ 9892656 „ 35, 48, 63.
(10) „ 9892741 „ 73, 86, 97.
(11) „ 50258880 „ 56, 54, 960.
(12) „ 6972020 „ 670, 8300.

(13) 100708610 ÷ 365.
(14) 1515899203 ÷ 497.
(15) 565157502 ÷ 819.
(16) 126448601 ÷ 936.
(17) 6083002 ÷ 2379.
(18) 712239990 ÷ 7854.
(19) 1234567 ÷ 3806.
(20) 64108528 ÷ 29584.

SUPPLEMENT

TO

SIMPLE MULTIPLICATION AND DIVISION.

25. In multiplying by 13, 14, &c., to 19, we might at once place under the given multiplicand the product by the right hand figure of the multiplier, observing to set the units figure of that product one place to the right; and then add. Similarly, in multiplying by 21, 31, 41, &c., we might at once place under the multiplicand the product by the left hand figure of the multiplier, observing to set the units figure one place to the left.

Ex. (1) 473256 × 17
3312792
8045352

Ex. (2) 508469 × 51
2542345
25931919

26. In multiplying by a composite number, viz. by a whole number that is the product of two whole numbers each greater than 1, we might multiply successively by the factors.

Thus, in the two examples following, we take $54=6\times 9$, and $64=8\times 8$.

Ex. (3) 784596 × 54
6
4707576
9
42368184

Ex. (4) 820657 × 64
8
6565256
8
52522048

Note.—This method of resolving a composite multiplier is generally of no utility in *Simple* Multiplication; but it is often expedient in the Compound Rule.

27. In dividing by a composite number, we might divide successively by the factors. But when, in using this method, a remainder occurs, multiply the last remainder by the first divisor, and to the product add the first remainder, if any, to get the entire remainder.

Ex. (5) Divide 431027 by 42, or by 7 × 6.

By the Rule.	Explanatory Form.
7)431027	7 *units*)431027 *units*
6) 61575 . . 2	6 *sevens*) 61575 *sevens*, and 2 *units* or 2 rem.
Ans. 10262, rem. 23	10262 times, and 3 *sevens* or 21 rem.
	Full rem. 23

Ex. 8.

(1) Multiply 8526093 by 15, 17, 19.
(2) „ 2815997 „ 41, 61, 71.
(3) „ 3183756 „ 48, 42, 72.
(4) Divide 3011575 „ 54, 64, 49.

28. It should be observed that, in Simple Division, any remainder left after dividing is really meant to be divided also by the divisor; and that, in order to make the quotient complete, no portion of the dividend should remain undivided. It is usual, therefore, to *signify* the division of a remainder by writing the divisor under it, with a line between them, and to annex this *fraction*, as it is called, to the units figure of the quotient.

Ex. (6) How often is 12 contained in 375103?

12)375103

Ans. 31258 $\frac{7}{12}$ *times.*

Here, if the dividend had been 375096, which is 7 less than the one given, we should have found 12 to be contained in it exactly 31258 times; so that that quotient ought to be increased by the number of times that 12 is contained in 7. Now 12 is only partly, or fractionally, contained in 7; and to represent the quotient of 7 ÷ 12, it is customary to use the form $\frac{7}{12}$, which is read 7 *twelfths*, and which here signifies the 12*th* part of 7 *times*, or 7 *twelfths* of *once*.

29. We have defined Simple Division to be a method of finding how often one number is contained in another.

This, it should be observed, does not imply an alteration of the value of the dividend. If it be asked, how many *twelves* are contained in 36, the quotient 3 signifies 3 times 12. But if we ask what is the *twelfth part* of 36, we propose an alteration of the value of the dividend, and though in this case the required operation is the same as before, viz. $36 \div 12 = 3$, the result 3 is not properly a *quotient*, that is, it is not here meant to express *how often* 12 is contained in 36, but it denotes that the quantity 3 is contained 12 times in 36, so that 12 is in strictness a *quotient given* to find what *divisor* would produce a given dividend.

Accordingly, if Ex. (6) had been, to find what quantity is contained 12 times in 375103, the strict process would have been, not 12 in 37 is contained 3 times, &c., but, the 12th part of 37 is 3, &c., and the fraction would have denoted the 12th part of 7, or 7 *twelfths* of 1.

30. We have shown by means of Ex. (5) that, when a divisor is a composite number, we may use its factors as successive divisors. This has anticipated the useful consideration that, when a dividend and its divisor (or the upper and lower numbers in a fractional form of expression) contain a common factor, that factor may be struck out, or cancelled, from both.

Compare the subjoined modes of dividing 5976 by 72.

9 *units*)5976 *units*
8 *nines*) 664 *nines*
83

Or, $\frac{5976}{72} = \frac{664 \text{ nines}}{8 \text{ nines}} = \frac{664}{8} = 83$

Or, cancelling in the usual form, $\frac{\overset{664}{\cancel{5976}}}{\underset{8}{\cancel{72}}} = 83$

Compare also the subjoined modes of dividing 4571 by 56.

8 *units*)4571 *units*
7 *eights*) 571 *eights*, and 3 *units* or 3 rem.
81 times, and 4 *eights* or 32 rem.

Ans. $81\frac{4}{8}$.

35 i. e. $\frac{35}{56} = \frac{5 \text{ sevens}}{8 \text{ sevens}} = \frac{5}{8}$

Or, 7 *units*)4571 *units.*
8 *sevens*) 653 *sevens.*
$81\frac{5}{8}$

Or, $\frac{4571}{56} = \frac{653 \text{ sevens}}{8 \text{ sevens}} = \frac{653}{8} = 81\frac{5}{8}$.

31. A dividend, or a divisor, or both, will often be found consisting of several given factors, to which cancelling may be applied. Thus, suppose the dividend to be $284 \times 84 \times 45$, and the divisor 240×455; adopting a usual form of division, we have—

$$\frac{284 \times 84 \times 45}{240 \times 455} = \frac{284 \times 7 \times 9}{20 \times 91} = \frac{71 \times 9}{5 \times 13} = \frac{639}{65} = 9\tfrac{54}{65}.$$

where we have first cancelled 12 and 5, each from both dividend and divisor, then 4 and 7; but the usual brief form of cancelling is as follows:—

$$\frac{\overset{71}{\cancel{284}} \times \overset{1}{\cancel{84}}\overset{7}{} \times \overset{9}{\cancel{45}}}{\underset{20}{\cancel{240}} \times \underset{91}{\cancel{455}}\underset{13}{} \underset{5}{}} = \frac{71 \times 9}{5 \times 13} = \frac{639}{65} = 9\tfrac{54}{65}.$$

The operation, without preliminary cancelling, would have given $1073520 \div 109200 = 9\frac{207}{1092}\frac{2}{0} = 9\frac{11}{13}\frac{34}{65} = 9\frac{378}{455} = 9\frac{54}{65}$.

Ex. 9.

Calculate the following expressions:—

(1) $\frac{63 \times 18}{28}$. (2) $\frac{315}{25 \times 49}$. (3) $\frac{132 \times 164}{154 \times 246}$ 4) $\frac{296 \times 385}{16 \times 15 \times 21}$.

(5) $\frac{75 \times 76 \times 280}{35 \times 12}$. (6) $\frac{76 \times 342 \times 50}{57 \times 57 \times 18 \times 45}$. (7) $\frac{637 \times 522 \times 58.}{26 \times 147}$

TABLES OF CONCRETE QUANTITIES.

32. NUMBERS are either *Abstract* or *Concrete*. They are called concrete when applied to the numeration of particular objects, as when we say 3 *trees*, 15 *shillings*, 12 *miles*: they are called abstract when used generally, as so many units, without any special denomination for the unit.

ities of a given denomination are often converted into higher or lower denominations of the same ; a number of miles may be converted into an number of yards.

34. In the transactions of commerce and science the following Tables of Equivalents are used. They should be committed to memory.

TABLE OF MONEY.

4 Farthings	=	1 Penny.
12 Pence	=	1 Shilling.
20 Shillings	=	1 Pound.

We denote Pounds, Shillings, and Pence, by the letters, £, *s*, *d*, respectively. Farthings are commonly expressed fractionally, in the denomination of Pence; thus $\frac{1}{4}d.$, meaning 1 fourth part of a penny, is put for 1 farthing; $\frac{1}{2}d.$, meaning 1 second part, or half, of a penny, for 2 farthings; $\frac{3}{4}d.$, meaning 3 fourth parts of a penny, for 3 farthings.

The above Table comprehends all our Moneys of Account. But we use various coins whose names do not occur in bills or accounts. The following are the coins now in general circulation in England.

Copper Coins.—A Farthing, the coin of lowest value; a Halfpenny = 2 Farthings; a Penny = 4 Farthings, or 2 Halfpennies.

Silver Coins.—A Threepenny Piece = 3 Pence; a Fourpenny Piece = 4 Pence; a Sixpence = 6 Pence; a Shilling = 12 Pence, or 2 Sixpences; a Florin = 2 Shillings; a Half-crown = 2 Shillings and 6 Pence; a Crown = 5 Shillings.

Gold Coins.—A Half-Sovereign = 10 Shillings; a Sovereign = 20 Shillings.

Formerly we had silver coins called a Groat and a Tester, or Testern, worth 4 pence and 6 pence, respectively. We had also the following gold coins;—a Noble=6*s.* 8*d.*, an Angel=10*s.*, a Half-Guinea=10*s.* 6*d.*, a Mark=13*s.* 4*d.*, a Guinea=21*s.*, a Moidore=27*s.*

The gold coins of this country are not made of pure

gold; in every 24 parts of the coinage or *standard* gold, 22 parts are pure gold and 2 parts are an alloy of copper.

In every 40 parts of *standard* silver, 37 parts are pure silver, and 3 parts are an alloy of copper.

The gold coinage is our standard of value, and the copper coinage is not *legal tender* for more than 12 pence, nor the silver coinage for more than 40 shillings.

TABLE OF TROY WEIGHT.

24 Grains	=	1 Pennyweight.
20 Pennyweights	=	1 Ounce.
12 Ounces	=	1 Pound.

This Table is used for weighing gold and silver, and other things of a costly or delicate nature. We write *grs.* for grains, *dwt.* for pennyweights, *oz.* for ounces, and *lb.* for pounds.

When gold is said to be so many *carats* fine, the meaning is that so many parts in every 24 parts of the metal are pure gold. Thus, standard gold is 22 carats fine; jewellers' gold is 18 carats fine.

From *a pound* Troy of standard gold are coined $46\frac{2}{10}$ sovereigns, value £46 14*s.* 6*d.*; hence, the Mint price of gold *per ounce* is the 12th part of £46 14*s.* 6*d.* = £3 17*s.* $10\frac{1}{2}$*d.*

From *a pound* Troy of standard silver are coined 66 shillings; hence the mint price of silver *per ounce* is the 12th part of 66 shillings = 5*s.* 6*d.*

TABLE OF AVOIRDUPOIS WEIGHT.

12 Drams	=	1 Ounce.
16 Ounces	=	1 Pound.
28 Pounds	=	1 Quarter.
4 Quarters	=	1 Hundredweight.
20 Hundredweights	=	1 Ton.

This Table is used for weighing tea, sugar, cheese, meat, and other things of a less costly nature than gold and silver. We write *drs.* for drams, *oz.* for ounces, *lb.* for pounds, *qrs.* for quarters, *cwt.* for hundredweights.

The cwt. is=112lb., and is often divided into eight equal parts, each 14lb., called *stones*.

1lb. Troy=5760 grains Troy; 1lb. Avoird.=7000 grains Troy.

TABLE OF APOTHECARIES' WEIGHT.

20 Grains Troy	=	1 Scruple.
3 Scruples	=	1 Dram.
8 Drams	=	1 Ounce Troy.
12 Ounces	=	1 Pound Troy.

TABLE OF LINEAL MEASURE.

3 Barley-corns	=	1 Inch.
12 Inches	=	1 Foot.
3 Feet (or 36 inches)	=	1 Yard.
5½ Yards	=	1 Rod or Pole.
40 Poles (or 220 yards)	=	1 Furlong.
8 Furlongs (or 1760 yards)	=	1 Mile.

This Table is used for measuring distance, length, breadth, height, depth and thickness. The abbreviations, *in.* for inches, *yds.* for yards, &c., are similar to those already mentioned.

The Chain used in measuring land is 22 yards long, and is divided into 100 links. The Hand, in measuring horses, is 4 inches. A Span is 9 inches, and a Cubit 18 inches. A League = 3 miles. A Degree of Latitude=60 geographical miles=69 $\frac{1}{10}$ English miles.

CLOTH MEASURE.

2¼ Inches	=	1 Nail.
4 Nails	=	1 Quarter.
4 Quarters	=	1 Yard.
5 Quarters (of a yd.)	=	1 English ell.

TABLE OF SURFACE OR SQUARE MEASURE.

144 Square Inches	=	1 Square Foot.
9 Square Feet	=	1 Square Yard.
30¼ Square Yards	=	1 Square Pole or Rod.
40 Square Poles	=	1 Rood of Land.
4 Roods	=	1 Acre of Land.

This Table is sometimes called that of Land Measure; but it is used in measuring all kinds of surfaces whose length and breadth can be determined. A square inch is equal to a square each of whose sides is 1 lineal inch; similar explanation applies to a square foot, &c.

Ten Square Chains, or 100000 Square Links make an Acre; also, 4840 Square Yards make an Acre; and 640 Acres are a Square Mile.

A Rod of Brickwork is 9 sq. ft. $\times 30\frac{1}{4} = 272\frac{1}{4}$ Square Feet, of the standard thickness of a brick-and-a-half.

TABLE OF SOLID OR CUBIC MEASURE.

1728 Cubic Inches	=	1 Cubic Foot.
27 Cubic Feet	=	1 Cubic Yard.

This Table is used in measuring volume or solidity, and applies to all kinds of volumes or bulks whose length, breadth, and depth or thickness, can be determined. A cubic inch is equal to a cube each of whose three dimensions is 1 lineal inch, or each of whose six faces is 1 square inch.

TABLE OF LIQUID MEASURE.

4 Gills	=	1 Pint.
2 Pints	=	1 Quart.
4 Quarts	=	1 Gallon.
63 Gallons, Wine, 54 Gallons, Beer,	=	1 Hogshead.
2 Hogsheads	=	1 Butt or Pipe.
2 Butts or Pipes	=	1 Tun.

TABLE OF DRY MEASURE.

2 Quarts	=	1 Pottle.
4 Quarts	=	1 Gallon.
2 Gallons	=	1 Peck.
4 Pecks	=	1 Bushel.
8 Bushels	=	1 Quarter.

3 Bushels of Coals = 1 Sack; 12 Sacks of Coals = 1 Chaldron.

TABLE OF TIME.

60	Seconds	= 1 Minute.
60	Minutes	= 1 Hour.
24	Hours	= 1 Day.
7	Days	= 1 Week.
365	Days	= 1 Common Year.
366	Days	= 1 Leap Year.
365¼	Days	= 1 Average Year.

In business, a Month is often regarded as the 12th part of a year; but the 12 Calendar Months vary as follows, save that in Leap Year February has 29 days.

January	31 days.	July	31 days.
February	28 „	August	31 „
March	31 „	September	30 „
April	30 „	October	31 „
May	31 „	November	30 „
June	30 „	December	31 „

Leap Year occurs once in every four years, the common year of 365 days being allowed until the extra quarter of a day in the average year shall amount to a whole day. If the number of a Christian year be exactly divisible by 4, it denotes a leap year: thus, 1864, being exactly divisible by 4, was a leap year, and 1868 is the next leap year following.

The average year of 365 days 6 hours (called, from Julius Cæsar, the Julian year) exceeds the true solar year by a small quantity, which would amount to about 3 days in 400 years, and which is allowed for by omitting the leap-year day in 3 out of every 4 centurial years; viz. in those centurial years of which the number of the century is not exactly divisible by 4, as in the years 1700, 1800, 1900, 2100, 2200, &c.; while, for the amount of error which had arisen up to the year 1582, Pope Gregory caused 10 days to be omitted from that year, thus introducing what is called the Gregorian or New Style, the Julian computation being called the Old Style. Old Christ-

mas now falls 12 days later than New Christmas. The Old Style, however, is still used in Russia.

TABLE OF CIRCULAR MEASURE.

60 Seconds = 1 Minute.
60 Minutes = 1 Degree.
360 Degrees = 1 Circumference.

1 degree is written, 1°; 1 minute, 1′; 1 second, 1″.

REDUCTION.

35. REDUCTION is the process of changing a number of one denomination into an equivalent number of another denomination.

Thus, to convert 120 shillings to £, since 20 shillings = £1, and is contained 6 times in 120 shillings, therefore 120*s.* = £6.

Thus, also, to convert 120 shillings to pence, since 1 shilling = 12 pence, therefore 120 shillings = 120 times 12*d.* = 120*d.* × 12 = 1440*d.*

To reduce from a lower denomination to a higher, requires division; to reduce from a higher denomination to a lower, requires multiplication.

Ex. (1) Reduce 3256 pence to £; 74 fourpences to shillings; and £29 13*s.* 10½*d.* to farthings.

12)3256*d.*	£29 13*s.* 10½*d.*
20) 27,1*s.* 4*d.*	20
Ans. £13 11*s.* 4*d.*	593*s.*
	12
	7126*d.*
3) 74*fo.*	4
Ans. 24*s.* 8*d.*	*Ans.* 28506*f.*

In the first of these conversions we change the given pence into the denomination next higher, viz. shillings, by dividing *by 12d.*; then the shillings into £, by dividing *by 20s.*; *in the* second, since every 3 fourpences make 1

shilling, we divide by 3, and find a remainder of 2 fourpences, viz. 8*d.*; in the third, we have £29=29 times 20*s.*, which, with the given 13*s.*, makes 593*s.*, and these shillings are=593 times 12*d.*, which, with the given 10*d.*, makes 7126*d.*, &c.

Ex. (2) Reduce 7797 oz. to cwt.; and 4 cwt. 1 qr. 11 lb. 5 oz. to oz.

By comparing the reductions of these two quantities, it will be observed that the quantities are equivalent, and that sums in Reduction may be verified by reducing the Answer to the denomination given. We resolve the divisors 16 and 28 into factors.

16 { 4)7797 oz.
4)1949 . . . 1 } 5 oz.
28 { 7) 487 lb. 1
4) 69 . . . 4 } 11 lb.
4) 17 qrs. 1

Ans. 4 cwt. 1 qr. 11 lb. 5 oz.

4 cwt. 1 qr. 11 lb. 5 oz.
4
17 qrs.
28
487 lb.
16
7797 oz. *Ans.*

Ex. (3) Reduce 42 Eng. ells 3 qrs. to yards.

42 ells 3 qrs.
5
4)213 qrs.
Ans. 53 yds. 1 qr.

Here we first reduce the ells to quarters of a yard by multiplying by 5, then the quarters to yards by dividing by 4.

Ex. 10.

Reduce

(1) 1560*s.*, 4739*s.*, 10029*s.*, each to £.
(2) 1560*d.*, 4739*d.*, 10029*d.*, each to £.
(3) 73280 *farth.*, and 67843 *farth.*, each to £.
(4) 10902 *farth.*, and 11869 *farth.*, each to £.
(5) 315 *hf. d.*, 430 *hf. d.*, 663 *hf. d.*, each to shillings.
(6) £76 15*s.* 7*d.*, £3 10*s.* 11*d.*, 19*s.* 5*d.*, each to pence.
(7) 12*s.* 10¾*d.*, 37*s.* 9½*d.*, £9 10*s.* 1*d.*, each to farth.
(8) £359 13*s.* 6¼*d.*, and £3000 10*s.*, each to farth.
(9) £89 18 11½*d.*, and £57 0*s.* 5*d.*, each to hf. pence.
(10) 9753 *hf. d.*, and 176401 *hf. d.*, each to £.
(11) 450 *guineas*, and 12*s.* 10½*d.*, each to pence.
(12) 4557*s.*, and 9828*d.*, each to guineas.

(13) £26 16*s.*, and £78 13*s.* 8*d.*, each to fourpences.

(14) 169*s.*, £85, and 1352 *farth.*, each to threepences.

(15) £56 8*s.* 6*d.*, and 1753 *farth.*, each to sixpences.

(16) 15330 *sixp.*, and 25767 *fourp.*, each to guineas.

(17) 2739 *hf. sovs.*, and 519 *hf. crs.*, each to pence.

(18) 5942 *fourp.* and 1009 *threep.*, each to crowns.

(19) 31175 *farth.*, £39 7*s.* 6*d.*, and £46 10*s.*, each to hf. crowns.

(20) 1789*d.*, 742 *threep.*, and £311 14*s.*, each to florins.

(21) 62252 *grs.*, and 19696 *grs.*, each to lb. Troy.

(22) 25 *lb.* 10 *oz.* 17 *dwt.*, and 11 *lb.* 12 *dwt.* 21 *grs.*, each to grains.

(23) 11 *oz.* 20 *grs.*, and 213 *lb.* 19 *grs.*, and 14 *lb.* 14 *dwt.* each to grains.

(24) 5375 *dwt.*, and 1364 *hf. oz.* Troy, and 4579 *dwt.*, each to Troy lb.

(25) 63 *cwt.* 3 *qrs.* 25 *lb.*, and 14 *t.* 14 *cwt.*, and 13 *cwt.* 8 *oz.*, each to lb.

(26) 1974 *lb. Av.*, and 65020 *oz. Av.*, each to cwt.

(27) 49 *cwt.* 2 *qrs.*, and 8 *cwt.* 1 *qr.* 27 *lb.* 7 *oz.*, and 3 *qrs.* 13 *lb.*, each to oz.

(28) 3 *t.* 13 *cwt.* 13 *oz.*, and 2 *t.* 7 *cwt.* 3 *qrs.* 15 *lb.*, each to oz.

(29) 234567 *oz. Av.*, and 34567 *lb. Av.*, each to tons.

(30) 510713 *drs.*, and 407365 *oz. Av.*, each to cwt.

(31) 47 *yds.* 2 *ft.* 10 *in*, and 6 *fur.* 106 *yds.* 1 *ft.*, each to inches.

(32) 3 *mi.* 5 *fur.* 62 *yds.*, and 23 *mi.* 323 *yds.* 2 *ft.* each to feet.

(33) 1040638 *yds.*, and 7635000 *in*, each to miles.

(34) 3265 *sq. yds.* 7 *ft.*, and 750 *cub. yds.* 20 *ft.* 100 *in.*, each to inches.

(35) 14 *ac.* 3 *ro.* 31 *po.*, and 263 *ac.* 2 *ro.*, and 7 *ac.* 17¼ *po.*, each to poles.

(36) 130708 *sq. po.*, and 275 *sq. miles*, each to acres.

(37) 478 *gals.*, and 47 *gals.* 3 *qts.*, each to pints.

(38) 6 *qrs.* 5 *bu.*, and 112 *pks.*, each to bushels.

(39) 49 *wks.* 6 *da.* 19 *hrs.*, and 365 *da.* 6 *hrs.*, each to minutes.

(40) 315 *yds.*, and 343 *yds.* 3 *qrs.*, each to English ells.

(41) 168 *Eng. ells*, and 209 *Eng. ells* 4 *qrs.*, each to yards.

(42) £431, £37 16*s.*, and £275 10*s.*, each to guineas.

(43) 57 *guin.*, and 180 *guin.*, each to £.

(44) 237 *lb.* Troy, and 1050 *oz.* Troy, each to lb. Av.

(45) 432 *lb.* Av., and 1175 *oz.* Av., each to lb. Troy.

COMPOUND ADDITION.

36. No quantities but such as are of like kind can be added together; but quantities of like kind may consist of various denominations. Thus, *cwt.*, *qr.*, *lb.*, are of like kind, but are various denominations.

COMPOUND ADDITION is the process of adding concrete quantities that are of like kind, but of various denominations.

Ex. (1) Add together £26 12*s.* 5¼*d.*, £75 9*s.* 6¼*d.*, 19*s.* 10½*d.*, £364 17*s.* 8½*d.*, and £86 7*s.* 5¾*d.*

	£	*s.*	*d.*
	26	12	5¼
	75	9	6¼
	0	19	10½
	364	17	8½
	86	7	5¾
Ans.	£554	7	0¼

Here, having arranged the given quantities in respective columns of £ *s.* *d.*, we add first the farthings, and reducing their sum, 9 farthings, to pence, by dividing it by 4, we get 2*d.* and ¼*d.* over; setting down, therefore, ¼*d.* we carry 2*d.* to the column of pence, which being added up amounts to 36*d.*, and this sum being reduced to shillings gives 3*s.* with no pence remaining; therefore, writing 0 under the column of pence, we carry 3*s.* to the shillings column, the sum of which is found to be 67*s.*; and this ÷ 20 gives £3 7*s.*; accordingly, we set down 7*s.* and carry £3 to the column of £, &c.

Note.—The most convenient method of summing a double column of shillings is, first, to add its units column, which, in the above example, amounts to 37, and to write down at once the units of that sum, 7; then, carrying the tens, 3, to the tens column, to divide the sum of that column by 2, which in this example gives 3 to be carried to the column of £; if there had been a remainder of 1, it would have been placed before the 7, making 17*s.*

Ex. (2) Add together 57 yds. 2 ft. 9 in., 37 yds. 2 ft. 11 in., 19 ft. 5 in., 5 ft. 7 in., and 23 yds. 10 in.

	yds.	ft.	in.
	57	2	9
	37	2	11
	6	1	5
	1	2	7
	23	0	10
Ans.	127	1	6

Here we have arranged the given quantities in respective columns of *yds.* *ft.* *in.*, resolving the given 19 ft. of the third quantity into its equivalent 6 yds. 1 ft., and the 5 feet of the fourth quantity into 1 yd. 2 ft.; the addition is then performed as in *Ex.* (1), 12 in. being used as a reducing divisor for the column of inches, and 3 ft. for that of feet, because *12 in. = 1 ft., and 3 ft. = 1 yd.*

Ex. 11.

	£	s.	d.
(1)	5	12	10
	6	16	6
	3	9	8
	5	13	2
	4	8	11
	6	13	7

	£	s.	d.
(2)	35	6	7
	24	13	11
	49	10	8
	36	5	11
	27	17	3
	96	1	6

	£	s.	d.
(3)	72	18	11½
	57	15	2¾
	68	12	7¾
	58	9	8
	85	13	9½
	16	12	5¼

	£	s.	d.
(4)	69	3	10
	72	16	10¾
	88	9	3½
	58	19	4½
	94	19	11
	20	4	9¼

	£	s.	d.
(5)	74	13	8¼
	60	9	10¼
	88	10	10½
	49	15	7
	56	6	3¾
	97	0	0
	8	18	9½
	5	0	5

	£	s.	d.
(6)	90	14	10
	57	16	9½
	9	15	7
	89	3	10¼
	87	6	4¼
	70	10	9½
	6	8	5
	16	7	2

	£	s.	d.
(7)	63	17	6
	73	18	6
	59	3	0
	95	11	7
	9	9	6
	7	4	11
	15	15	6
	9	18	10

	£	s.	d.
(8)	39	12	10½
	47	5	6
	38	6	9¾
	54	10	10½
	87	13	5½
	69	7	3½
	95	9	8¾
	67	14	7¾

	£	s.	d.
(9)	9	17	11½
	64	14	6
	78	1	10
	7	11	8
	76	7	5¾
	10	12	6
	59	9	10½
	5	17	6
	43	16	7

	£	s.	d.
(10)	46	3	7
	49	12	6¼
	73	19	11¾
	8	7	4¾
	7	8	10¾
	84	10	9½
	5	17	6
	66	0	3¾
	98	6	8½

	£	s.	d.
(11)	72	11	10¾
	50	8	8
	48	5	9½
	99	19	11¾
	84	6	6¾
	9	12	8
	9	10	6¼
	75	1	7½
	76	10	7½

	£	s.	d.
(12)	99	0	7
	46	17	6½
	68	4	11¾
	90	5	8
	77	11	11½
	69	7	1¼
	70	0	10
	85	0	3¼
	96	13	11

	£	s.	d.
(13)	2567	16	11¾
	689	13	8½
	758	9	7
	1695	10	6¾
	947	8	10½
	76	15	9
	365	17	3¾
	6068	4	5
	899	19	6¾
	1752	18	7½

	£	s.	d.
(14)	6096	8	2
	1587	13	11¾
	695	3	6
	2345	6	7
	876	12	10½
	758	9	9
	567	8	9
	1728	17	5¾
	69	8	10
	84	8	4

	£	s.	d.
(15)	989	18	6
	698	12	9
	765	15	10
	996	17	5
	577	9	8
	898	9	7
	466	14	6
	789	6	11
	974	12	3
	703	1	7

	cwt.	qrs.	lb.
(16)	26	3	23
	30	3	16
	59	2	25
	19	0	18
	42	3	20

	lb.	oz.	drs.
(17)	25	12	0
	78	15	13
	67	8	10
	9	9	14
	19	13	7

	t.	cwt.	qrs.	lb.	oz.
(18)	23	18	1	12	10
	63	7	3	6	15
	8	19	3	27	11
	70	17	2	18	8
	25	8	1	22	7

	lb.	oz.	dwt.
(19)	36	11	14
	29	3	17
	18	10	19
	7	7	6
	47	9	12

	yds.	ft.	in.
(20)	63	2	10
	43	1	8
	59	2	7
	15	0	11
	25	2	6

	mi.	fur.	yds.	ft.	in.
(21)	46	3	136	2	7
	9	7	201	2	0
	91	5	11	1	2
	8	0	174	0	1
	86	2	36	1	1

	ac.	ro.	po.
(22)	32	3	36
	59	2	26
	17	3	9
	48	2	17
	50	1	20

	sq. yds.	ft.	in.
(23)	93	2	122
	49	8	67
	38	4	60
	55	7	59
	49	0	100

	c. yds.	ft.	in.
(24)	216	23	1657
	98	12	748
	144	16	505
	87	3	31
	86	10	1000

	gal.	qts.	pts.
(25)	33	2	1
	79	0	0
	97	3	1
	9	2	1½
	6	1	0½

	qrs.	bu.	pks.
(26)	60	7	3
	45	3	2
	55	3	3
	67	5	2
	37	7	3

	da.	ho.	min.	sec.
(27)	223	16	35	30
	141	19	43	57
	287	10	13	40
	149	5	56	56
	258	9	11	23

(28) Find the sum of £75 16*s.* 8½*d.* + £769 12*s.* 7½*d.* + £94 17*s.* 5*d.* + 19*s.* 8¼*d.* + 17*s.* 10½*d.* + £17 8*s.* 10¾*d.* + £365 9*s.* 8¾*d.*

(29) Collect into one sum £2754 6*s.* 9*d.*, £144 15*s.*, £1360 13*s.* 4*d.*, £5789 7*s.* 11*d.*, £854 19*s.* 2*d.*, £864 0*s.* 5*d.*, £299 15*s.* 10*d.*

(30) Find in £ *s.* *d.* the sum total of 936 guineas, 553 half-sovereigns, 7363 crowns, and 365 half-crowns.

COMPOUND SUBTRACTION.

37. COMPOUND SUBTRACTION is the process of finding the difference of two concrete quantities that are of like kind, but of various denominations.

Ex. From £180 13*s.* 7¼*d.* subtract £18 16*s.* 2½*d.*

£	*s.*	*d.*
180	13	7¼
18	16	2½
Rem. £161	17	4¾

Here, placing the less amount under the greater, and beginning the subtraction at the place of farthings, we say 2 farthings from 1 farthing cannot be taken, therefore add 1 penny, = 4 farthings, to the 1 farthing, and subtract 2 from 5, which leaves 3 farthings, or ¾*d.*; then carry, or add, 1 penny to the 2*d.* before subtracting from 7; 3*d.* from 7*d.* leaves 4*d.*; then, 16*s.* from 13*s.* cannot be taken, therefore increase the 13*s.* by £1, or 20*s.*, and take 16*s.* from 33*s.*, which leaves 17*s.*; carry £1 to £8, &c.

Note.—It will often be found more convenient for subtracting the shillings in such an example as the present, first to subtract the lower number from 20*s.*, and then to add the remainder to the upper number; thus, 16 from 20 leaves 4, and 13 added makes 17.

Ex. 12.

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(1)	7	18	9¾	(2)	46	11	6¾	(3)	60	18	2	(4)	58	10	7½
	2	10	8½		30	14	8½		12	9	9		8	11	0¾
(5)	66	10	10	(6)	74	3	1¼	(7)	48	17	10½	(8)	91	1	11¼
	56	19	10½		25	12	10¼		17	18	4		0	9	11¾

(9)	157	6	6	(10)	81	12	2¼	(11)	349	0	5	(12)	43	19	3½
	47	9	9¼		4	16	0½		278	19	5¾		6	19	8¼

	lb.	oz.	dwt.		oz.	dwt.	gr.		cwt.	qr.	lb.		yd.	ft.	in.
(13)	26	10	11	(14)	25	13	21	(15)	121	1	11	(16)	61	0	3
	0	11	16		15	9	22		105	3	20		31	0	9

	ac.	ro.	po.	yds.		mi.	yd.	ft.		da.	ho.	min.
(17)	130	1	22	28	(18)	100	609	0	(19)	365	6	0
	0	1	31	15		20	900	2		78	14	11

(20) Subtract $6\frac{1}{2}d.$ from £100, and then subtract 19*s.* 6*d.* from the remainder.

(21) From £2111 1*s.* 3*d.* subtract 19 guineas and 6*d.*

(22) I have now 26 sovereigns 26 shillings and 26 pence; what shall I have left, if I spend 9 guineas and 9 pence of that sum?

COMPOUND MULTIPLICATION.

38. Compound Multiplication is a short method of finding the amount of a compound quantity that is to be repeated any proposed number of times.

Ex. (1) Multiply £34 9*s.* $8\frac{3}{4}d.$ by 9, by 56, and by 132.

£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
34	9	$8\frac{3}{4}$		34	9	$8\frac{3}{4}$		34	9	$8\frac{3}{4}$
		9				8				12
Ans. £310	7	$6\frac{3}{4}$		275	17	10		413	16	9
						7				11
			Ans.	£1931	4	10	*Ans.*	£4552	4	3

In the first of the above examples, we have $3f. \times 9 = 27f. = 6\frac{3}{4}d.$, set down $\frac{3}{4}d.$ and carry 6*d.*; $8d. \times 9 = 72d.$, and 6*d.* carried, makes $78d. = 6s.\ 6d.$, set down 6*d.* and carry 6*s.*; $9s. \times 9 = 81s.$, and 6*s.* carried, makes $87s. = £4\ 7s.$, set down 7*s.* and carry £4, &c. In the next Example we resolve 56 into its factors 8 and 7; and in the one following, we take $132 = 12 \times 11$.

Ex. (2) Find the amount of 17 times £29 13*s.* $7\frac{1}{2}d.$

£	*s.*	*d.*			£	*s.*	*d.*		
29	13	$7\frac{1}{2}$	$\times (8 \times 2 + 1)$	Or,	29	13	$7\frac{1}{2}$	$\times (12 + 5)$	
		8					12		
237	9	0			356	3	6	= 12 times.	
		2			148	8	$1\frac{1}{2}$	= 5 times.	
474	18	0		*Ans.*	£504	11	$7\frac{1}{2}$	= 17 times.	

£504 11 $7\frac{1}{2}$ *Ans.*

In this example, 17 not being a composite number, we take 16, the Table composite number next lower $=8\times2$, and to the product by 16 we add the top line, which, of course, will give 17 times the top line. A different method is exemplified on the right, and it is self-explanatory; but its expedient application is limited to the numbers 13, 17, 19, 23, which are respectively equal to $12+1$, $12+5$, $12+7$, $12+11$.

Ex. (3) Multiply £137 11*s.* 11¼*d.* by 93.

£	*s.*	*d.*	
137	11	11¼	× (10 × 9 + 3)
		10	
1375	19	4½	
		9	
12383	14	4½	
Add 412	15	9¾	= 3 times.
£12796	10	2¼	*Ans.*

Or,

137	11	11¼	× (12 × 8 − 3)
		12	
1651	3	3	
		8	
13209	6	0	
Sub. 412	15	9¾	= 3 times.
£12796	10	2¼	*Ans.*

£	*s.*	*d.*	
Or, 137	11	11¼	× 93
		93	
£12796	10	2¼	*Ans.*

4)93 *f.*
23 . . . 1 *f.*
1023
12)1046 *d.*
87 . . . 2*d.*
1023
2,0)111,0 *s.*
55 . . . 10*s.*
411
1233
£12796

Of the three methods here presented, the first on the left is derived from 90 being the Table composite number next lower than 93; the second on the left is adapted to the consideration that 96 is the Table composite number next higher than 93, in which case 3 times the top line must be subtracted. The method on the right exhibits the process of multiplying each term of the given multiplicand separately by 93; this method is often preferred by expert calculators, and should generally be employed in the case of a multiplier exceeding 150 and not resolvable into convenient factors; the working in this case, if considered accurate, need not be preserved when the answer is completed.

Ex. 13.

		£	s.	d.		
(1)	Multiply	46	8	$9\frac{3}{4}$	separately by	2, 3, 4.
(2)	„	95	6	$5\frac{1}{2}$	„	3, 4, 5.
(3)	„	87	16	$11\frac{1}{4}$	„	2, 6, 8.
(4)	„	60	9	10	„	5, 7, 9.
(5)	„	75	13	$7\frac{1}{2}$	„	6, 8, 9.
(6)	„	294	17	$8\frac{1}{4}$	„	3, 7, 9.
(7)	„	459	12	$6\frac{3}{4}$	„	6, 7, 8.
(8)	„	583	7	$10\frac{1}{4}$	„	10, 11, 12.
(9)	„	796	15	$4\frac{3}{4}$	„	10, 11, 12.
(10)	„	368	0	$11\frac{1}{2}$	„	10, 11, 12.

	£	s.	d.				£	s.	d.		
(11)	29	17	$6\frac{1}{4}$	×	18	(12)	57	14	$10\frac{1}{2}$	×	20
(13)	130	5	10	×	45	(14)	20	18	$6\frac{1}{4}$	×	64
(15)	65	13	$7\frac{1}{2}$	×	108	(16)	139	6	9	×	132
(17)	264	12	$9\frac{1}{4}$	×	19	(18)	225	17	$11\frac{3}{4}$	×	98
(19)	64	7	11	×	105	(20)	119	8	$3\frac{3}{4}$	×	124
(21)	89	10	$10\frac{3}{4}$	×	280	(22)	79	12	$7\frac{3}{4}$	×	173
(23)	365	9	$2\frac{1}{2}$	×	365	(24)	284	13	6	×	247

(25) 15 cwt. 3 qrs. 23 lb. × 9
(26) 3 tons. 13 cwt. 2 qrs. 25 lb. × 12
(27) 26 cwt. 0 qrs. 19 lb. 13 oz. × 13
(28) 5 lbs. 9 oz. 17 dwt. 16 grs. × 25
(29) 17 ac. 2 ro. 37 po. × 69
(30) 9 da. 11 ho. 43 min. 33 sec. × 73

COMPOUND DIVISION.

39. Compound Division is of two kinds:—

(i.) This Rule teaches a short method of finding how often one given concrete quantity is contained in another of the same kind, either or both of the given quantities being compound.

It is generally necessary to reduce both the given concrete quantities to the same single denomination before dividing.

Ex. (1) How often is 13*s.* 9¾*d.* contained in £2 16*s.* 5½*d.*?

	£2 16*s.* 5½*d.*
	20
13*s.* 9¾*d.*	56
12	12
165	677
4	4
663*f.*	)2710*f.* (4 $\frac{58}{663}$ *ti.*
	2652

Ans. 4 *ti.*, and a rem. = 58 *f.* = 14½*d.*

Proof.

	£	*s.*	*d.*	
13*s.* 9¾*d.*)	2	16	5½	(4
	2	15	3	
	£0	1	2½	

Here, the lowest given denomination being farthings, we have reduced both dividend and divisor to farthings; then proceeding as in simple division, we find the required quotient to be 4 times, with a remainder of 58 *farth.* = 1*s.* 2½*d.*; or, according to the more usual way of treating a remainder, we may write *Ans.* $4\frac{58}{663}$ *times.* In this particular example, it might have been readily perceived without reduction that the quotient would be 4, since 13*s.* is evidently contained in £2 very little more than 3 times, and in 16*s.* very little more than 1 time: *see* the proof.

Ex. (2) Divide 7969 ac. 2 ro. by 75 ac. 3 ro. 24 po.

75 ac. 3 ro. 24 po.	7969 ac. 2 ro.
4	4
303	31878
40	40
12144	)1275120(105
	12144
	60720
	60720

Here, the lowest given denomination being poles, we reduce both the given quantities to poles, and we then find the required quotient to be 105, exactly.

Ex. 12.

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(1)	28	1	3	÷	1	17	5	(2)	68	7	11	÷	0	19	6½
(3)	261	12	6	÷	2	17	6	(4)	1	6	3¼	÷	0	3	9½
(5)	626	7	0	÷	26	1	11½	(6)	384	3	6	÷	23	5	8
(7)	321	10	3¾	÷	5	12	9¾	(8)	100	0	0	÷	11	12	6
(9)	8	0	11½	÷	1	19	6	(10)	123	12	2	÷	4	8	3½
(11)	399	0	6¾	÷	12	17	5¼	(12)	988	13	0½	÷	30	0	10

(13) 1136 cwt. 2 qrs. 2 lb. ÷ 35 cwt. 2 qrs. 1 lb. 13 oz.

(14) 10 tons 18 cwt. 2 qrs. 14 lb. ÷ 12 cwt. 1 qr. 14 lb.

(15) 17 mi. 3 fur. 162 yds. 9 in. ÷ 9 ft. 11 in.

(16) 12245 qrs. of wheat ÷ 61 qrs. 5 bu. 3 pks.

(ii.) This Rule also teaches how to find a concrete quantity that is contained a given number of times in a given concrete quantity, either or both of the concrete quantities being compound.

Ex. (1) What sum of money is contained 7 times in £556 12*s.* 6*d.*?

	£	s.	d.
7)556	12	6	
Ans. £79	10	$4\frac{2}{7}$	

Here, we first find the *7th* part of the given £s to be £79, with a remainder of £3 to be reduced to shillings in order that we may be able to continue the division by 7; we say, therefore, £3 = 60*s.*, which with 12*s.* more in the dividend makes 72*s.*, the *7th* part of which is 10*s.* with a remainder of 2*s.*; similarly, 2*s.* = 24*d.*, and 6*d.* more makes 30*d.*, the *7th* of which is $4\frac{2}{7}d.$

Ex. (2) Find the 105th part of 7969*ac.* 2*ro.*

105)7969*ac.* 2*ro.* (75*ac.* 3*ro.* 24*po.* *Ans.*

735
619
525
94
4
378 *ro.*
315
63
40
2520 *po.*
210
420
420

Otherwise, since 105 is resolvable into the three factors, 5, 3, 7, we may divide by these successively.

		ac.	ro.	po.
	5)7969	2	0	
105	3)1593	3	24	
	7) 531	1	8	
Ans.	75	3	24	

Ex. (3) Divide £434 11*s.* $2\frac{1}{4}d.$ by 45.

	£	s.	d.
5)434	11	$2\frac{1}{4}$	
9) 86	18	$2\frac{3}{4}$	.... $2f.$
Ans. £9	13	$1\frac{1}{2}$	*rem.* $27f.$
or, £9	13*s.*	$1\frac{1}{2}d.$	$\frac{3}{5}f.$

Or,

	£	s.	d.
5)434	11	$2\frac{1}{4}$	
9) 86	18	2	.... $4\frac{1}{4}d.$
Ans. £9	13	1	*rem.* $29\frac{1}{4}d.$
or, £9	13*s.*	$1\frac{13}{20}d.$	

In each of the above methods, we have resolved 45 into factors 5 and 9. By the first method, however, we carry out the division to farthings, and since the respective divisions leave remainders $2f.$ and $5f.$, we multiply the second remainder by the first divisor, which gives $25f.$ and then adding the first remainder we have $27f.$ or $6\frac{3}{4}d.$ as the full remainder. In the second method, we carry out the division only to pence, which is the usual practice; we thus have as successive remainders $4\frac{1}{4}d.$ and $5d.$; hence, to get the complete remainder, $5d. \times 5 + 4\frac{1}{4}d. = 29\frac{1}{4}d.$

Note.—The remainders $27f.$ and $29\frac{1}{4}d.$, which in strictness ought to be divided by 45, would, if so divided, assume the fractional forms $\frac{27}{45}f.$ and $\frac{29\frac{1}{4}}{45}d.$, as formerly explained. The latter of these forms, being of a complex character, can be simplified by multiplying both its terms by 4, producing the equivalent fraction $\frac{117}{180}$; and further, we have $\frac{27}{45} = \frac{3}{5}$, and $\frac{117}{180} = \frac{13}{20}$, by cancelling 9 from the upper and lower terms of each fraction.

Ex. (4) Divide £731 14*s.* 7*d.* by 100.

£7,31 14*s.* 7*d.*
20
6,34*s.*
12
4,15*d.* $= 4\frac{15}{100}$
Ans. £7 6*s.* $4\frac{3}{20}d.$

We here exemplify the usual contracted process of dividing by 100. Thus, 731 is 7 hundreds and 31 over, therefore simply point off the two right-hand figures 31 as a remainder of £ to be reduced to shillings; then dividing again by 100, we say 634 is 6 hundreds and 34 over, therefore point off the two right-hand figures 34 as remaining shillings to be brought to pence, and so forth. The final remainder 15*d.* divided by 100 is $\frac{15}{100}d. = \frac{3}{20}d.$

The general procedure in division by 10, 100, 1000, &c., is to point off from the right as many figures as the divisor contains ciphers.

Ex. 15.

	£	*s.*	*d.*				£	*s.*	*d.*		
(1)	59	17	8	÷	2	(2)	73	11	7	÷	2
(3)	116	9	$5\frac{1}{4}$	÷	3	(4)	2	13	8	÷	3
(5)	230	15	6	÷	4	(6)	47	11	$6\frac{3}{4}$	÷	5
(7)	57	1	$7\frac{1}{2}$	÷	6	(8)	100	3	$2\frac{1}{4}$	÷	7

	£	s.	d.				£	s.	d.		
(9)	29	11	8	÷	8	(10)	230	7	10½	÷	9
(11)	101	10	1	÷	11	(12)	36	11	6	÷	12
(13)	75	4	4	÷	16	(14)	175	9	8	÷	18
(15)	900	13	7	÷	19	(16)	80	0	5	÷	23
(17)	0	17	11½	÷	28	(18)	114	19	4½	÷	36
(19)	10	6	5½	÷	34	(20)	537	11	3	÷	47
(21)	136	14	10½	÷	54	(22)	100	0	0	÷	84
(23)	1	15	7½	÷	10	(24)	364	7	6	÷	100
(25)	57	11	5	÷	100	(26)	701	13	3	÷	120
(27)	316	12	4½	÷	180	(28)	93	14	8½	÷	144
(29)	524	1	0	÷	235	(30)	984	14	9½	÷	365
(31)	251	2	0¾	÷	252	(32)	596	17	1¾	÷	569

(33) 207 cwt. 3 qrs. 19 lb. ÷ 29
(34) 541 cwt. 14 lb. ÷ 35
(35) 18 yds. 3 qrs. 1 na. ÷ 21
(36) 1695 da. 14 ho. 40 min. ÷ 483
(37) 9075 ac. 4 po. ÷ 100
(38) 18396 mi. 7 fur. ÷ 2800

SUPPLEMENT

TO

COMPOUND MULTIPLICATION AND DIVISION.

40. It is expedient here to anticipate some portion of the department of Vulgar Fractions.

Since, as already explained (28), the expression ¾ signifies the 4th part of 3, it follows that to take ¾ of a number, or to multiply it by ¾, is to take a 4th part of 3 times the number; hence, ¾ of £1 $= \frac{£3}{4}$ = the 4th part of £3 = 60*s.* ÷ 4 = 15*s.* And hence, generally, to calculate a fractional part of any quantity, we may multiply the quantity by the upper number of the fraction, and divide the product by the lower number.

Ex. (1) Calculate $\frac{3}{4}$ of £13 15*s.* 6*d.*, and $\frac{2}{5}$ of 47yds. 2ft. 5in.

£	*s.*	*d.*
13	15	6
		3
4)41	6	6
Ans. £10	6	$7\frac{1}{2}$

yds.	ft.	in.
47	2	5
		2
5)95	1	10
Ans. 19	0	$4\frac{2}{5}$

To take $17\frac{3}{4}$ times a number is to take 17 times the number, and add it to $\frac{3}{4}$ of the number. Otherwise, as will generally be more convenient at this stage of the learner's progress, we may first express the multiplier altogether in *fourths*: thus, since $17 = \frac{17}{1} = \frac{68}{4}$, therefore, $17\frac{3}{4} = \frac{68+3}{4} = \frac{71}{4}$; and hence, to take $17\frac{3}{4}$ times a number is the same as to take $\frac{71}{4}$ of the number, or the 4th part of 71 times the number; as will appear from the following example:—

Ex. (2) Multiply £13 15*s.* 6*d.* by $17\frac{3}{4}$.

	£	*s.*	*d.*
	13	15	6
			$17\frac{3}{4}$
	4)41	6	6
Product by $\frac{3}{4}$ =	10	6	$7\frac{1}{2}$
„ 17 =	234	3	6
Ans.	£244	10	$1\frac{1}{2}$

£	*s.*	*d.*
13	15	6
		71
4)978	0	6
Ans. £244	10	$1\frac{1}{2}$

The learner should now be made familiar with the fact, that multiplying a number consisting of a whole number and a fraction by the lower number of the fraction will remove the fraction. Thus, $17\frac{3}{4} \times 4 = 68 + 3 = 71$; and $16\frac{2}{3} \times 3 = 48 + 2 = 50$. Hence, to divide $17\frac{3}{4}$ by 18, we may simplify $\frac{17\frac{3}{4}}{18}$ by multiplying the upper and lower lines by 4, which gives $\frac{71}{18 \times 4}$ or $\frac{71}{72}$.

Ex. (3) Multiply £3 11*s.* 7½*d.* by $\frac{7}{12}$.

£	s.	d.
3	11	7½
		7
12)25	1	4½
Ans. £2	1	9⅜

Here, in the division by 12, the pence dividend is 112½*d.*, which divided by 12 gives 9 with a remainder of 4½ to be divided by 12; hence we have $\frac{4\frac{1}{2}}{12} = \frac{9}{24} = \frac{3}{8}d.$ If the multiplier had been $3\frac{7}{12}$, we should have used it in the form $\frac{43}{12}$, to avoid the necessity of addition of fractions, with which the pupil is presumed to be unacquainted.

Ex. (4) Reduce 2ac. 1ro. 33po. 13yds. 13in. to square inches.

2ac. 1ro. 33po. 13yds. 13in.
4
9 *ro.*
40
393 *po.*
30¼
98¼
11803
11901¼ *sq. yds.*
9
107111¼ *sq. ft.*
12
1285335
12
15424033 *sq. in.*

Here, in reducing the sq. po. to sq. yds, we add ¼ of 393 to 30 times 393, and include the given 13 yds.; then multiplying by 9, to reduce the sq. yds. to sq. ft., we say 9 times ¼ is $\frac{9}{4} = 2\frac{1}{4}$; set down ¼ and carry 2, &c. Lastly, multiplying by 144, or 12 times 12, to reduce the sq. ft. to sq. inches, we say 12 times ¼ is $\frac{12}{4} = 3$, and therefore carry 3. The 13 inches are not to be included till we are multiplying by the second 12.

Ex. 16.

	£	s.	d.				£	s.	d.		
(1)	237	8	6	×	$\frac{5}{9}$	(2)	9	17	6	×	$\frac{7}{8}$
(3)	0	16	0½	×	$\frac{13}{21}$	(4)	27	17	9½	×	$8\frac{1}{2}$
(5)	179	10	11¼	×	$7\frac{2}{3}$	(6)	360	6	0¾	×	$9\frac{3}{7}$
(7)	0	3	6¼	×	$12\frac{10}{13}$	(8)	78	11	11½	×	$19\frac{9}{20}$

(9) Multiply 9yds. 2ft. 9½in. by $\frac{4}{17}$.
(10) Multiply 18cwt. 3qrs. 10lb. by $21\frac{3}{14}$.
(11) Reduce 7mi. 3fur. 27po. 3yds. to inches.
(12) Reduce 3ac. 2ro. 31po. 23sq. yds. to square inches.
(13) Reduce 12ac. 3ro. 13sq yds. 5sq. ft. to square feet.
(14) Reduce 5ac. 18po. 25sq. yds. to square inches.

41. Since $\frac{2}{3}$ signifies the 3rd part of 2, it follows that to divide a number by $\frac{2}{3}$ will produce 3 times as much as if it were divided by 2; so that $16 \div \frac{2}{3} = 16 \div \frac{1}{2} \times 3 = 48 \div 2 = 24$. Similarly, $28 \div 4\frac{2}{3} = 28 \div \frac{14}{3} = \frac{28}{14} \times 3 = 84 \div 14 = 6$; otherwise, if the upper and lower numbers of $\frac{28}{4\frac{2}{3}}$ be both multiplied by 3, we shall have $\frac{84}{14} = 6$. Hence, when a divisor contains a fraction, multiply both dividend and divisor by the lower number of the fraction, and then divide.

Ex. (1) Divide 23cwt. 1qr. 23lb. by $\frac{4}{7}$, and by $2\frac{4}{7}$.

cwt.	qr.	lb.		cwt.	qr.	lb.
$0\frac{4}{7})23$	1	23	$2\frac{4}{7})23$	1	23	
7		7	7		7	
4)164	0	21	18)164	0	21	
Ans. 41	0	$5\frac{1}{4}$	*Ans.* 9	0	$13\frac{11}{18}$	

Ex. (2) Reduce 15424033 square inches to the higher denominations.

144 { 12)15424033
12) 1285336 ... 1
9) 107111 ft. 49 in.
$30\frac{1}{4})$ 11901 yds. 2 ft.
4 4
121 { 11)47604 qr. yds.
11) 4327 ... 7
393 po. 51 qr. yds., or 393 po. $12\frac{3}{4}$ yds.

Now, $\frac{3}{4}$ of a sq. yd. = $\frac{3}{4}$ of 9 sq. ft. = $6\frac{3}{4}$ ft. = 6 ft. 108 in.

393 po. 12 sq. yds. 6 ft. 108 in.
2 49

Ans. 2 ac. 1 ro. 33 po. 13 sq. yds. 0 ft. 13 in.

Ex. 17.

	£	s.	d.			£	s.	d.		
(1)	11	13	7	+	$\frac{3}{8}$	(2) 146	16	$3\frac{3}{4}$	+	$\frac{11}{12}$
(3)	4	19	$10\frac{3}{4}$	+	$7\frac{1}{2}$	(4) 69	0	6	+	$8\frac{4}{5}$
(5)	417	12	$7\frac{1}{2}$	+	$10\frac{1}{9}$	(6) 78	17	$9\frac{1}{2}$	+	$15\frac{5}{6}$
(7)	0	16	$2\frac{1}{4}$	+	$2\frac{39}{40}$	(8) 700	13	$2\frac{3}{4}$	+	$27\frac{7}{27}$

(9) 173 lb. 9 oz. 2 dwt. + $12\frac{3}{7}$ (10) 12 da. 4 ho. 55 min. + $10\frac{3}{13}$

(11) Reduce 47212 sq. yds. to acres, roods, &c.
(12) Reduce 6912000 sq. in. to acres, roods, &c.
(13) Reduce 15790875 sq. in. to acres, roods, &c.
(14) Reduce 2010650 minutes to years of $365\frac{1}{4}$ days.

42. Miscellaneous Examples Worked Out.

(1) If I spend as much of 14 sovereigns as can buy an exact number of boxes of tea, each 5 lb., at 3*s.* 9*d.* a lb., how many boxes shall I receive, and what change out?

Cost of each box = 3*s.* 9*d.* × 5 = 45*d.* × 5 = 225*d.*; therefore the no. of boxes to be received = the greatest no. of times that 225*d.* can be subtracted from £14 or 3360*d.*

225*d.*) 3360*d.* (14 boxes and 17*s.* 6*d.* change. *Ans.*
rem. 210*d.* = 17*s.* 6*d.*

(2) If a gentleman's yearly income be £537 15*s.* 4*d.*, and he lays up £47 2*s.* 6*d.* every quarter, what does he spend, on an average, per week, reckoning a year = 52 weeks?

£537 15*s.* 4*d.* received per annum.
£47 2*s.* 6*d.* × 4 = 188 10 0 laid up „ „
52) 349 5 4 spent „ „
Ans. £6 14*s.* 4*d.* spent per week.

(3) Add together the quantities 5*s.* $5\frac{3}{4}$*d.*, 10*s.* $6\frac{1}{2}$*d.*, 17*s.* $10\frac{1}{2}$*d.*, and 9*s.* $1\frac{1}{2}$*d.*, and divide their amount by a third of the sum of the greatest and least of them.

5*s.* $5\frac{3}{4}$*d.*
10 $6\frac{1}{2}$
17 $10\frac{1}{2}$
9 $1\frac{1}{2}$

5*s.* $5\frac{3}{4}$*d.*
17 $10\frac{1}{2}$

Sum of all = 43*s.* $0\frac{1}{4}$*d.*; and sum of 1*st* and 3*rd* = 23*s.* $4\frac{1}{4}$*d.*

$$\frac{43s.\ 0\frac{1}{4}d.}{\frac{1}{3}\ of\ 23s.\ 4\frac{1}{4}d.} = \frac{43s.\ 0\frac{1}{4}d. \times 3}{23s.\ 4\frac{1}{4}d.} = \frac{6195f.}{1121f.} = 5\tfrac{590}{1121} = 5\tfrac{10}{19}.$$ Ans.

(4) Express in the ordinary money of account the sum of 31 half-guineas, 31 crowns, and 31 florins.

The sum is = 31 times (10*s.* 6*d.* + 5*s.* + 2*s.*)
= 17*s.* 6*d.* × 31 = £27 2*s.* 6*d.* *Ans.*

(5) Divide £27 2*s.* 6*d.* into three equal numbers of half-guineas, crowns, and florins, respectively.

Here we are to find how many times (10*s.* 6*d.* + 5*s.* + 2*s.*) will amount to £27 2*s.* 6*d.*; therefore, 542*s.* 6*d.* ÷ 17*s.* 6*d.* = 1085 ÷ 35 = 31 of each denomination. *Ans.*

(6) If 9lb. of isinglass cost 7 guineas, what should a dozen lb. cost?

Here we have 12 lb. = a third as much again as 9 lb.; so that the cost required will be a third as much again as 7 guineas; hence, 21*s.* × 7 × $1\frac{1}{3}$ = 28*s.* × 7 = 196*s.* = £9 16*s.* *Ans.*

Otherwise: We are to find the cost of 12 lb. at the rate of £7 7*s.* for 9 lb. Now, 12 lb. reckoned at £7 7*s.* *per lb.*, would evidently amount to 9 times the cost required; therefore, we may first assume 12 lb. at £7 7*s.* per lb., and then divide by 9; thus,

$$\frac{\text{£7 7s.} \times 12}{9} = \frac{\text{£88 4s.}}{9} = \text{£9 16s.}\ \textit{Ans.}$$

Note.—Questions of this kind, in which a certain value of one quantity is given, and a proportional or corresponding value of another quantity of like kind is required, are usually assigned to that part of Arithmetic called the *Rule of Three.* A good general method of solving them is that sometimes called the *Unitary* method, from its first determining the value of 1 of those things whose total value is required. Thus, for the present example, we have

Cost of 9 lb. = 147*s.*
∴ do. of 1 lb. = $\frac{1}{9}$ of 147*s.*
∴ do. of 12 lb. = $\frac{12}{9}$ of 147*s.* = $\frac{147s. \times 12}{9}$ = *Ans.* as above.

It may be remarked that the sign ∴ here employed conventionally denotes *therefore.*

(7) What weight of silver, at 5*s.* 6*d.* an ounce, would be equal in value to $13\frac{3}{4}$ ounces of gold at £3 17 $10\frac{1}{2}d.$ an ounce?

The whole value of the gold is £3 17*s.* $10\frac{1}{2}d. \times 13\frac{3}{4}$, and that amount will buy a number of ounces of silver equal to the number of times it contains 5*s.* 6*d.* Accordingly,

$$\frac{£3\ 17s.\ 10\frac{1}{2}d. \times 13\frac{3}{4}}{5s.\ 6d.} = \frac{1869\ hf.\ d. \times \overset{5}{\cancel{55}}}{\underset{12}{\cancel{132}}\ hf.\ d. \times \cancel{4}} = \frac{9345}{48} = \frac{3115}{16}\ \text{oz.}$$

$$= 194 \text{ oz. } 13 \text{ dwt. } 18 \text{ grs. } \textit{Ans.}$$

(8) A wine-merchant buys 24 gallons of spirits at 29*s.* a gallon, 15 at 25*s.*, and 27 at 19*s.*, and, mixing them together, he sells the mixture at 27*s.* 6*d.* a gallon; what is the amount of his profit?

24 gallons	@ 29*s.*	=	696*s.*
15 „	@ 25*s.*	=	375
27 „	@ 19*s.*	=	513
Cost of 66 gall. of mixture		=	1584*s.*
„ 1 gall.	„	=	1584*s.* ÷ 66 = 24*s.*

Gain by selling 1 gall. = 27*s.* 6*d.* − 24*s.* = 3*s.* 6*d.*

Gain on 66 gall. = 3*s.* 6*d.* × 66 = 7*s.* × 33 = 11 guin. *Ans.*

Note.—When, in a question of this kind, the aggregate parts of the mixture all contain a common factor, that factor has no effect on the average cost of the gallon, and may, therefore, be struck out. Thus, 3 being a common factor in 24, 15, and 27, may be cancelled, by which the quantities become 8, 5, and 9; the procedure will then be as follows:—

8 gallons	@ 29*s.*	=	232*s.*
5 „	@ 25*s.*	=	125
9 „	@ 19*s.*	=	171
22			) 528*s.*

Cost of mixture per gall. 24*s.*

(27*s.* 6*d.* − 24*s.*) × 66 = £11 11*s.* *Ans.*

(9) I gained £2 12*s.* $10\frac{1}{2}d.$ by selling, at 14*s.* 9*d.* a yard, a quantity of silk velvet which I bought at 10*s.* 10*d.*; what was the quantity?

The gain on each yard was 14*s.* 9*d.* — 10*s.* 10*d.* = 3*s.* 11*d.*, and

the gain on the whole no. of yards was 52*s.* 10½*d.*; therefore, there were in all as many yards as the no. of times that 52*s.* 10½*d.* contains 3*s.* 11*d.*, which is = 1269 *hf. d.* ÷ 94 *hf. d.* = 13½ yds. *Ans.*

(10) How much alloy must be mixed with 15 ounces of gold, worth £4 4*s.* 6*d.* an ounce, to reduce the value to £3 10*s.* 5*d.* an ounce, neglecting the value of the alloy?

The mixed metal is to be altogether worth £4 4*s.* 6*d.* × 15, and will consist of as many ounces as the no. of times that that value contains £3 10*s.* 5*d.* Accordingly,

$$\frac{£4\ 4s.\ 6d. \times 15}{£3\ 10s.\ 5d.} = \frac{1014d. \times 15}{845d.} = \frac{1014 \times 3}{169} = 6 \times 3 = 18 \text{ oz.};$$

and 18 oz. mixture − 15 oz. gold = 3 oz. alloy. *Ans.*

(11) Divide 6*s.* 6*d.* between Jane and Ellen, so that Jane may receive 3*s.* more than Ellen.

Here it is evident that if the 6*s.* 6*d.* were placed on a table, and Jane first took up 3*s.*, and then shared the remainder equally with Ellen, Jane's share would be 3*s.* more than Ellen's. Or, thus:

Ellen receives 1 share
Jane „ 1 such share + 3*s.*

so that the whole, 6*s.* 6*d.*, = 2 of Ellen's shares + 3*s.*

hence, 6*s.* 6*d.* − 3*s.*, or 3*s.* 6*d.* = 2 of Ellen's shares;

or, 3*s.* 6*d.* ÷ 2 = 1*s.* 9*d.* to Ellen
6*s.* 6*d.* − 1*s.* 9*d.* = 4*s.* 9*d.* to Jane } *Ans.*

Note.—This question is a particular example of the general problem—*Given the sum and the difference of two quantities, to find each of the quantities.*

(12) Divide £5 6*s.* 8*d.* among three persons, giving the second 1*s.* 6*d.* more than the first, and the third 4*s.* 10*d.* less than the second.

The 2*nd* gets 1 share
1*st* „ 1 share − 1*s.* 6*d.*
3*rd* „ 1 share − 4*s.* 10*d.*

The whole £5 6*s.* 8*d.* = 3 of the 2*nd*'s share − 6*s.* 4*d.*

hence, £5 6*s.* 8*d.* + 6*s.* 4*d.*, or £5 13*s.* = 3 shares of the 2*nd*

∴ £5 13*s.* ÷ 3 = £1 17*s.* 8*d.* to the 2*nd*
2*nd* − 1*s.* 6*d.* = £1 16*s.* 2*d.* to the 1*st*
2*nd* − 4*s.* 10*d.* = £1 12*s.* 10*d.* to the 3*rd* } *Ans.*

(13) Divide 10 guineas among A, B, and C, giving B half as much again as A, and C a third as much again as B.

If we represent A's share by 2 equal sums
B's share will be $2 \times 1\frac{1}{2} = 3$ such sums
C's „ „ $3 \times 1\frac{1}{3} = 4$ „ „
Therefore, the whole £10 10*s.* = 9 „ „
and 1 of the equal sums = £10 10*s.* ÷ 9 = £1 3*s.* 4*d.*

hence £1 3*s.* 4*d.* × 2 = £2 6*s.* 8*d.* to A
× 3 = £3 10*s.* 0*d.* to B
× 4 = £4 13*s.* 4*d.* to C
} *Ans.*

(14) Divide £25 13*s.* 6*d.* among 6 men, 8 women, and 9 boys, giving each woman half as much again as a boy, and each man twice as much as a woman.

9 boys will have 9 shares,
8 women, = 8 × 1½ „ „ 12 such shares,
6 men, = 6 × 2 women, = 12 × 1½ „ „ 18 „
In all 39 boys' shares;

hence, each boy's share, £25 13*s.* 6*d.* ÷ 39 = 13*s.* 2*d.*
each woman's, 13*s.* 2*d.* × 1½ = 19*s.* 9*d.*
each man's, 19*s.* 9*d.* × 2 = 39*s.* 6*d.*
} *Ans.*

(15) Suppose that there are 24 parcels, each weighing 13½ lb., 23 each 12 lb., and 21 each 10 lb.; that the 68 parcels are to be carried, in equal loads, on 3 trucks, F, G, H; that the trucks are to receive each its full load in the order F, G, H, and with parcels taken in the given order of weight:—How many parcels will each truck contain?

24 parcels each 13½ lb. = 13½ × 24 = 324 lb.
23 „ „ 12 „ = 12 × 23 = 276 „
21 „ „ 10 „ = 10 × 21 = 210 „
The 68 parcels weigh altogether . . 810 „

hence each truck is to receive a load = 810 ÷ 3 = 270 lb.

The first truck, therefore, will receive its load from the 324 lb.; the second will receive the remaining 54 lb., together with 216 lb. out of the 276 lb., &c. Accordingly, the no. of parcels for each truck may be ascertained as follows:—

F will contain 270 lb. ÷ 13½ lb. = 20 parcels.
324
270

G will contain 54 lb. ÷ 13½ lb. = 4 parcels,
and 216 „ ÷ 12 „ = 18
276 270 „ = weight of G's 22 parcels.
216

H will contain 60 lb. ÷ 12 lb. = 5 parcels,
and 210 „ ÷ 10 „ = 21
270 „ = weight of H's 26 parcels.

Ans. F 20 parcels, G 22, H 26.

MISCELLANEOUS EXERCISES.

Ex. 18.

(1) Add together, 1526, 2986, 957, and 3879; and multiply the 19*th* part of the sum by 76.

(2) Add together £38 8*s.* 6¼*d.*, £19 18*s.* 6¼*d.*, £34 7*s.* 10½*d.*, and £57 9*s.* 8*d.*; and divide the total of these sums by the difference of the greatest and least of them.

(3) How many dozen copies of a book, at 2*s.* 5½*d.* per copy, can be bought with £13 5*s.* 6*d.*?

(4) From 190 crowns, 11 florins, and 10 pence, subtract 30 guineas, 9 half-crowns, and 7½ pence: give the answer in £ *s.* *d.*

(5) Calculate the whole cost of 497 stones of hay, at 5¾*d.* per stone, and 19½ tons of potatoes at £1 18*s.* 10*d.* per ton.

(6) How often can £1 9*s.* 11*d.* be paid away out of £1359 13*s.* 2*d.*? and what is the value of the remainder?

(7) Reduce £563 12*s.* 6*d.* to pieces of the value of 16*s.* 7½*d.* each; and express the remainder in current English money.

(8) A man's income is £219 a year, and he spends 8*s.* 6*d.* a day; in what time would his savings enable him to buy a Thirty-Guinea pianoforte?

(9) If a farm of 144 a. 3 r. 8¾ p. were enlarged to $\frac{8}{11}$ as much again, what would then be the area?

Ex. 19.

(1) Add together £589 12*s.*, £78 16*s.*, and £599 9*s.*; from the sum subtract £79 19*s.*; and multiply the 14*th* part of the remainder by 21.

(2) Add 12 *thousand* 12 *hundred* and 12 to 11 *thousand* 11 *hundred* and 11; from the sum subtract 925; and divide the remainder by 242.

(3) Subtract 31 half-crowns, 17 florins, 1 shilling, and 3 farthings from the 19*th* part of £1425 1*s.* 2¼*d.*; and divide the remainder by 37.

(4) A quantity of sugar, weighing 123 cwt. 3 qrs. 12 lb., is reduced by successive subtractions of 18 cwt. 1 qr. 16 lb., until less than that quantity remains. What would the remainder amount to at 4¾*d.* a lb.?

(5) How many guineas in all would purchase 17 bushels of wheat at 44*s.* a quarter, and 408 stones of straw at 2½*d.* a stone?

(6) Add together £14 10*s.* 6*d.*, £14 13*s.* 9*d.*, £16 9*s.* 4¼*d.*, and £14 10*s.* 10½*d.*, and divide the total of these sums by half the difference between the greatest and least of them.

(7) How many yards of cambric at 9*s.* 7*d.* a yard would amount to just as much money as 65 yards of silk at 7*s.* 8*d.* a yard?

(8) What should be a lady's average daily expenditure, if her income is £360 18*s.* 9*d.* a quarter, and she intends to lay by 864 guineas yearly?

(9) Divide £101 10*s.* 4*d.* by $3\frac{5}{9}$.

Ex. 20.

(1) If I buy 315 yards of cloth at 19*s.* 9*d.* a yard, and sell it at 23*s.* 6*d.*, what will my profit amount to?

(2) When geese were selling at 6*s.* 4*d.* each, I found that the sum which would buy 87 geese was also the cost of 232 pairs of rabbits; what was the price of rabbits a pair?

(3) How many boxes of vermicelli, each containing 5 lb., may be bought for 2½ guineas, at the rate of 10½*d.* a lb.?

(4) What is the weight of 63 qu. 6 bu. of wheat, at 64 lb. a bushel? and what money would it produce at 37*s.* 4*d.* a quarter?

(5) Express in ordinary form the sum of 47 yds. 47 ft. 47 in.; and find the three equal numbers of yards, feet, and inches, of which the sum is 5 mi. 902 yds.

(6) A wine-merchant has spirits worth 26*s.* a gallon; if he put $2\frac{1}{2}$ gallons of water to every 17 gallons of spirits, what will a gallon of the mixture be worth?

(7) How much water must be added to 51 gallons of whisky, worth 19*s.* 6*d.* a gallon, to reduce the value to 18*s.* a gallon?

(8) Divide 14*s.* between two sisters, giving the elder 14*d.* more than the younger.

(9) Reduce 29 sq. po. 22 sq. yds. to square feet.

Ex. 21.

(1) How many yards of cloth at 11*s.* $8\frac{1}{4}$*d.* would £8 0*s.* $10\frac{1}{2}$*d.* buy?

(2) If a gentleman's expenditure is £8 15*s.* 6*d.* weekly, and he lays up £193 14*s.* yearly; what is his annual income, reckoning a year = 52 weeks?

(3) From 23 times 4*s.* 10*d.* subtract the 19th part of £6 13*s.* $9\frac{1}{2}$*d.*; divide the remainder by 49, and multiply the result by 84.

(4) Valuing a napoleon, or 20-franc piece, at 15*s.* 8*d.*, and a rupee at 1*s.* $11\frac{1}{2}$*d.*, how many rupees are equivalent to 365 napoleons?

(5) Divide 30*s.* between William and Mary, giving William thrice as much as Mary.

(6) I bought a quantity of raisins at 8*d.* a lb., the same quantity of currants at 5*d.*, and the same of almonds at 18*d.*; and in paying for the whole, I put down a sovereign and received 7*s.* 1*d.* change. How many lbs. of each did I buy?

(7) The sum of two numbers is 567, and their difference is 234: find them.

(8) If 8 lbs. of corn cost $4\frac{1}{2}$*d.*, and 7 lbs. of hay cost $2\frac{3}{4}$*d.*, what does the food of a horse amount to in 17 days, supposing him to consume, on an average, 12 lbs. of corn and 28 lbs. of hay per day?

(9) Multiply £76 19*s.* $8\frac{5}{13}$*d.* by $5\frac{4}{7}$.

Ex. 22.

(1) Divide £14 × 273 × $2\frac{1}{4}$ by 156 × 81.

(2) Convert 7225 francs to British money, supposing the exchange to be 20 shillings for $25\frac{1}{2}$ francs.

(3) Divide £10 13*s.* 2*d.* between A and B, giving B £1 6*s.* 7*d.* less than A.

(4) Change 46 cwt. 2 qrs. 9 lbs. into an equivalent quantity consisting of cwt. qrs. lbs., all expressed by the same number.

(5) To 72 gallons of brandy, worth 36*s.* 8*d.* a gallon, 5 gallons of water are added; what is the worth of $3\frac{1}{2}$ gallons of the diluted spirits?

(6) Divide 134 ac. 5 po. between C and D, so that C's part may be half as much again as D's.

(7) If a horse consume daily 7 lbs. of oats and 35 lbs. of hay, what sum will buy his food for 6 weeks, when hay is 5*d.* a stone, and oats $7\frac{1}{2}$*d.*?

(8) If 6 lbs. of tea at 4*s.* 8*d.* a lb., 84 lbs. at 4*s.* 3*d.*, and 57 lbs. at 3*s.* 8*d.*, are to be divided among A, B, C, so that they shall have shares of equal value, but have precedence as to the quality of tea in the order A, B, C; how many lbs. will each receive?

(9) Divide 52 yds. 1 ft. $10\frac{1}{2}$ in. by $3\frac{12}{17}$.

Ex. 23.

(1) How many gowns can be made with 358 yds. 2 qrs. 2 na. of silk, if 18 yds. 3 qrs. 2 na. be required for each?

(2) A grocer buys 25 dozen pots of marmalade at 6*s.* 4*d.* per dozen, and retails it at 9*d.* a pot; what is the amount of his profit?

(3) Divide 12*s.* $10\frac{1}{2}$*d.* × 1078 by 15*s.* 9*d.* × 21 × $5\frac{1}{2}$.

(4) What sum will remain after subtracting as often as is practicable the combined value of 26 crowns, 23 florins, and 9 pence, from that of 351 half-crowns, 31 sixpences, and 3 pence?

(5) How much water must be added to 54 gallons of spirits, worth 20*s.* a gallon, so that 4 gallons of the diluted spirits may be worth £3 15*s.*?

(6) Two fields together measure 21 ac. 1 ro. 10 po., one of them containing 9 ac. 1 ro. 28 po. more than the other. Find the area of the larger field.

(7) I bought nutmegs at 4*s.* $2\frac{1}{2}$*d.* a lb., and by selling them at 4*s.* 10*d.* my amount of profit was £2 3*s.* 9*d.*; what quantity did I buy?

(8) Four salesmen, A, B, C, D, are to divide among themselves 282 salmon, worth a uniform price per lb. Suppose that 60 of the salmon weigh each $15\frac{3}{4}$ lbs., 50 each 14 lbs., and the rest each $8\frac{3}{4}$ lbs., and that the salesmen are to have shares equal in value, but to count out the heavier fish first, and serve full shares in the order A, B, C, D; how many salmon will each salesman receive? *and what will be the* value of his share at $10\frac{1}{2}$*d.* a lb.?

(9) Reduce 724060 square inches to the higher denominations.

Ex. 24.

(1) Multiply 27 cwt. 3 qrs. 12 lbs. by $6\frac{1}{4}$, and divide the product by 50.

(2) How often would a coach wheel, 15 ft. 7 in. in circumference, turn in passing over a distance of 85 miles?

(3) Suppose that the money taken in a day, from foot-passengers, each paying a halfpenny, for bridge toll, is in all equivalent to 7 half-sovereigns 7 half-crowns and 7 florins; what has been the number of passengers?

(4) A wine-merchant mingles 20 gallons of spirits at 15*s.* with 12 at 17*s.*, 32 at 20*s.*, and 24 at 22*s.* What is a gallon of the mixture worth?

(5) Calculate the expression $\frac{27 \text{ cwt. } 19 \text{ lbs.} \times 145\frac{1}{2}}{4\frac{1}{4} \times 485}$.

(6) 57 gallons of spirits at 7*s.* 10*d.* are mingled with 17 gallons at 9*s.* 6*d.* and 2 gallons of water. What is the mixture worth per gallon?

(7) Divide £45 among A, B, C, giving B 26*s.* 6*d.* more than A, and C 26*s.* 6*d.* more than B.

(8) Divide 365 days into three periods, such that the second may be 29 times as long as the first, and the third 4 times as long as the second.

(9) If 11 ro. 34 po. 1 sq. yd. 1 sq. ft. 108 sq. in. were divided into seven equal allotments, what would be the extent of each?

Ex. 25.

(1) A maid-servant is hired at £13 a year. In how many weeks will she earn £5? (a year = 52 weeks.)

(2) If the fore wheel of a carriage be 9 ft. 4 in. in circumference, and the hind wheel 15 ft. 9 in.; how many revolutions will each wheel make in passing over 84 miles?

(3) If 15 gallons of gin at 17*s.* 4*d.* be mingled with 28 at 18*s.* 7*d.* and 9 at 19*s.* 8*d.*, how must the mixture be sold so as to make a profit of 3*s.* 2*d.* on the average cost of a gallon?

(4) Divide £6 19*s.* 7*d.* into three sums, of which the second shall be 4 times as much as the first, and the third 5 times as much as the second.

(5) Divide £5 among A, B, C, so that B may receive half-a-sovereign more than A, and C half-a-guinea less than B.

(6) A wine-merchant pays £45 1*s.* 4*d.* for a quantity of spirits, at 17*s.* 4*d.* a gallon; but losing some of it by leakage, he sells the remainder at 18*s.* 6*d.* a gallon, and on the whole gains £1 12*s.* 11*d.* What quantity leaked out?

(7) Suppose that the whole amount of toll levied at Waterloo Bridge, in one day, for carriages and foot-passengers, was £12 14*s.* 2*d.*, the toll for a carriage being 2*d.*, and for a foot passenger ½*d.*; and that the amount from the carriages exceeded that from the foot-passengers by 12*s.* 6*d.*; how many foot-passengers and carriages were there?

(8) At a bazaar 30 fancy articles are sold at the average price of 3*s.* 7*d.* each, and the amount is divided among 23 men and 17 women, each man receiving thrice as much as each woman. What did each person receive?

(9) Multiply 3 mi. 5 fur. 21 po. 1 yd. 2½ ft. by 45.

Ex. 26.

(1) A wheel makes 880 revolutions in passing over 2 mi. 1430 yds.; what is its circumference?

(2) How much water must be mingled with 54 gallons of spirits at 9*s.* 7½*d.* and 63 at 8*s.* 11½*d.*, to reduce the price to 8*s.* 7¼*d.* a gallon?

(3) How many English ells of cloth, at 6*s.* 3*d.* per ell, should be given for 52½ yards of cloth worth 6*s.* 8*d.* per ell?

(4) Calculate $\frac{60° 3' 36'' \times 12\frac{1}{2} \times 40\frac{1}{4}}{2° 6' \times 6\frac{1}{2} \times 126\frac{1}{2}}$.

(5) Divide £2 7*s.* 3*d.* among 5 men, 9 youths, and 12 women, giving each man twice as much as a youth, and each youth half as much again as a woman.

(6) What is the value of 8 packages of silk, each package containing 9 parcels, each parcel 25 pieces, and each piece 63 yards, at the rate of 32*s.* for 5 yards?

(7) If 81 packages of cloth, each containing 8 parcels, each parcel 35 pieces, and each piece 84 yards, were re-sorted to 30 lots, each lot containing 7 packages, each package 9 parcels, and the parcels consisting of equal numbers of pieces, each 42 yards long; how many pieces would there be in a parcel?

(8) The total amount of wages earned in a week by a certain number of men, twice as many women, and thrice as many boys,

is £77 19*s.* 3*d.*; each man receives 3*s.* 8*d.*, each woman 1*s.* 8*d.*, and each boy 1*s.* 2*d.*, per day; but on Saturday they work only half the day. How many men, women, and boys are there?

(9) Divide 20 ac. 3 ro. 13 po. 24 sq. yds. between A and B, so that B's share may be $\frac{9}{10}$ as much as A's.

PROPORTION.

43. That relation between two quantities which signifies how often one of them is contained in the other is called the ratio of the quantities, or the ratio between them.

Thus, the ratio between 12 and 6 is the fractional form that expresses how often 12 contains 6, which is $\frac{12}{6}$ or $\frac{2}{1}$, or it is that which expresses how often 6 contains 12, which is $\frac{6}{12}$ or $\frac{1}{2}$. We might state the ratio between 12 and 6 by simply saying that one is twice the other, or that one is half the other; but the more usual mode of naming ratios being by means of two numbers, we should say that the ratio between 12 and 6 is $\frac{2}{1}$ or $\frac{1}{2}$.

If, instead of asking generally the ratio *between* 12 and 6, we ask the particular ratio of 12 *to* 6, then the answer is $\frac{2}{1}$, which may be read 2 to 1, and which may be written 2 : 1. Similarly, the ratio of 6 to 12 is $\frac{1}{2}$, or 1 : 2, which is read 1 to 2.

And any two other numbers of which one is double the other, as, for instance, 20 and 10, or 5 and $2\frac{1}{2}$, have, of course, the same ratio to one another as 12 and 6.

Again: the ratio between 12 and 8 is $\frac{12}{8}=\frac{3}{2}$, or $\frac{8}{12}=\frac{2}{3}$; that of 12 to 8 being $\frac{3}{2}$, or 3 : 2, and that of 8 to 12 being $\frac{2}{3}$, or 2 : 3.

And any two other numbers of which one is half as much again as the other, as, for instance, 18 and 12, or $7\frac{1}{2}$ and 5, have, of course, the same ratio to one another as 12 and 8.

We can easily obtain, then, a great variety of pairs of numbers, all expressing one and the same ratio. For suppose the ratio of 28 to 21, viz. $\frac{28}{21}$ or $\frac{4}{3}$, is to be expressed

by other pairs of numbers; we shall evidently obtain as many as we please by multiplying the upper and lower numbers of $\frac{4}{3}$ by any one number, as by 2, 3, 4, 5, &c. producing $\frac{8}{6}$, $\frac{12}{9}$, $\frac{16}{12}$, $\frac{20}{15}$, &c., all of which express the ratio of 28 : 21.

44. When we write $\frac{28}{21}=\frac{20}{15}$, we write two expressions for the same ratio, calling one of the expressions equal to the other. The four numbers thus arranged form what is called a *Proportion*, and may be presented also thus, 28 : 21 = 20 : 15, or, according to general practice, 28 : 21 :: 20 : 15; this is read 28 *is to* 21 *as* 20 *is to* 15; the meaning of which statement is, that 28 contains 21 just as often as 20 contains 15, viz., $1\frac{1}{3}$ times.

Suppose, then, that we have three terms of the above proportion, as thus, 28 : 21 :: (?) : 15, the 3*rd* term being not given, how may we find it? We know that 28 contains 21 as often as the required term contains 15; and we have accordingly to find what number contains 15 as often as 28 contains 21. Now the number of times that 28 contains 21 is $\frac{28}{21}$; therefore, taking so many times 15, we have $15\times\frac{28}{21}=\frac{15\times28}{21}=\frac{420}{21}=20$, the term required.

But it is generally the 4*th*, or last, term which is to be found; the first three being given. Let it be required, then, to complete the proportion 28 : 21 :: 20 : (?). Here the required number is to contain 20 as often as 21 contains 28. Now, 21 contains 28 the fractional part of a time expressed by $\frac{21}{28}$; therefore $20\times\frac{21}{28}=\frac{420}{28}=15$, the required 4*th* term.

Hence we derive the following *Rule* to find the 4*th* term of a proportion, when the first three terms are given:—Multiply the 3*rd* term by the 2*nd*, and divide the product by the 1*st*.

45. When the ratio of two concrete quantities is required, they must be expressed in one common denomination. Thus, the ratio of 3 *miles* to 6 *furlongs* is not that of 3 to 6, which would falsely denote the second distance to be double the first; but it is that of 8 *fur.* × 3 to 6 *fur.*, or of 24 : 6, or of 4 : 1, which truly denotes the second

distance to be one-quarter of the first. So, the ratio of 10 yards to 6 English ells is that of 4 qrs. × 10 to 5 qrs. × 6, or of 40 : 30, or of 4 : 3.

When two concrete quantities are in one denomination, the ratio between them is quite independent of the denomination, and is identical with that of the numbers taken abstractly; thus, the ratio of 24 *furlongs* to 6 *furlongs*, viz. $\frac{24\ fur.}{6\ fur.}$ is evidently $= \frac{24}{6}$, for the value of each of the expressions is 4 *times*. We may, therefore, at pleasure, assign any one denomination to the first two terms of an abstract proportion statement, and any to the last two; thus, the four following proportions all state virtually the same thing.

28 : 21	::	20 : 15.
28 *days* : 21 *days*	::	20 *tons* : 15 *tons*.
28 *men* : 21 *men*	::	£20 : £15.
£1 8*s*. : 1 *guinea*	::	1 *lb*. 4 *oz*. *Av*. : 15 *oz*. *Av*.

The correspondence of the last of these statements with each of the preceding is not obvious, until we have expressed its 1*st* and 2*nd* terms in shillings, viz. 28*s*. and 21*s*., and its 3*rd* and 4*th* terms in ounces, viz. 20 oz. and 15 oz.; then the statement becomes 28*s*. : 21*s*. :: 20 oz. : 15 oz.

46. Before applying to concrete quantities the Rule for finding the 4*th* term of a proportion, we must have the 1*st* and 2*nd* terms expressed in one common denomination, that we may be at liberty to convert them into abstract numbers, the 2*nd* to be used as a multiplier of the 3*rd* term, and the 1*st* as a divisor of the product. Whether the 3*rd* term also should be reduced, is a question of convenience, to be decided according as the 2*nd* term is, or is not, an easy multiplier for compound multiplication; but when the 3*rd* term is given concrete, we must not change it to an abstract number.

Ex. (1). Find the last term of the proportion in which the 1*st* term is £19 17*s.*, the 2*nd* is £165 8*s.* 4*d.*, and the 3*rd* is 3*ac.* 1*ro.* 32*po.*

£19 17*s.*	:	£165 8*s.* 4*d.*	::	3*ac.* 1*ro.* 32*po.*
20		20		4
3 *fourp.* × 397		3308		13
		3		40
		9925 *fourp.*		552 *po.*

$$\frac{552\,po. \times 9925}{3 \times 397} = \frac{46 \times 12 \times 1985 \times 5}{3 \times 397} = 46\,po. \times 100 = 4600\,po.$$

and 4600 *po.* reduced gives 28*ac.* 3*ro.* *Ans.*

In the above process, we have reduced the 1*st* and 2*nd* terms to the common denomination of *fourpences*, avoiding, however, the actual multiplication of the shillings in the 1*st*, as there are no given pence interfering, and as facility of cancelling may be promoted by reserving the factors 3 and 397. It is often, indeed, a good expedient to resolve large numbers into such factors as they are readily seen composed of, when we mean to try what cancelling may be employed; thus, since it is readily seen that 552 is divisible by 12, and 9925 by 5, we have resolved 552 into 46 × 12, and 9925 into 1985 × 5. We might have further resolved 1985 into 397 × 5, which would have exposed the cancelling factor 397.

Ex. (2). 1 *cwt.* 3 *qr.* 20 *lb.* : 1 *cwt.* 2 *qr.* :: £3 13*s.* 8¼*d.* : *what quantity?*

1*cwt.* 3*qr.* 20*lb.*	:	1*cwt.* 2*qr.*	::	£3 13*s.* 8¼*d.*
4		4		
7		28*lb.* × 6		
28				
216*lb.*				

$$\frac{£3\ 13s.\ 8\tfrac{1}{4}d. \times 28 \times 6}{216} = \frac{£3\ 13s.\ 8\tfrac{1}{4}d. \times 7 \times 24}{24 \times 9}$$

$$= £25\ 15s.\ 9\tfrac{3}{4}d. \div 9 = £2\ 17s.\ 3\tfrac{3}{4}d.\ \textit{Ans.}$$

Ex. (3). 3*lb.* 5¼*oz. Av.* : 8*lb.* 14*oz. Av.* :: 6*ft.* 8¾*in.* : *what quantity?*

$$\begin{array}{ccc} 3lb.\ 5\tfrac{1}{4}oz.\ Av. & : & 8lb.\ 14oz.\ Av. \\ 16 & & 16 \\ \hline 53\tfrac{1}{4}oz. & & 142oz. \end{array} :: \begin{array}{c} 6ft.\ 8\tfrac{3}{4}in. \\ 12 \\ \hline 80\tfrac{3}{4}in. \end{array}$$

$$\frac{80\frac{3}{4}in. \times 142}{53\frac{1}{4}} = \frac{323in. \times 142}{213} = \frac{323in. \times 2}{3} = 17ft.\ 11\tfrac{1}{3}in.\ \textit{Ans.}$$

Note.—When the first three terms are concrete quantities all of one kind, the 2*nd* and 3*rd* terms, being intended to be multiplied together, may, if we choose, be made to exchange places, without affecting the result. This will occasionally save some trouble of reduction.

Ex. (4). £24 16*s.* 6¾*d.* : 50 *guin.* :: 9*s.* 5½*d.* : *what sum?*

Here, reducing 1*st* and 3*rd* to farthings, and transposing 2*nd* and 3*rd*, we have

$$23835f. : 454f. :: 21s. \times 50$$

$$\frac{21s. \times 50 \times 454}{23835} = \frac{7s. \times 3 \times 50 \times 227 \times 2}{227 \times 5 \times 7 \times 3} = 20s.\ \textit{Ans.}$$

Ex. 27.

(1) 21 : 28 .: 15 : (what number?)
(2) 20 : 12 :: 85 : (what number?)
(3) 24 : 132 :: 26 : (what number?)
(4) 117 : 52 :: 17*s.* 3*d.* : (what sum?)
(5) 9 *men* : 24 *men* :: 15 *days* : (what time?)
(6) 51 : 30 :: 2*ft.* 10*in.* : (what length?)
(7) £27 15*s.* : £33 6*s.* :: 100 : (what number?)
(8) 9*s.* 6*d.* : 12*s.* :: £40 17*s.* : (what sum?)
(9) 10*cw.* 2*qrs.* 14*lbs.* : 4*cwt.* 2*lbs.* :: 14*s.* 10½*d.* : (what sum?)
(10) 61*s.* 4*d.* : 65*s.* 2*d.* :: 2*lbs.* 14*oz. Av.* : (what weight?)
(11) £97 3*s.* 4*d.* : £1 :: £21 17*s.* 3*d.* : (what sum?)
(12) 27*gal.* 3*qts.* : 6½ *pints* :: £27 15*s.* : (what sum?)
(13) £8 17*s.* 7½*d.* : £5 18*s.* 11½*d.* :: 23*cwt.* 2*qrs.* 3*lbs.* : (what weight?)
(14) 23° 13′ 31″ : 25 20′ 12″ :: 1604*mi.* 6*fur.* 206*yds.* : (what length?)

SIMPLE RULE OF THREE.

47. Under this Rule, which is often called *Simple Proportion*, are comprehended all arithmetical problems in which the data furnish the first three terms of a simple proportion, the required answer being the fourth term. But many Rule of Three questions that refer to certain subjects having peculiar features, such as Interest, Discount, &c., are commonly and conveniently treated under separate Rules.

It should be observed that Rule of Three problems can be solved, without proportion statement, by the first principles of Arithmetic, that is, by an immediate demonstrative application of multiplication and division. We shall exemplify both methods, and give directions for them.

48. *Directions for Simple Proportion Statement.*

Ask yourself what kind of thing the question requires you to find, name it, and write the given quantity of that kind as the *3rd* term, prefixing a double colon. Then ask yourself whether the required answer must (from the nature of the problem) be less, or more, than the *3rd* term; and if you see it must be *less*, write as the *2nd* term the *less* of the two other given quantities between which a comparison is proposed: if *more*, the *greater*. The other of those two quantities will form the *1st* term, a colon being placed between it and the *2nd*.

The *4th* term is now to be calculated as explained in the preceding article.

Ex. (1). A train, running at uniform speed, passes over 171 miles of railway in 6 hours; in what time will it run 285 miles?

What kind of thing is here required?—*Time.* Therefore, the given time, 6 hrs., will be the *3rd* term. Will the required time be less, or more, than 6 hours?—*More*; since 285 miles is more than the distance run in 6 hours. Therefore, 285 miles, the greater

of the two given distances, will be the 2*nd* term, leaving 171 miles for the 1*st* term.

$$171mi. : 285mi. :: 6hrs.$$

$$\frac{6hrs. \times 285}{171} = \frac{6hrs. \times 57 \times 5}{57 \times 3} = 10hrs. \quad Ans.$$

Ex. (2). If 27 men can do a piece of work in 16 days, how many such men would require 36 days to do it?

What kind of thing is here required?—*Men.* Therefore, 27 men will be the 3*rd* term. Will the required number of men be less, or more, than 27?—*Less*; because longer time is to be allowed. Therefore 16, the less number of days, will be the 2*nd* term, leaving 36 days for the 1*st* term.

$$36da. : 16da. :: 27\ men.$$

$$\frac{27m. \times 16}{36} = \frac{3m. \times 9 \times 16}{4 \times 9} = 3m. \times 4 = 12\ men. \quad Ans.$$

49. *Directions for the Method of First Principles.*

Ask yourself what kind of thing the question requires you to find, and state briefly in one line what is said to correspond to the given quantity of that kind; ending the line with that quantity preceded by (=), the sign of equality, and beginning the line with a phrase describing the quantity, such as 'No. of lbs.,' 'Cost,' 'Time,' &c.

Next, write a second line, in which the corresponding number on the left is altered to 1, and the quantity on the right is modified to suit that alteration.

Then, write a third line, in which the 1 on the left is altered to that given quantity which is to correspond to the required answer, and again modify the right hand expression to suit that further alteration. The result now arrived at represents the required answer.

The first of the preceding examples may be worked, according to this method (which is sometimes called

the *Unitary* method: *see Note* under Ex. 6, p. 43), as follows:—

No. of hours for 171 miles = 6*hrs.*
" " for 1 mile = $\frac{1}{171}$ of 6*hrs.*
" " for 285 miles = $\frac{285}{171}$ of 6*hrs.*

hence, $\frac{6hrs. \times 285}{171} = 10hrs.$, as before.

The second example may be worked as follows:—

No. of men required for 16 days = 27*m.*
" " for 1 day = 16 times 27*m.*
" " for 36 days = $\frac{1}{36}$ of 16 times 27*m.*

hence, $\frac{27m. \times 16}{36} = 12$ men, as before.

Ex. (3). If 23*lb.* 12*oz.* of black pepper be worth 33*s.* 3*d.*, what quantity is worth a guinea?

(i) *By Proportion Statement.*

$33\frac{1}{4}s. : 21s. :: 23lb.\ 12oz.$

$$\frac{23lb.\ 12oz. \times 21}{33\frac{1}{4}} = \frac{95lb. \times 21}{133} = 15lb. \quad Ans.$$

(ii) *By First Principles.*

No. of ounces for 399*d.* = 380*oz.*
" " for 1*d.* = $\frac{1}{399}$ of 380*oz.*
" " for 252*d.* = $\frac{252}{399}$ of 380*oz.*

hence, $\frac{380oz. \times 252}{399} = \frac{20oz. \times 21 \times 12}{21} = 240oz. = 15lb.$ *Ans.*

Ex. (4). If $6\frac{1}{2}lb.$ of bread cost a certain sum, when wheat is 63*s.* a quarter, what quantity of bread should be got for the same sum when wheat is 58*s.* 6*d.*?

(i) *By Proportion Statement.*

$58\frac{1}{2}s. : 63s. :: 6\frac{1}{2}lb.$

$$\frac{6\frac{1}{2}lb. \times 63}{58\frac{1}{2}} = \frac{13lb. \times 63}{117} = \frac{13lb. \times 7 \times 9}{13 \times 9} = 7lb. \quad Ans.$$

(ii) *By First Principles.*

Weight to be received, when wheat is at $63s.=6\frac{1}{2}lb.$

" " if wheat were at $1s.=63$ times $6\frac{1}{2}lb.$

" " when wheat is at $58\frac{1}{2}s.=\frac{1}{58\frac{1}{2}}$ of 63 times $6\frac{1}{2}lb.$

$$\text{hence, } \frac{6\frac{1}{2}lbs.\times 63}{58\frac{1}{2}}=\frac{13lb.\times 63}{117}=\frac{63lb.}{9}=7lbs. \quad Ans.$$

Ex. (5). How many English ells of cloth, at 6*s.* 3*d.* an ell, should be given for $52\frac{1}{2}$ yards of cloth at 5*s.* 4*d.* a yard?

Here, as ells are required, we may substitute $52\frac{1}{2}$ *ells* at 5*s.* 4*d.* *an ell* for $52\frac{1}{2}$ *yds.* at 5*s.* 4*d.* a yard, the amounts of both being the same.

(i) *By Proportion Statement.*

6*s.* 3*d.* : 5*s.* 4*d.* :: $52\frac{1}{2}$ *ells*

12 12

75 64

$$\frac{52\frac{1}{2}\ ells\times 64}{75}=\frac{105\ ells\times 32}{75}=\frac{224\ E.}{5}=44\ E.\ 4qrs. \quad Ans.$$

(ii) *By First Principles.*

$52\frac{1}{2}$ yds. at $64d. = 32d. \times 105.$

No. of ells for $75d. = 1$ *ell*;

" " " $1d. = \frac{1}{75}$ *ell*;

" " " $32d. \times 105 = \frac{1}{75} \times 32 \times 105$;

&c. as before.

Ex. 28.

(1) What is the cost of 25*lb.* of arrowroot, if 18*s.* is the cost of 15*lb.*?

(2) What quantity of rice can be got for 16*s.* when 63*lb.* cost 28*s.*?

(3) Find the amount paid for 65 yards of silk, at the rate of £9 for 26 yards.

(4) A coach-wheel makes 2388 revolutions in passing over 84 miles; how often would it revolve in passing over 91 miles?

(5) If 85 copies of a book cost 35*s.*, how many of them cost 28*s.*?

(6) What is the weight of $62\frac{1}{2}$ quarters of grain, if 75 quarters weigh 294*cwt.*?

(7) If a piece of work can be done by 24 men in 21 days, in what time could 28 such men do it?

(8) If the grazing of a sheep cost a certain fixed sum per day, how many sheep for 18 days' grazing would cost as much as 12 sheep for $13\frac{1}{3}$ days?

(9) If for a certain sum 18 sheep may be grazed 20 days, how many days may 30 sheep be grazed for the same sum?

(10) What do 105 oranges cost, when 13*s.* 9*d.* is given for 154?

(11) When apples are sold at $10\frac{1}{2}d.$ a dozen, what should I pay for 20?

(12) When apples are sold at the rate of 20 for 1*s.* $5\frac{1}{2}d.$, how many should I get for 1*s.* 9*d.*?

(13) If $22\frac{1}{2}$ quarters of wheat cost £57, what quantity costs £41 16*s.*?

(14) If $13\frac{1}{2}$*lbs.* of coffee cost 17*s.* 6*d.*, how much may be bought for 20*s.* 5*d.*?

(15) If a man do a piece of work in 15 days by working $10\frac{1}{2}$ hours a day, in how many days of 9 hours should he do it?

(16) If a train, running at uniform speed, pass over 17 miles 176 yds. of railway in 19 minutes, in what time will it have passed over 75 miles 1056 yards?

(17) What was my whole profit on an outlay of £3 13*s.* 8*d.*, if the proportion of profit on £2 19*s.* 6*d.* was a guinea?

(18) On an outlay of £4 7*s.* I lost 18*s.* $8\frac{3}{4}d.$, what amount of outlay would have incurred a corresponding loss of 19*s.* $4\frac{1}{2}d.$?

(19) A has pepper at 1*s.* 9*d.* a lb.; B has ginger at 1*s.* $5\frac{1}{2}d.$ If they should make an exchange, how much ginger should be given for $12\frac{1}{2}$*lb.* of pepper?

(20) A contractor finds that 256 navvies would be required to complete an excavation in the due time, if working $10\frac{1}{2}$ hours a day; but he can obtain only 224 navvies: how many hours a day must they work?

(21) If the worth of 6*oz.* 15*dwt.* 3*grs.* be £4 5*s.* $3\frac{1}{2}d.$, what is the corresponding worth of 2*oz.* 18*dwt.* 18*grs.*?

(22) Suppose I had formerly to pay £10 7*s.* 8*d.* income tax, being at the rate of 7*d.* in the £, and that I have now only £5 18*s.* 8*d.* to pay, on a return of the same income; what is now the rate per £? and what is the amount of my taxable income?

(23) If silver were worth 4*s.* $10\frac{1}{2}d.$ an ounce, and gold £3 17*s.* 9*d.*

an ounce, what would be the value of a bar of gold equal in weight to one of silver worth £3.5*s.* ?

(24) Reckoning silver worth 5*s.* 4*d.* an ounce, and gold worth 77*s.* 6*d.*, what would be the weight of a bar of gold equal in value to 33*lb.* 7*oz.* of silver ?

(25) If the weight of a mass of silver and that of a mass of gold of the same bulk are as 10 to 19, what is the weight of a bar of silver equal in bulk to a bar of gold weighing 28½ ounces ?

(26) I bought 3*cwt.* 2*qr.* 7*lb.* of goods, amounting, per invoice, to £15 10*s.* 6*d.*, and paid for freight and other charges £1 10*s.* 4*d.* At what must I sell the goods per cwt., so as to gain £2 15*s.* on the whole outlay ?

(27) A merchant's liabilities amount to £3868 5*s.*, and his assets to £902 11*s.* 10*d.* ; how much can he pay in the £ ?

(28) What is the amount of a bankrupt's estate, if it yields 3*s.* 9*d.* in the £, the amount of his debts being £2010 15*s.* 4*d.* ?

(29) If a bankrupt owes £15960 14*s.* 7*d.*, and his effects amount to £5382 13*s.*, how much will a creditor receive to whom he owes £488 11*s.* 10½*d.* ?

(30) What average daily expenditure for the year 1864 would have amounted to the same total as an average daily expenditure of 6*s.* 4¼*d.* for the year 1865 ?

(31) A garrison of 1600 men have provisions sufficient for 38 days ; if they be now reinforced with 490 more men, how long will the same provisions last ?

(32) If 1400 men have provisions that will serve for 38 days, how many of the men must be sent away, that the same provisions may serve, for the remaining men, for 56 days ?

(33) If a company of 380 men have provisions for 15 days, but 120 of the men are removed from the company at the end of 2 days, how long thereafter will the provisions last ?

(34) Suppose 27 men were engaged in a piece of work which they could perform in 40 days, and that, after they had wrought at it for 12 days, 8 men were added to their number ; how soon after this would the work be finished ?

(35) A certain sum of money is divided among 16 men 5 women and 13 boys, so that a man receives thrice as much as a woman, and a woman twice as much as a boy ; and it is observed that 6 men 4 women and 5 boys together receive £12 10*s.* 3*d.* What was the whole sum ?

COMPOUND RULE OF THREE.

50. This Rule, commonly called *Compound Proportion*, and often, but less properly, the *Double Rule of Three*, proposes to find the 4*th* term of a proportion, when the 1*st* and 2*nd* terms are each compounded of two or more quantities. Thus—

$$2cwt. \times 3 \times 7 \ : \ 5cwt. \times 8 \times 5\tfrac{1}{4} \ :: \ £2\ 10s. \ : \ (?)$$

is a Compound Rule of Three statement, the required 4th term of which may of course be found by multiplying the 3*rd* term by all the factors of the 2*nd* and then dividing by all the factors of the 1*st*. But the result is better worked out as follows :—

$$\frac{£2\ 10s. \times 5 \times 8 \times 5\frac{1}{4}}{2 \times 3 \times 7} = \frac{£2\ 10s. \times 5 \times 42}{42} = £12\ 10s.$$

For this Rule the method of Proportion Statement is generally more convenient than that of First Principles.

51. *Directions for Compound Proportion Statement.*

Determine and write the 3*rd* term, just as in Simple Rule of Three questions. Then, subdivide the problem into the two or more Simple Proportion questions of which it is composed, and, for each of these, ask whether the answer must be less or more, and state the 1*st* and 2*nd* terms accordingly.

The 4th term may now be calculated by multiplying the 3*rd* term by the continued product of the 2*nd* terms, and then dividing by that of the 1*st* terms. But the concise method, above exemplified, should generally be employed.

Ex. (1). If 18 men earn £36 12*s.* in 13 days, by working 10 hours a day, what number of men should earn £27 9*s.* in 15 days, by working 9 hours a day?

Here, a number of men is the thing wanted, and therefore the *3rd term is 18 men.* Now, the problem involves three distinct *ratios, each affecting* the result sought.

(i) So far as *the earnings* are concerned, the question is—If 18 men earn £36 12*s.*, how many men will earn £27 9*s.*?—*Fewer*; therefore, the 1*st* ratio is that of £36 12*s.* to £27 9*s.*

(ii) So far as *the days* are concerned, the question is—If 18 men earn a sum in 13 days, how many men will earn it in 15 days?—*Fewer*; therefore, the 2*nd* ratio is that of 15 days to 13 days.

(iii) So far as *the hours* are concerned, the question is—If 18 men earn a sum by working 10 hours a day, how many men will earn it by working 9 hours a day?—*More*; therefore, the 3*rd* ratio is that of 9 hours to 10 hours.

Accordingly, the statements are as follows:—

$$\left.\begin{array}{r} £36\ 12s. : £27\ 9s. \\ 15da. : 13da. \\ 9hrs. : 10hrs. \end{array}\right\} :: 18\ men.$$

And the ratio compounded of these ratios gives—

$$732s. \times 15 \times 9 : 549s. \times 13 \times 10 :: 18\ men.$$

which may be worked out thus:—

$$\frac{18m. \times 549 \times 13 \times 10}{732 \times 15 \times 9} = \frac{18m. \times 61 \times 9 \times 13 \times 10}{61 \times 12 \times 15 \times 9} = \frac{\overset{3}{\cancel{18}} \times 13 \times \overset{2}{\cancel{10}}}{\underset{2}{\cancel{12}} \times \underset{3}{\cancel{15}}}$$

$$= 13m.\ \textit{Ans.}$$

Ex. (2). If 24*lb.* 15*oz.* of bread can be bought for 5*s.* when wheat is 45*s.* a quarter, what sum should buy 14*lb.* 4*oz.* of bread when wheat is 2 *guineas* a quarter?

Here, price of bread is the thing wanted; so that 5*s.* will be the 3*rd* term. Then, we say—(i) If 5*s.* buy 24*lb.* 15*oz.*, what sum will buy 14*lb.* 4*oz.*?—*Less.* (ii) If bread cost 5*s.* when wheat is at 45*s.*, what will bread cost when wheat is at 2 *guineas*?—*Less.* Hence the following statement and operation:—

$$\left.\begin{array}{r} 24lb.\ 15oz. : 14lb.\ 4oz. \\ 45s. : 2gu. \end{array}\right\} :: 5s.$$

or, $399oz. \times 45 : 228oz. \times 42 :: 5s.$

$$\frac{5s. \times 228 \times 42}{399 \times 45} = \frac{5s. \times 57 \times 4 \times \overset{6}{\cancel{42}}}{\underset{7}{\cancel{399}} \times \underset{1}{\cancel{45}}} = \frac{4s. \times 6}{9} = 2s.\ 8d.\ \text{Ans.}$$

Ex. (3) Assuming that £42 1*s.* 8*d.* would gain £5 9*s.* 5*d.* in 4 years, what outlay will gain £23 8*s.* 3½*d.* in 7 years, at the same annual rate of profit?

Here, outlay being wanted, the 3*rd* term will be £42 1*s.* 8*d.* Then, we say—(i) If £42 1*s.* 8*d.* gain £5 9*s.* 5*d.*, what outlay will gain £23 8*s.* 3½*d.*?—*More.* (ii) If £42 1*s.* 8*d.* gain a certain sum in 4 years, what outlay will gain the same in 7 years?—*Less.* Hence we state and work as follows:—

$$\left.\begin{array}{r} £5\ 9s.\ 5d. : £23\ 8s.\ 3\tfrac{1}{4}d. \\ 7yrs. : \quad 4yrs. \end{array}\right\} :: £42\ 1s.\ 8d.$$

or, $4f. \times 1313 \times 7 : 22477f. \times 4 :: 10100d.$

$$\frac{10100d. \times 22477 \times 4}{4 \times 1313 \times 7} = \frac{101d. \times 100 \times 1\cancel{7}\cancel{2}\cancel{9} \times 13}{101 \times 13 \times \cancel{7}_{1}^{247}} = 24700d.$$

$$= £102\ 18s.\ 4d.\ \textit{Ans.}$$

Ex. (4). If 15 men, of one-half better than average expertness, can perform a piece of work in 13 days, how many men, of one-fourth better than average expertness, could perform a work half as great in a third of the time?

Here, before proceeding to state terms, we may simplify the given fractional quantities. First, in order to suit the fractions referring to expertness, viz. ½ and ¼, let us assume 4 degrees of ability to represent average expertness; then one-half better will be 6 degrees, and one-fourth better will be 5 degrees. Secondly, the comparison of one piece of work with another half as great may be represented by 2 measures of work and 1 measure. Thirdly, the comparison of 13 days with a third of 13 days may be represented by 3 periods and 1 period.

The question will now take the following form:—If 15 men of 6 degrees of ability can do 2 measures of work in 3 periods, how many men of 5 degrees of ability can do 1 measure of work in 1 period?—Hence the following statement:—

$$\left.\begin{array}{l} 5deg. : 6deg. \\ 2mea. : 1mea. \\ 1per. : 3per. \end{array}\right\} :: 15m. : \frac{15m. \times 6 \times 3}{5 \times 2} = 27\ men.\ \textit{Ans.}$$

Ex. (5). If 44oz. of gold be worth 623*oz.* of silver, and 267*oz of silver worth 210* bushels of wheat, and 147 bushels of wheat

worth £51 3*s.*, and 10*s.* 4*d.* be the value of 13 francs: How many francs should buy 3*oz.* of gold?

Here, a no. of francs being required, we make 13*fr.* the 3*rd* term. Then we say—(i) If 13*fr.*=124*d.*, how many fr. = 12*d.* ×1023?—*More.* (ii) If more francs = 147*bu.*, how many fr. = 210*bu.*?—Still *more.* (iii) If more francs = 267*oz.* silver, how many fr. = 623*oz.*?—Still *more.* (iv) If more francs = 44*oz.* gold, how many fr. = 3*oz.* gold?—*Less.* Hence the following statement:—

$$\left.\begin{array}{ll} 124d. & : 12d. \times 1023 \\ 147bu. & : 210bu. \\ 267oz. & : 623oz. \\ 44oz. & : 3oz. \end{array}\right\} ::13fr.: \frac{3 \quad 93 \quad 10 \quad 7}{13fr. \times \cancel{12} \times \cancel{1023} \times \cancel{210} \times \cancel{623} \times 3} \\ \cancel{124} \times \cancel{147} \times \cancel{267} \times \cancel{44} \\ 31 \quad 7 \quad 3 \quad 4$$

$$= 292\tfrac{1}{2}fr. \quad Ans.$$

Note.—Problems of this kind are very often worked by what is called the *Chain Rule*, according to which the above question would be stated as follows:—

How many francs? = 3*oz.* gold
when 44*oz.* gold = 623*oz.* silver
267*oz.* silver = 210*bu.* wheat
147*bu.* wheat = 12*d.* ×1023
and 124*d.* = 13 francs

The statement here is virtually the same as that given above; and the answer is found by dividing the continued product of the right hand by that of the left hand terms.

Ex. 29.

(1) If 6 men can plough 9 acres in 14 hours, how many men can plough 12 acres in 8 hours?

(2) How many acres could 10 men plough in 14 hours, if 5 men plough 6 acres in 10½ hours?

(3) What time would 10 men take to plough 5¼ acres, if 8 men would plough 9 acres in 15 hours?

(4) A trader's rate of profit is such, that with a capital of £1600 he can gain £48 in 5 months; in what time could he gain 60 guineas with a capital of £2800.

(5) If £100 gain £4 10*s.* in a year, what will be the corresponding gain on £265 in 219 days?

(6) If £34 11s. 8d. would gain £4 9s. 11d. in 4 years, what

would £15 16*s.* 8*d.* gain, at the same annual rate of profit, in 7 years?

(7) If £100 gain £4 5*s.* in a year, what capital will gain £3 17*s.* 6½*d.* in 60 days?

(8) Assuming that when wheat is 48*s.* a quarter, 22lb. 5oz. of bread should cost 4*s.* 8*d.*, what would be the price of wheat when 38½lbs. of bread should cost 8*s.* 2*d.*?

(9) If 37lb. 2oz. of bread can be bought for 8*s.* 8*d.*, when wheat is 52*s.* a quarter, what quantity of bread should be bought for 9*s.* 11*d.*, when wheat is 51*s.* a quarter?

(10) Supposing that the sixpenny loaf should weigh 5¾lb., if wheat were 4*s.* 4½*d.* a bushel, what should be the price of a 3½lb. loaf, if wheat were 4*s.* 9½*d.* a bushel?

(11) If 57 men must work 10 hours a day for 6 weeks to earn 411 guineas, how many hours a day must 20 men work to earn £119 17*s.* 6*d.* in 5 weeks?

(12) If £100 gain £3 15*s.* in a year, in what time will £243 6*s.* 8*d.* gain £10 5*s.*

(13) If 38 men, of 4 times ordinary expertness, can earn £263 in 5 weeks by working 8 hours a day, how many men, of 3 times ordinary expertness, can earn 535½ guineas in 12 weeks by working 10½ hours a day?

(14) If £40 10*s.* 8*d.* is requisite to provide 16 horses with hay for 8 weeks, when hay is 5*d.* a stone, what sum would be requisite to provide 15 horses for 10½ weeks, when hay is 6*d.* a stone?

(15) Of what weight per foot should 52 cubic feet of marble be, that the payment of £1 13*s.* for carrying it 27 miles may correspond to a payment of £2 3*s.* 9*d.*, for 39 miles carriage of 150 cubic feet of timber, weighing 3½ stone per foot?

(16) If 35 labourers, of one-half more than average ability, would be necessary to excavate 346½ cubic yards in 8¼ hours, of what ability must 15 labourers be, in order that they may excavate 102 cubic yards in 8½ hours?

(17) If 32 men dig a trench 126 yards long, 3¼ feet wide, and 4 feet deep, in 11¼ hours, how many men, more expert by one-half than the former, would dig a trench 117 yards long, 3½ feet wide, and 4½ feet deep, in 10 hours?

(18) If 48 men, of 3 times average ability, dig a trench 112 *yards long, 5 feet wide*, and 4½ feet deep, in 6¼ hours, what will *be the depth of a trench* 135 yards long, and 4 feet wide, which

can be dug in $7\frac{1}{2}$ hours by 72 men of one-fourth more than average ability?

(19) When 7lb. of tea are worth 17lb. of coffee, and $22\frac{1}{2}$lb. of coffee worth 59lb. of sugar, and $88\frac{1}{2}$lb. of sugar worth 18 pecks of flour, and 17 pecks of flour worth $47\frac{1}{2}$lb. of cheese, and 19lb. of cheese worth 21lb. of bacon, how many lb. of bacon are equivalent to 10lb. of tea?

(20) When $8\frac{1}{2}$lb. of tea are worth 18lb. of coffee, and 63 sides of bacon, each weighing on an average 55lb., are worth 649lb. of tea, what should be the average weight of a side of bacon, when 51 sides, of the same quality as the former, are worth 1180lb. of coffee?

(21) If 42 cubic feet of mahogany be equal in weight to 38 feet of oak, 45 of oak to 21 of Portland stone, 57 of Portland stone to 52 of marble, 28 of marble to 27 of granite, and 39 of granite to 35 of iron-stone, what weight of mahogany will be equal in bulk to a ton of iron-stone?

VULGAR FRACTIONS.

52. A FRACTION is an expression denoting one or more of any equal parts into which a unit is divided.

Thus, if unity is divided into 5 equal parts, 3 of these parts will be 3 *fifths* of unity, which is written $\frac{3}{5}$ of unity, or simply $\frac{3}{5}$; if £1 is divided into 5 equal parts, 3 of these parts will be 3 *fifths* of £1, which may be written $£\frac{3}{5}$.

53. It should be remembered that the form $\frac{3}{5}$ primarily denotes the quotient of 3 divided by 5: it is an example of a conventional way of representing a quotient, by writing the dividend over the divisor with a line between them. But, as $\frac{1}{5}$ similarly denotes 1 divided by 5, or a fifth of a unit, so 3 times 1 divided by 5, or $\frac{3}{5}$, must denote 3 *fifths* of a unit, as well as 1 *fifth* of 3 units. On this account, the lower number of a fraction is commonly called the *Denominator* (the naming term), as expressing the denomination of parts into which the unit is divided; and the upper number is called the *Numerator* (the numbering term), as expressing the *number of parts taken.*

54. *Vulgar Fractions*, that is, fractions of the common or general form, as $\frac{3}{5}$, is a name occasioned by the invention of a particular kind of fractions called *Decimals*.

55. A *Proper Fraction* is one whose numerator is less than the denominator. An *Improper Fraction* is one whose numerator is not less than the denominator. When the value of a fraction is less than 1, that is, when the result of the division signified by a numerator and denominator is not so much as 1, the fractional form is essential, and the expression is a proper fraction. When the value of a fraction is not less than 1, that is, when the quotient signified by a numerator and denominator equals or exceeds 1, the expression is not a proper fraction, and may be changed to another expression that is either not at all, or else not entirely, fractional.

Thus, $\frac{3}{7}$, $\frac{12}{4}$, $\frac{8}{8}$, $\frac{14}{5}$, are all called fractions, but only the first of them is a proper fraction, 7 not being contained in 3 so much as once, whereas the second fraction is $= 3$, the third $= 1$, the fourth $= 2\frac{4}{5}$.

A number partly integral, partly fractional, as $2\frac{4}{5}$, is called a *Mixed Number*, or a *Mixed Fraction*.

56. Any integer may have its value expressed in the form of a fraction, with any assigned denominator. Thus, since $3 = \frac{3}{1}$, if we multiply both numerator and denominator by 5, or 7, or 8, &c., &c., we shall have $\frac{15}{5}$, or $\frac{21}{7}$, or $\frac{24}{8}$, &c., that is $\frac{3 \textit{ fives}}{1 \textit{ five}}$, $\frac{3 \textit{ sevens}}{1 \textit{ seven}}$, &c., each of which is the same as $\frac{3}{1}$, or 3 divided by 1. (*See* § 30.)

Unity may evidently be expressed by $\frac{3}{3}$, $\frac{10}{10}$, &c., that is, by any fraction whose upper number is the same, or of the same value, as the lower.

Any mixed number may have its value expressed in a form entirely fractional, by changing the integral part into a fraction having the same denominator as the fractional part, and then combining the two parts. Thus, let it be required to convert $2\frac{4}{5}$ into an improper fraction.

$2 = \frac{2}{1} = \frac{10}{5}$; therefore, $2\frac{4}{5} = \frac{10}{5}$ together with $\frac{4}{5} = \frac{14}{5}$. This process suggests the Rule—Multiply the integer by the given denominator, adding to the product the given numerator; then, the sum written over the given denominator will be the improper fraction required.

Ex. 30.

(1) Express in proper form the fractions $\frac{16}{9}$, $\frac{13}{5}$, $\frac{10}{2}$, $\frac{12}{12}$, $\frac{80}{9}$, $\frac{136}{17}$.
(2) Reduce 1 to 10*ths*, 6 to 9*ths*, 8 to 7*ths*, and 18 to 18*ths*.
(3) Reduce to improper fractions $11\frac{2}{3}$, 15, $8\frac{11}{12}$, $17\frac{1}{7}$, $260\frac{17}{20}$.

Greatest Common Measure.

57. In fractional arithmetic, it is often requisite to obtain what is called the *Greatest Common Measure* of a numerator and denominator.

When one quantity is contained in another an exact number of times, the former is called a *measure* of the latter; thus, 7 is a measure of 84, for 7 is contained in 84 exactly 12 times; also, $2\frac{1}{2}$ is a measure of $22\frac{1}{2}$, for $2\frac{1}{2}$ is contained in $22\frac{1}{2}$ exactly 9 times.

When one quantity is contained in each of two others an exact number of times, it is called a *common measure* of the two; thus, 7 is a common measure of 84 and 98; also, $2\frac{1}{2}$ is a common measure of $22\frac{1}{2}$ and 30.

The G. C. M. (*greatest common measure*) of two quantities is the highest number that will exactly divide them both. Though 7 is *a* common measure of 84 and 98, we should find that 14 is *the greatest* common measure of them. Also, though $2\frac{1}{2}$ is *a* common measure of $22\frac{1}{2}$ and 30, we should find that $7\frac{1}{2}$ is *the greatest* common measure of them.

58. *To find the Greatest Common Measure of two whole numbers*:—

Rule.—Divide the greater number by the less, and use the remainder as a divisor of the first divisor; if a *remainder again* arises, use that as a divisor of the second

divisor, and so on, till there is no remainder. The divisor last used is the G. C. M. required.

Ex. (1). Find the G. C. M. of 527 and 1836.

527)1836(3
1581
255)527(2
510
Ans. 17)255(15
17
85
85

By the annexed process, which will be easily traced by means of the preceding Rule, the G. C. M. of the two given numbers is found to be 17.

In the ordinary form of working out the G. C. M. there is occasioned a waste of space, which might be avoided by writing each remainder under the preceding divisor, as in the second method of working the following Example :—

Ex. (2). What number is the G. C. M. of 143 and 182?

143)182(1
143
39)143(3
117
26)39(1
26
Ans. 13)26(2

Or thus :—

143	182	1
39	143	3
	117	
26	39	1
Ans. 13	26	2

To demonstrate strictly the Rule according to which the two preceding examples have been worked, is the province of Algebra rather than of ordinary Arithmetic. But the following illustration of the 1st example may suffice for the arithmetical student.

If 1836 be an exact multiple of 527 (that is, an exact no. of times 527), then 527 is evidently the G. C. M. of the two numbers; but, on dividing 1836 by 527, we find there is 255 in excess of an exact multiple of 527; but the G. C. M. must also measure the 255; therefore, the required G. C. M. *must be that of* 255 and 527. If 527 be an exact multiple *of 255, then 255 is* evidently the G. C. M. of 255 and 527;

but, on dividing 527 by 255, we find there is 17 in excess of an exact multiple of 255; therefore, the G. C. M. sought must be that of 17 and 255. If 255 be an exact multiple of 17, then 17 is the G. C. M. sought; and, on dividing 255 by 17, we find that 255 is an exact multiple of 17; therefore, 17 is the G. C. M. required: it will be found contained 31 times in 527, and 108 times in 1836.

59. When two numbers are divided by their G. C. M. the results are said to be *prime* to each other. A prime number is one that is not the product of any two integral factors greater than unity, as 31; and although 108 is, therefore, in itself, not a prime but a composite number, it is prime to 31, because there is no common measure of 31 and 108 greater than unity.

How to find the G. C. M. of two numbers, one or both of them being fractional, will be taught in a subsequent article (67).

60. To find the G. C. M. of three given numbers, first find that of any two of them, then take the result along with the remaining number, and find the G. C. M. of these two, which will be that of the three given numbers.

Ex. 31.

Find the Greatest Common Measure of—

(1) 24 and 30.	(2) 36 and 8.	(3) 21 and 34.
(4) 32 and 124.	(5) 48 and 64.	(6) 98 and 63.
(7) 169 and 208.	(8) 84 and 189.	(9) 245 and 105.
(10) 1008 and 1026.	(11) 219 and 1825.	(12) 3421 and 1866.
(13) 1225 and 2247.	(14) 2247 and 1391.	(15) 4305 and 52070.
(16) 4004 and 5772.	(17) 1924 and 200170.	(18) 859529 and 9394.

Least Common Multiple.

61. It is also often requisite, in fractional arithmetic, to obtain what is called the *Least Common Multiple* of two or more denominators.

A *common multiple* of two or more numbers is one number *that each* of them exactly measures; and their least

common multiple is the lowest number that each of them measures. Thus, 108 is *a* common multiple of 6, 9, and 4; but 72 is a lower common multiple of the same numbers; and 36 is a still lower common multiple of them. No number lower than 36 is exactly measured by 6, 9, and 4; and, therefore, 36 is said to be the L. C. M. (*least common multiple*) of 6, 9, and 4.

62. *To find the Least Common Multiple of two whole numbers.*

RULE.—Divide their product by their greatest common measure.

Ex. (1) Find the L. C. M. of 6 and 15.

Here, the G. C. M. of 6 and 15 being 3, we have

$$\tfrac{1}{3} \text{ of } 6 \times 15 = 6 \times 5 = 30. \quad \textit{Ans.}$$

To see the reason of this, observe, that 2×5, or 10, is of course a comm. mult. of 2 and 5, and that, as 2 and 5 are prime to each other, 10 is the *least* comm. mult. of 2 and 5; hence it easily appears that 3 times 10, or 30, must be the L. C. M. of 3 times 2 and 3 times 5, or of 6 and 15. In like manner, 7 times 10, or 70, will be the L. C. M. of 7 times 2 and 7 times 5, or of 14 and 35.

How to find the L. C. M. of two numbers, one or both of them being fractional, will be taught in a subsequent article (67).

63. To find the L. C. M. of more than two whole numbers, obtain the L. C. M. of the first two of them; then find the L. C. M. of that result and the third number; and so forth.

Ex. (2) Required the L. C. M. of 16, 28, 42, and 20.

G. C. M. of 16 and 28 = 4;

$\therefore$ L. C. M. of 16 and 28 $= \tfrac{1}{4}$ of $16 \times 28 = 112$;

G. C. M. of 112 and 42 = 14;

$\therefore$ L. C. M. of 112 and 42 $= \tfrac{1}{14}$ of $112 \times 42 = 336$;

G. C. M. of 336 and 20 = 4;

$\therefore$ L. C. M. of 336 and 20 $= \tfrac{1}{4}$ of $336 \times 20 = 1680$;

Or, the L. C. M. of the given numbers is 1680. *Ans.*

64. But when several numbers are thus proposed, the *following more* convenient method may be employed:—

Arrange the given numbers in a line, omitting any one that is an exact measure of one of the others. On the left of the line, before a dividing curve, place any factor common to two of the numbers. Then, divide each of the numbers by the G. C. M. of itself and the factor. From the new line thus formed strike out any number that is an exact measure of any of the others in that line, and on the left of the line place any factor common to two of its numbers, and proceed as before, until no two numbers have a common factor greater than unity. The continued product of all the numbers in the last line, and factors on the left of the lines, will be the L. C. M. of the given numbers.

Ex. (3) Find the L. C. M. of 18, 14, 20, 35, 16, and 42.

4)18. 20. 35. 16. 42
3)9. 5. 35. 4. 21
3. 35. 4. 7

Here we omit 14, as it exactly measures 42; then, placing 4 on the left, as being a factor of 20 and 16, we divide 18 by the G. C. M. of 4 and 18, viz. 2, then 20 by the G. C. M. of 4 and 20, viz. 4, then 35 by the G. C. M. of 4 and 35, viz. 1; and so forth.

$4 \times 3 \times 3 \times 35 \times 4 = 5040$. *Ans.*

Ex. 32.

Find the Least Common Multiple of—

(1) 12, 21.
(2) 36, 20.
(3) 81, 18.
(4) 9, 6, 27.
(5) 24, 8, 6.
(6) 16, 22, 24.
(7) 24, 21, 28.
(8) 12, 20, 32, 48.
(9) 35, 30, 84, 45.
(10) 12, 21, 15, 18.
(11) 25, 36, 33, 12, 45.
(12) 7, 9, 12, 28, 48, 62.
(13) 52, 25, 30, 65, 75.
(14) 34, 28, 12, 42, 51, 63.
(15) 10, 9, 39, 4, 27, 65.
(16) 18, 8, 28, 27, 24, 81, 91.
(17) 1, 2, 3, 4, 5, 6, 7, 8, 9.
(18) 12, 14, 16, 18, 20, 22, 24.

65. *To reduce a fraction to its lowest terms.*

RULE.—Divide the numerator and denominator by their G. C. M.

It appears from § 30 that a dividend and its divisor, and therefore the numerator and denominator of a fraction, *may be both* multiplied, or both divided, by any one number

without affecting the value of the quotient or fraction. Thus, $\frac{6}{2} = \frac{6\ dozen}{2\ dozen} = \frac{72}{24}$; and $\frac{54}{36} = \frac{6\ nines}{4\ nines} = \frac{6}{4}$; and $\frac{42}{54} = \frac{7\ sixes}{9\ sixes} = \frac{7}{9}$. A fraction, therefore, is reduced to its lowest terms, when its numerator and denominator have their G. C. M. struck out.

Ex. (1) Reduce $\frac{105}{280}$ to its lowest terms.

Here we need not resort to the Rule of G. C. M., as we can, by inspection, easily detect the factors common to the two terms. Dividing each of them by 5, and then each term of the resulting expression by 7, we have $\frac{105}{280} = \frac{21}{56} = \frac{3}{8}$. *Ans.*

Ex. (2) Reduce $\frac{5893}{6059}$ to its lowest terms.

5893	6059	1
166	5893	35
	5810	
83	166	2

Here, as no common factor can be readily perceived, we apply the Rule of G. C. M., and find that 83 is the highest factor; accordingly, dividing each term of the given fraction by 83, we find

$$\frac{5893}{6059} = \frac{71 \times 83}{73 \times 83} = \frac{71}{73}.$$ *Ans.*

Fractions should always be expressed in their lowest terms, when there is no necessity or expediency in having them otherwise.

Ex. 33.

Reduce to their lowest terms—

(1) $\frac{63}{70}$.	(2) $\frac{84}{108}$.	(3) $\frac{112}{96}$.	(4) $\frac{112}{147}$.
(5) $\frac{78}{126}$.	(6) $\frac{198}{253}$.	(7) $\frac{222}{185}$.	(8) $\frac{343}{588}$.
(9) $\frac{611}{799}$.	(10) $\frac{781}{891}$.	(11) $\frac{976}{366}$.	(12) $\frac{2147}{133}$.
(13) $\frac{5207}{6150}$.	(14) $\frac{347}{1326}$.	(15) $\frac{2370}{3397}$.	(16) $\frac{2345}{4221}$.
(17) $\frac{3551}{7685}$.	(18) $\frac{119144}{1113}$.	(19) $\frac{1053}{8991}$.	(20) $\frac{31119}{2806}$.

66. *To convert fractions to equivalent ones having a common denominator.*

Rule.—Take the L. C. M. of the denominators for the *new denominator* of each fraction; then multiply the

numerator of each fraction by the number of times its denominator is contained in the common denominator.

Ex. Reduce $\frac{5}{9}$, $\frac{7}{12}$, $\frac{1}{15}$, $\frac{7}{16}$, to equivalent fractions with a common denominator.

Here the L. C. M. of 9, 12, 15, 16, will be found to be 720. Therefore the multipliers of the numerators will be, respectively, $720 \div 9$, $720 \div 12$, &c., or, 80, 60, 48, 45, producing $\frac{400}{720}$, $\frac{420}{720}$, $\frac{48}{720}$, $\frac{315}{720}$. *Ans.*

It is obvious that each of the given fractions has thus had its terms both multiplied by the same number, and therefore has not been altered in value.

67. The process of reducing fractions to a common denominator is used in determining the order of magnitude of fractions, in comparing them, in finding their G. C. M. and L. C. M., and in the addition and subtraction of them.

Thus; if it be required to determine whether $\frac{15}{28}$ or $\frac{18}{35}$ is the greater fraction:—we find 140 the L. C. M. of 28 and 35; hence, the given fractions respectively equal $\frac{75}{140}$ and $\frac{72}{140}$, and it now appears that $\frac{15}{28}$ is the greater fraction. And, if it be required to compare the fractions, that is, to state in the simplest form the ratio of one to the other, we observe that the first is to the second as 75 to 72, or as 25 to 24.

Thus, also; if it be required to find the G. C. M. and the L. C. M. of $\frac{8}{9}$ and $\frac{10}{21}$:—reducing to common denominators gives $\frac{56}{63}$ and $\frac{30}{63}$; and the G. C. M. and L. C. M. of 56 and 30 being, respectively, 2 and 840, we have $\frac{2}{63}$ as the G. C. M. of the given fractions, and $\frac{840}{63}$, or $13\frac{1}{3}$, as their L. C. M.

Ex. 34.

Reduce to a common denominator—

(1) $\frac{5}{14}$, $\frac{11}{18}$, $\frac{10}{21}$. (2) $\frac{12}{25}$, $\frac{15}{26}$, $\frac{1}{80}$, $\frac{31}{39}$. (3) $\frac{10}{63}$, $\frac{11}{24}$, $\frac{12}{49}$, $\frac{13}{56}$.

Arrange in order of magnitude—

(4) $\frac{3}{4}$, $\frac{101}{134}$, $\frac{53}{71}$. (5) $\frac{5}{17}$, $\frac{75}{256}$, $\frac{61}{204}$. (6) $\frac{23}{86}$, $\frac{333}{1304}$, $\frac{1771}{7009}$.

Compare—

(7) $\frac{12}{25}$ and $\frac{11}{35}$. (8) $\frac{16}{27}$ and $\frac{52}{87}$. (9) $\frac{284}{561}$ and $\frac{655}{1394}$.

Find the G. C. M. of—

(10) $\frac{12}{25}$ and $\frac{16}{45}$. (11) $10\frac{1}{5}$ and 34. (12) $6\frac{7}{18}$ and $28\frac{5}{6}$.

Find the L. C. M. of—

(13) $13\frac{1}{8}$, 10, $3\frac{3}{4}$. (14) $9\frac{1}{3}$, $10\frac{2}{3}$, $24\frac{5}{9}$. (15) $\frac{7}{20}$, $\frac{4}{35}$, $\frac{9}{25}$.

ADDITION OF VULGAR FRACTIONS.

68. As a number of feet and a number of inches cannot be added together into one sum unless they are first made to be of one denomination, so fractions that are of different denominations of parts cannot be added till they are reduced to a common denominator.

RULE.—Reduce (if necessary) the given fractions to a common denominator; then add the numerators, and under the sum write the common denominator. If integers are also given, find their sum separately, and add it to whatever integer may arise from the addition of the fractions.

Ex. (1) $\frac{2}{9} + \frac{5}{9} = \frac{7}{9}$.

Ex. (2) $\frac{2}{9} + \frac{5}{6} = \frac{4}{18} + \frac{15}{18} = \frac{19}{18} = 1\frac{1}{18}$.

Ex. (3) Add together $3\frac{7}{8}$, $14\frac{5}{12}$, $\frac{1}{10}$, and $5\frac{13}{18}$.

The sum of the fractions is =

$$\frac{315}{360} + \frac{150}{360} + \frac{36}{360} + \frac{260}{360} = \frac{761}{360} = 2\frac{41}{360}.$$

The sum of the integers is $3 + 14 + 5 = 22$

Ans. $24\frac{41}{360}$.

Ex. 35.

Find the value of—

(1) $\frac{3}{8} + \frac{5}{18}$. (2) $\frac{5}{6} + \frac{1}{16}$. (3) $\frac{7}{16} + \frac{11}{12}$.

(4) $\frac{7}{8} + \frac{8}{9} + \frac{9}{10}$. (5) $\frac{3}{10} + \frac{3}{14} + \frac{17}{35}$. (6) $\frac{2}{11} + \frac{14}{19} + 1\frac{9}{17}$.

(7) $3\frac{1}{2} + 3\frac{3}{4} + \frac{2}{5} + \frac{1}{10}$. (8) $\frac{8}{9} + \frac{13}{15} + \frac{11}{12} + \frac{1}{20}$.

(9) $5\frac{3}{14} + 6\frac{9}{16} + 8 + \frac{13}{21}$. (10) $\frac{25}{52} + \frac{31}{63} + \frac{48}{49} + \frac{25}{39}$.

(11) $8\frac{10}{33} + 28\frac{15}{44} + \frac{5}{6} + \frac{4}{11}$. (12) $\frac{18}{25} + \frac{23}{45} + \frac{47}{60} + \frac{5}{18}$.

(13) $68\frac{7}{24} + 86\frac{9}{16} + \frac{17}{20} + 84\frac{3}{10}$. (14) $\frac{5}{57} + \frac{5}{26} + \frac{63}{76} + \frac{33}{65} + \frac{23}{60}$.

(15) $\frac{29}{132} + 9\frac{17}{198} + \frac{55}{144} + 8\frac{28}{33}$. (16) $\frac{71}{124} + \frac{72}{125} + \frac{73}{126} + \frac{81}{155} + \frac{61}{189}$.

Perform the addition of the following:—

(17) £	s.	d.	(18) £	s.	d.	(19) £	s.	d.	(20) £	s.	d.
8	13	$10\frac{5}{8}$	3	5	$6\frac{1}{6}$	5	12	$11\frac{3}{4}$	2	13	$4\frac{11}{16}$
3	17	$8\frac{1}{2}$	3	18	$10\frac{2}{3}$	2	6	$10\frac{1}{21}$	9	11	$5\frac{1}{4}$
4	9	$7\frac{7}{12}$	7	16	$8\frac{3}{4}$	4	6	$9\frac{7}{15}$	3	7	$6\frac{2}{3}$
6	10	$11\frac{5}{6}$	5	6	$7\frac{8}{9}$	0	15	$8\frac{5}{7}$	1	14	$9\frac{1}{12}$
1	7	$9\frac{3}{16}$	8	19	$11\frac{3}{10}$	0	18	$6\frac{3}{10}$	5	5	$7\frac{1}{8}$
0	8	$9\frac{5}{32}$	2	8	$9\frac{1}{2}$	7	9	5	2	6	$0\frac{3}{16}$

SUBTRACTION OF VULGAR FRACTIONS.

69. RULE.—Reduce (if necessary) the given fractions to a common denominator; then take the difference of the numerators, and under the difference write the common denominator.

Ex. (1) From $\frac{9}{20}$ subtract $\frac{3}{8}$.

Here 40 is the L. C. M. of the denominators; and, reducing the fractions to 40*ths*, we have $\frac{18}{40} - \frac{15}{40} = \frac{3}{40}$. *Ans.*

70. *To subtract a proper fraction from a whole number*, suppose one unit of the whole number to be expressed in the form of a fraction, having the same denominator as the fraction to be subtracted, and then annex to the remaining integer the difference of the fractions.

Ex. (2) From 75 subtract $\frac{3}{16}$.

Here 75 is $= 74\frac{16}{16}$; from which $\frac{3}{16}$ being taken, the remainder is $74\frac{13}{16}$. *Ans.*

71. *To subtract a proper fraction from a mixed number*, increase or diminish the whole number by the difference of the proper fractions, according as the difference of the fractions is in favour of the mixed number, or of the fraction to be subtracted.

Ex. (3) From $6\frac{5}{9}$ subtract $\frac{2}{5}$.

$$6\tfrac{25}{45} - \tfrac{18}{45} = 6\tfrac{7}{45}. \quad \textit{Ans.}$$

Ex. (4) From $6\frac{2}{5}$ subtract $\frac{5}{9}$.

$$6\tfrac{18}{45} - \tfrac{25}{45} = 6 - \tfrac{7}{45} = 5\tfrac{38}{45}. \quad \textit{Ans.}$$

72. *To subtract one mixed number from another*, first subtract the less integer, and then its annexed fraction.

Ex. (5) Subtract $13\frac{5}{12}$ from $60\frac{7}{8}$.

$$60\tfrac{7}{8} - 13\tfrac{5}{12} = 47\tfrac{21}{24} - \tfrac{10}{24} = 47\tfrac{11}{24}. \quad \textit{Ans.}$$

Ex. (6) Subtract $21\frac{7}{8}$ from $22\frac{5}{12}$.

$$22\tfrac{5}{12} - 21\tfrac{7}{8} = 1\tfrac{10}{24} - \tfrac{21}{24} = 1 - \tfrac{11}{24} = \tfrac{13}{24}. \quad \text{Ans.}$$

Ex. 36.

Find the value of—

(1) $\frac{5}{6} - \frac{2}{9}$. (2) $\frac{11}{18} - \frac{7}{12}$. (3) $\frac{9}{10} - \frac{1}{15}$.
(4) $\frac{7}{39} - \frac{7}{65}$. (5) $\frac{104}{105} - \frac{23}{35}$. (6) $\frac{20}{51} - \frac{14}{85}$.
(7) $17 - \frac{3}{20}$. (8) $1 - \frac{11}{19}$. (9) $300 - \frac{8}{17}$.
(10) $12\frac{1}{2} - \frac{1}{9}$. (11) $27\frac{5}{12} - \frac{4}{15}$. (12) $88\frac{11}{14} - \frac{17}{35}$.
(13) $18\frac{2}{3} - \frac{7}{8}$. (14) $20\frac{1}{9} - \frac{11}{18}$. (15) $4\frac{33}{34} - 3\frac{50}{51}$.
(16) $22\frac{3}{7} - 16\frac{1}{8}$. (17) $16\frac{1}{8} - 5\frac{3}{14}$. (18) $201\frac{3}{5} - 102\frac{6}{25}$.
(19) $1001\frac{3}{5} - 23\frac{112}{175}$. (20) $180\frac{1}{2004} - 99\frac{25}{26052}$. (21) $101\frac{1}{561} - 91\frac{3}{836}$.

Perform the subtraction of the following:—

	£	s.	d.		Cwts.	qrs.	lbs.		Ac.	ro.	po.
(22)	45	10	$9\frac{5}{6}$	(23)	27	2	17	(24)	13	1	$15\frac{4}{11}$
	25	17	$10\frac{3}{4}$		9	2	$22\frac{2}{9}$		6	3	$14\frac{7}{10}$.

MULTIPLICATION OF VULGAR FRACTIONS.

73. Rule.—Multiply the numerators together for the numerator of the product, and the denominators for its denominator.

This Rule is general, a whole number being regarded as a numerator over 1 as denominator, and mixed numbers being supposed to be reduced to improper fractions.

In multiplying any quantity by a fraction, we multiply by the numerator of that fraction, and divide by its denominator.

Ex. (1) Multiply 23 by $\frac{6}{7}$.

According to the Rule, we have $\frac{23}{1} \times \frac{6}{7} = \frac{138}{7} = 19\frac{5}{7}$; which may be explained thus:—Since $\frac{6}{7}$ signifies the *7th* part of 6, therefore, to multiply 23 by $\frac{6}{7}$ is to obtain the *7th* part of 6 times 23, which is $= \frac{23 \times 6}{7} = 19\frac{5}{7}$. *Ans.*

Note.—To multiply a fraction by a whole number, we, of course, multiply only the numerator; thus, $\frac{6}{7} \times 23 = \frac{6 \times 23}{7} = 19\frac{5}{7}$.

Ex. (2) Multiply $\frac{4}{5}$ *by* $\frac{6}{7}$.

Here we are to find the 7th part of 6 times $\frac{4}{5}$. Now 6 times $\frac{4}{5}$ is

$\frac{24}{5}$, and the *7th* part of $\frac{24}{5}$ will evidently be 24 parts each 7 times as small as a *5th*, that is $\frac{24}{35}$. Whence it appears that the product of one fraction by another is the product of the numerators written over that of the denominators.

But the product can often be obtained in simpler form by striking out or cancelling factors that are common to the upper and under lines.

Ex. (3) Multiply $\frac{6}{7}$ by $\frac{8}{9}$.

$$\frac{6}{7} \times \frac{8}{9} = \frac{\overset{2}{\cancel{6}} \times 8}{7 \times \underset{3}{\cancel{9}}} = \frac{16}{21}. \quad \textit{Ans.}$$

Ex. (4) Multiply $6\frac{3}{10}$ by $1\frac{1}{14}$.

Here it is expedient to reduce the given quantities to improper fractions before multiplying: $6\frac{3}{10} = \frac{63}{10}$, and $1\frac{1}{14} = \frac{15}{14}$; hence, $\frac{63 \times 15}{10 \times 14} = \frac{7 \times 9 \times 5 \times 3}{2 \times 5 \times 7 \times 2} = \frac{9 \times 3}{2 \times 2} = \frac{27}{4} = 6\frac{3}{4}$. *Ans.*

Ex. (5) Find the continued product of 27, $\frac{4}{7}$, $4\frac{1}{6}$, and $\frac{8}{15}$.

$$\frac{27}{1} \times \frac{4}{7} \times \frac{25}{6} \times \frac{8}{15} = \frac{27 \times 4 \times \overset{5}{\cancel{25}} \times 8}{7 \times \underset{3}{\cancel{6}} \times \underset{3}{\cancel{15}}} = \frac{3 \times 4 \times 5 \times 4}{7}$$

$= 34\frac{2}{7}$. *Ans.*

To multiply a fraction by the number which expresses its denominator, obtains the numerator as product. Thus, $\frac{7}{9} \times 9 = \frac{7 \times \cancel{9}}{\cancel{9}} = 7$.

74. Very frequently the word *of*, placed between two fractions, is used to signify the multiplication of the second by the first; thus, just as 4 *of* £5 shares amount to £5 × 4, so $\frac{4}{9}$ *of* $\frac{5}{8}$ means $\frac{5}{8} \times \frac{4}{9}$. The word is also used to indicate a continued product; thus, $\frac{2}{7}$ *of* $\frac{4}{5}$ *of* $\frac{3}{8}$ *of* $\frac{5}{9}$ may be written $\frac{2}{7} \times \frac{4}{5} \times \frac{3}{8} \times \frac{5}{9}$; the strict meaning is $\frac{5}{9} \times \frac{3}{8} \times \frac{4}{5} \times \frac{2}{7}$, but all the fractions here being abstract, the factors may be taken in any order; if, however, the last of the given quantities had been a concrete quantity, as £$\frac{5}{9}$, it would have claimed to be placed first when the word *of* was changed to the sign ×, *unless £1* were placed first, and the concrete fraction were made abstract.

Ex. **37.**

Find the value of—

(1) $\frac{4}{9} \times \frac{5}{7}$. (2) $\frac{7}{10} \times \frac{5}{8}$. (3) $\frac{6}{7} \times \frac{5}{9}$.
(4) $\frac{11}{12} \times \frac{8}{15}$. (5) $\frac{8}{15}$ of $\frac{9}{10}$. (6) $\frac{7}{9}$ of 12.
(7) $\frac{9}{16} \times 60$. (8) $\frac{11}{16} \times \frac{12}{13}$. (9) $\frac{18}{25}$ of $2\frac{1}{2}$.
(10) $2\frac{2}{3} \times 5\frac{1}{4}$. (11) $2\frac{2}{9} \times 9$. (12) $1\frac{2}{7}$ of $5\frac{5}{6}$.
(13) $12\frac{7}{9} \times 8$. (14) $5\frac{2}{15}$ of $11\frac{4}{11}$. (15) $4\frac{3}{28} \times \frac{49}{69}$.
(16) $\frac{7}{16} \times \frac{10}{27} \times \frac{22}{35}$. (17) $\frac{20}{21} \times \frac{35}{36} \times \frac{9}{25}$. (18) $5\frac{1}{5} \times \frac{34}{39} \times \frac{32}{51}$.
(19) $\frac{8}{9}$ of $\frac{9}{10}$ of $1\frac{1}{4}$. (20) $37 \times \frac{37}{48}$ of $\frac{32}{55}$. (21) $\frac{57}{75}$ of $7\frac{1}{2} \times 12\frac{1}{4}$.
(22) $2\frac{23}{26} \times 6 \times \frac{5}{9} \times 1\frac{23}{42} \times 1\frac{13}{50}$. (23) $758\frac{5}{6} \times 29\frac{2}{29}$.
(24) $32 \times \frac{32}{33}$ of $\frac{6}{7}$ of $17\frac{1}{9} \times \frac{83}{85}$ of $29\frac{3}{4}$. (25) $320\frac{37}{57} \times 38$.

DIVISION OF VULGAR FRACTIONS.

75. RULE.—Make the numerator and denominator of the divisor exchange places, and then proceed as in multiplication.

This Rule is general, a whole number being regarded as a numerator over 1 as denominator, and mixed numbers being supposed to be reduced to improper fractions.

In dividing any quantity by a fraction, we multiply by the denominator of that fraction, and divide by its numerator.

Ex. (1) Divide 23 by $\frac{6}{7}$.

By the Rule we have $\frac{23}{1} \div \frac{6}{7} = \frac{23}{1} \times \frac{7}{6} = 26\frac{5}{6}$. To illustrate this:—Since a dividend and its divisor quote the same value after being both multiplied by any one number, let us multiply both dividend and divisor here by 7; we have then to divide 23×7 by $\frac{6}{7} \times 7$, or 7 times 23 by 6, which gives $\frac{23 \times 7}{6} = 26\frac{5}{6}$. *Ans.*

Ex. (2) Divide $\frac{6}{7}$ by 23.

By the Rule, $\frac{6}{7} \div \frac{23}{1} = \frac{6}{7} \times \frac{1}{23} = \frac{6}{161}$. For, if we multiply both dividend and divisor by 7, we have then to divide 7 times $\frac{6}{7}$ by 7 times 23, or 6 by 161, which gives $\frac{6}{161}$. *Ans.*

Ex. (3) Divide $\frac{18}{25}$ by 6.

By the Rule, $\frac{18}{25} \div \frac{6}{1} = \frac{18}{25} \times \frac{1}{6} = \frac{3}{25}$. For, if we multiply both *dividend and divisor by 25,* we have then to divide 25 times $\frac{18}{25}$ by *25 times 6, or 18 by* 6×25, which gives $\frac{3}{25}$. *Ans.* So that, to

divide a fraction by a whole number that is a factor of the numerator, we may simply divide the numerator by the whole number.

Ex. (4) Divide $\frac{5}{9}$ by $\frac{6}{7}$.

By the Rule, $\frac{5}{9} \div \frac{6}{7} = \frac{5}{9} \times \frac{7}{6} = \frac{35}{54}$. For, multiplying both fractions by 9, we have 5 to be divided by 9 times $\frac{6}{7}$; then, multiplying dividend and divisor by 7, we have 7 times 5 to be divided by 9 times 6, which is $= \frac{5 \times 7}{9 \times 6} = \frac{35}{54}$. *Ans.*

Ex. (5) Divide $1\frac{5}{9}$ by $8\frac{13}{15}$.

Reducing to improper fractions, we have

$$\frac{14}{9} \div \frac{133}{15} = \frac{\overset{2}{\cancel{14}}}{\underset{3}{\cancel{9}}} \times \frac{15}{\underset{19}{\cancel{133}}} = \frac{2 \times 5}{3 \times 19} = \frac{10}{57}. \quad \textit{Ans.}$$

76. Under the present Rule may be included the reduction of a *complex* fraction to a simple form: a complex fraction being one that has for its numerator, or its denominator, or each of these, a fractional number; as

$$\frac{3\frac{1}{4}}{10}, \quad \frac{8}{5\frac{1}{2}}, \quad \frac{1\frac{2}{3}}{3\frac{3}{4}}, \quad \frac{\frac{7}{8}}{\frac{9}{10}}, \quad \frac{3\frac{1}{5}}{\frac{4}{7}}.$$

The meaning of these examples, and the simplification of them, may evidently be presented as follows:—

$$3\tfrac{1}{4} \div 10 = \tfrac{13}{4} \times \tfrac{1}{10} = \tfrac{3}{8}.$$
$$8 \div 5\tfrac{1}{2} = \tfrac{8}{1} \times \tfrac{2}{11} = \tfrac{16}{11} = 1\tfrac{5}{11}.$$
$$1\tfrac{2}{3} \div 3\tfrac{3}{4} = \tfrac{5}{3} \times \tfrac{4}{15} = \tfrac{4}{9}.$$
$$\tfrac{7}{8} \div \tfrac{9}{10} = \tfrac{7}{8} \times \tfrac{10}{9} = \tfrac{35}{36}.$$
$$3\tfrac{1}{5} \div \tfrac{4}{7} = \tfrac{16}{5} \times \tfrac{7}{4} = \tfrac{28}{5} = 5\tfrac{3}{5}.$$

Ex. 38.

Find the value of—

(1) $\frac{5}{8} \div \frac{4}{9}$. (2) $\frac{3}{8} \div \frac{5}{6}$. (3) $\frac{14}{15} \div \frac{21}{25}$. (4) $\frac{8}{9} \div 6$.

(5) $6\frac{1}{4} \div \frac{5}{6}$. (6) $5\frac{1}{4} \div 14$. (7) $14 \div 8\frac{3}{4}$. (8) $18 \div \frac{12}{13}$.

(9) $\frac{3}{7}$ of $\frac{8}{9}$ of $6 \div \frac{11}{24}$. (10) $\frac{7}{12}$ of $21 \div \frac{8}{13}$ of $8\frac{1}{4}$. (11) $3\frac{1}{5} \div \frac{7}{8}$ of $\frac{3}{11}$ of $3\frac{1}{7}$.

(12) $\frac{23}{28}$ of $2\frac{3}{5}$ of $\frac{11}{12} \div \frac{46}{49}$ of $6\frac{1}{2}$. (13) $\frac{19}{20}$ of $3\frac{2}{5} \div 1\frac{1}{10}$ of $\frac{34}{35}$ of 133.

(14) $\dfrac{20}{22\frac{1}{2}}$. (15) $\dfrac{3\frac{4}{7}}{10}$. (16) $\dfrac{6\frac{1}{3}}{8\frac{4}{9}}$. (17) $\dfrac{4\frac{7}{12}}{6\frac{3}{16}}$. (18) $\dfrac{\frac{7}{8}}{13}$.

(19) $\dfrac{1\frac{3}{25}}{2\frac{1}{10}}$ of $\dfrac{\frac{5}{7}}{1\frac{5}{9}} \div \dfrac{8}{8\frac{1}{6}}$. (20) $\dfrac{36\frac{2}{5}}{36\frac{1}{2}} \times \dfrac{17}{17\frac{1}{3}} \div \dfrac{2}{9}$ of $\dfrac{8\frac{1}{2}}{9\frac{1}{3}}$

SUPPLEMENT

TO

MULTIPLICATION AND DIVISION OF VULGAR FRACTIONS.

77. When a dividend is a mixed number of which the integral part is large, it is generally inexpedient to reduce it to an improper fraction before dividing.

Ex. (1) Divide $81073\frac{2}{7}$ successively by 12, 6, and $4\frac{1}{8}$.

$12)81073\frac{2}{7}$
$6)6756\frac{3}{28}$
$4\frac{1}{8})1126\frac{1}{56}$
$8 \quad 8$
$33)9008\frac{1}{7}$
Ans. $272\frac{75}{77}$.

Here, the first remainder is $1\frac{2}{7}$, and $\frac{1\frac{2}{7}}{12} = \frac{9}{84} = \frac{3}{28}$; the second remainder is $\frac{3}{28}$, which divided by 6 gives $\frac{1}{56}$. In the third division, after multiplying both dividend and divisor by 8 to render the divisor integral, the remainder is $32\frac{1}{7}$, which divided by 33 gives $\frac{75}{77}$.

78. When a multiplicand is a mixed number of which the integral part is large, it is generally inexpedient to reduce it to an improper fraction before multiplying.

Ex. (2) Multiply, separately, $5873\frac{23}{35}$ by 14, $11\frac{3}{8}$, and $8\frac{8}{9}$.

$5873\frac{23}{35} \times 14$
14
$82231\frac{1}{5}$ *Ans.*

$5873\frac{23}{35} \times 11\frac{3}{8}$
91
$8)534502\frac{4}{5}$
$66812\frac{17}{20}$ *Ans.*

$5873\frac{23}{35} \times 8\frac{8}{9}$
8
$9)46989\frac{9}{35}$
$5221\frac{1}{35}$
$52210\frac{2}{7}$. *Ans.*

In the first of the above sums, we take 14 times $\frac{23}{35} = \frac{23 \times 14}{35} = \frac{46}{5} = 9\frac{1}{5}$; we therefore set down $\frac{1}{5}$ and carry 9, &c. In the second sum, we take $11\frac{3}{8} = \frac{91}{8}$, and multiplying by 91 we have 91 times $\frac{23}{35} = \frac{23 \times 91}{35} = \frac{299}{5} = 59\frac{4}{5}$, &c. In the third sum we take first 8 times, then the *9th* of 8 times, and add the results to make $8\frac{8}{9}$ times; instead of this, since the given multiplier is equal to $9 - \frac{1}{9}$, *we might take first* 9 times and then $\frac{1}{9}$ of the given multiplicand, *and subtract the* latter result from the former.

79. We will now exemplify the manner of simplifying certain complex expressions.

Ex. (3) Simplify the expression $\frac{6\frac{7}{8} - \frac{1}{7}}{5\frac{2}{9} + 2\frac{5}{6}}$.

$$\frac{6\frac{49}{56} - \frac{8}{56}}{7\frac{4}{18} + \frac{15}{18}} = \frac{6\frac{41}{56}}{8\frac{1}{18}} = \frac{377 \times 18}{140 \times 56} = \frac{13 \times 9}{5 \times 28} = \frac{117}{140}. \quad \textit{Ans.}$$

Ex. (4) Simplify the expression

$$4\left(\frac{1}{2\frac{1}{2}} + \frac{1}{4\frac{1}{2}} + \frac{1}{7\frac{1}{2}}\right) \div \left(\tfrac{1}{2} + \tfrac{3}{4} + \tfrac{4}{7}\right).$$

Note.—It should be observed here, that the sign ( ), or { }, called a *bracket*, is used to denote the collective value of the whole expression within it; and that a number placed immediately before a bracketed expression, as 4 in the present example, denotes a multiplier of the enclosed value.

The given expression, then, is equal to

$$8\ (\tfrac{1}{5} + \tfrac{1}{9} + \tfrac{1}{15}) \div (\tfrac{1}{2} + \tfrac{3}{4} + \tfrac{4}{7}) = \tfrac{17}{45} \times 8 \div \tfrac{51}{28} = \tfrac{17}{45} \times \tfrac{8}{1} \times \tfrac{28}{51}$$
$$= \frac{8 \times 28}{45 \times 3} = \frac{224}{135} = 1\tfrac{89}{135}. \quad \textit{Ans.}$$

Ex. 39.

Find the value of—

1. $975\frac{3}{14} \div 9$.
2. $2794\frac{3}{5} \div 7$.
3. $5489\frac{12}{13} \div 11$.
4. $2679\frac{7}{19} \div 12$.
5. $473\frac{101}{119} \div 12$.
6. $5109\frac{11}{24} \div 16$.
7. $6007\frac{13}{15} \div 28$.
8. $4222\frac{1}{8} \div 88$.
9. $220\frac{9}{28} \div 93$.
10. $7365\frac{13}{27} \times 15$.
11. $495\frac{3}{49} \times 28$.
12. $1507\frac{14}{65} \times 64$.
13. $269\frac{3}{26} \times 8\frac{1}{8}$.
14. $1440\frac{17}{20} \times 8\frac{12}{13}$.
15. $783\frac{19}{105} \times 14\frac{7}{24}$.
16. $4567\frac{8}{9} \div 18\frac{3}{8}$.
17. $7017\frac{13}{36} \div 16\frac{25}{28}$.
18. $968\frac{18}{365} \div 30\frac{18}{65}$.
19. $\dfrac{21\frac{3}{7} + 4\frac{1}{2}}{24\frac{1}{8} - 7\frac{3}{7}}$.
20. $\dfrac{5\frac{13}{15} + \frac{2}{9} \text{ of } 17\frac{1}{3}}{\frac{3}{10} \text{ of } 63 + 5\frac{7}{10}}$.
21. $\dfrac{\frac{7}{16} \text{ of } 5\frac{1}{4} \text{ of } 12.}{\frac{2}{3}\,(8\frac{2}{3} - 1\frac{1}{5})}$
22. $\dfrac{4\frac{1}{2} - 9\frac{1}{2} \div \frac{4}{7}\,(18\frac{2}{5} + 4\frac{7}{8})}{(9\frac{1}{4} - \frac{2}{9}) \times (7\frac{3}{7} + \frac{12}{13})}$.
23. $\dfrac{10\frac{1}{2} + 5\frac{1}{8}\,(4\frac{1}{4} - 3\frac{1}{6})}{\frac{1}{3} - \frac{1}{4}\,\{\frac{1}{5} - \frac{1}{6}\,(\frac{1}{7} - \frac{1}{8})\}}$.
24. $\dfrac{\frac{8\frac{1}{3}}{11\frac{3}{4}} \text{ of } \frac{15\frac{1}{3}}{23\frac{3}{4}} \text{ of } \frac{29\frac{1}{8}}{32\frac{1}{6}} \text{ of } \left(\frac{3\frac{7}{8}}{3\frac{2}{7}} - \frac{5}{7} \text{ of } \frac{2\frac{1}{2}}{4\frac{1}{2}}\right)}{\{9 - 8 \div 7\,(6 - 5 \div 4\}}$.

80. We will now exemplify the multiplication and division of concrete quantities by vulgar fractions.

Ex. (1) Multiply £3 17*s.* $8\frac{7}{9}$*d.* by $12\frac{3}{4}$.

£	*s.*	*d.*	
3	17	$8\frac{7}{9}$	× $12\frac{3}{4}$
		$12\frac{3}{4}$	
4) 11	13	$2\frac{1}{3}$	
2	18	$3\frac{7}{12}$	
46	12	$9\frac{1}{3}$	

£49 11*s.* $0\frac{11}{12}$*d.* *Ans.*

Otherwise.

£	*s.*	*d.*	
3	17	$8\frac{7}{9}$	× $12\frac{3}{4}$
		51	
4) 198	4	$3\frac{2}{3}$	

£49 11*s.* $0\frac{11}{12}$*d.* *Ans.*

In the first of these forms of multiplication, we begin by taking $\frac{3}{4}$ of the given multiplicand, and to the result we add 12 times the multiplicand. In the second form, we use the multiplier as = the improper fraction $\frac{51}{4}$. It will be easy to determine which of these forms is to be preferred for any particular example. In the following example the first form is chosen, because the multiplier 28 is 7 times the multiplier 4.

Ex. (2) Multiply 21 yds. 2 ft. $11\frac{1}{2}$ in. by $28\frac{4}{11}$.

			$28\frac{4}{11}$	
11)	87	2	10	= 4 times the multiplicand.
	7	2	$11\frac{9}{11}$	= $\frac{4}{11}$ of the multiplicand.
	615	1	10	= 7 times the first product.

623yds.1 ft. $9\frac{9}{11}$ in. *Ans.*

Ex. (3) Divide £112 10*s.* $3\frac{3}{4}$*d.* by $5\frac{2}{5}$.

				5	
27	3)	562	11	$6\frac{3}{4}$	= the dividend ÷ $\frac{1}{5}$.
	9)	187	10	$6\frac{1}{4}$	= the dividend ÷ $\frac{3}{5}$.
Ans.		£20	16*s.*	$8\frac{25}{36}$	= the dividend ÷ $\frac{27}{5}$, or $5\frac{2}{5}$.

Ex. (4) Divide 37 ac. 2 ro. 19 po. $16\frac{1}{4}$ sq. yds. by $\frac{23}{35}$.

				35	
23)	1316	3	3	$24\frac{1}{4}$	

57 ac. 1 ro. 0 po. 5 yds. *Ans.*

Ex. 40.

Find the value of—

1. $\frac{4}{9}$ of £1; and $\frac{5}{8}$ of a yard.
2. $1\frac{11}{13}$ of 2 ac. 3 ro. 15 po.
3. £12 15*s.* $8\frac{1}{2}d.$ × $\frac{17}{32}$.
4. £49 15*s.* 3*d.* × $\frac{8}{81}$.
5. 3 mi. 5 fur. $21\frac{1}{2}$ po. × $6\frac{6}{7}$.
6. 23 gall. 3 qts. $2\frac{25}{26}$ pts. × $33\frac{4}{5}$.
7. £8 7*s.* $11\frac{8}{15}d.$ × $35\frac{5}{12}$.
8. 13 yrs. 37 da. $17\frac{7}{8}$ hrs. × $10\frac{9}{14}$.
9. £5 16*s.* $9\frac{3}{4}d.$ ÷ $\frac{5}{11}$.
10. 11 ac. 2 ro. $12\frac{6}{7}$ po. ÷ $1\frac{5}{13}$.
11. £370 9*s.* $3\frac{11}{27}d.$ ÷ $9\frac{1}{10}$.
12. 3 lbs. Tr. 10 oz. $17\frac{16}{35}$ dwt. ÷ $\frac{20}{21}$.
13. £16 0*s.* $3\frac{1}{8}d.$ ÷ $109\frac{1}{8}$.
14. 48 qu. 5 bu. $0\frac{17}{81}$ pk. ÷ $\frac{233}{240}$.
15. 6 fur. 17 po. 4 yds. $2\frac{2}{5}$ ft. × $20\frac{1}{7}$.
16. 2 ro. 39 po. 30 sq. yds. $2\frac{1}{8}$ sq. ft. × $6\frac{7}{8}$.

REDUCTION OF CONCRETE VULGAR FRACTIONS.

81. *To reduce a given concrete quantity to an equivalent fraction of another concrete quantity of like kind:*—

Rule. Write the former quantity as a dividend over the latter as a divisor; let both terms be expressed in the same denomination, and then simplify as may be required.

Ex. (1) Reduce $£\frac{5}{8}$ to the fraction of a guinea.

$$\frac{20s. \times \frac{5}{8}}{21s.} = \frac{20 \times 5}{21 \times 8} = \frac{25}{42} \text{ of a guin. } \textit{Ans.}$$

Here, according to the Rule, we have written $£\frac{5}{8}$, reduced to shillings, over a guinea reduced to shillings, and then discarding the common denomination of shillings, and simplifying, we have obtained $\frac{25}{42}$ as the equivalent fraction of a guinea. The procedure may be explained thus: 1*s.* = $\frac{1}{21}$ of a guin., ∴ £1 = $\frac{20}{21}$ of a guin., ∴ $£\frac{5}{8}$ = $\frac{20}{21}$ of a guin. × $\frac{5}{8}$; or thus $£\frac{5}{8}$ = $\frac{1}{8}$ of £5 = $\frac{1}{8}$ of $\frac{20 \times 5}{21}$ guin. = $\frac{20 \times 5}{21 \times 8}$ of a guin.

Ex. (2) Reduce 3*s.* $1\frac{1}{2}d.$ to the fraction of 6*s.* 8*d.*

1*d.* = $\frac{1}{80}$ of 6*s.* 8*d.*; ∴ $37\frac{1}{2}d. = \frac{37\frac{1}{2}}{80} = \frac{75}{160} = \frac{15}{32}$ of 6*s.* 8*d.* Ans.

Ex. (3) What fraction of $\frac{2}{3}$ of a ton is $5\frac{5}{8}$ of $\frac{7}{10}$ of a cwt.?

$$\frac{5\frac{5}{8} \text{ of } \frac{7}{10} \text{ cwt.}}{20 \text{ cwt.} \times \frac{2}{3}} = \frac{45 \times 7 \times 3}{20 \times 2 \times 80} = \frac{189}{640}. \quad \textit{Ans.}$$

Note.—It should be observed that by the present Rule we obtain the ratio of the two given quantities; thus, the ratio of $£\frac{2}{5}$ to 1 guinea is $\frac{25}{42}$, or 25 : 42; that of 3*s.* $1\frac{1}{2}$*d.* to 6*s.* 8*d.* is 15 : 32; &c.

Ex. 41.

Reduce

(1) $\frac{1}{8}$ of a penny to the fraction of a shill.; and $\frac{7}{12}$ guin. to the fr. of £1.

(2) $\frac{33}{35}$ of a yard to the fr. of a mile; and $\frac{5}{11}$ ac. to the fr. of a square yd.

(3) $1\frac{3}{17}$ of a crown to the fr. of £1; and $2\frac{2}{5}$ farthings to the fr. of a crown.

(4) $\frac{13}{16}$ of a moidore to the fr. of 1 guin.; and $\frac{5}{18}$ guin to the fr. of $\frac{1}{2}$ a crown.

(5) $\frac{24}{25}$ dwt. to the fr. of a lb. Tr.; and $\frac{1}{350}$ of a ton to the fr. of a lb. Av.

(6) $\frac{15}{26}$ of $\frac{1}{2}$ a sov. to the fr. of a moid.; and $1\frac{1}{9}$ guin. to the fr. of $\frac{1}{4}$ a sov.

(7) $\frac{14}{27}$ of an oz. Av. to the fr. of a cwt.; and $\frac{17}{96}$ of a lb. Tr. to the fr. of 1 gr.

(8) $\frac{3}{80}$ of 1 lb. Av. to the fr. of 1 lb. Tr.; and $\frac{7}{11}$ of 1 lb. Tr. to the fr. of 1 oz. Av.

(9) $£\frac{35}{81}$ to the fr. of 8*s.* 9*d.*; and $\frac{3}{8}$ of a crown to the fr. of 5*s.* 3*d.*

(10) $1\frac{4}{7}$ of $\frac{1}{2}$ a cr. to the fr. of £2 10*s.*; and $\frac{7}{8}$ of $\frac{1}{2}$ a moid. to the fr. of 18*s.* $9\frac{3}{4}$*d.*

(11) $\frac{31}{38}$ fur. to the fr. of 4950 yds.; and $2\frac{9}{20}$ mi. to the fr. of 2 mi. 330 yds.

(12) $1\frac{29}{45}$ Eng. ell to the fr. of $5\frac{1}{2}$ yds.; and $2\frac{3}{10}$ yds. to the fr. of 5 Eng. ells, 5 in.

(13) £3 10*s.* 6*d.* to the fr. of £6; and 18*s.* 5*d.* to the fr. of 15*s.* 7*d.*

(14) 1*s.* 7*d.* to the fr. of 7*s.* $1\frac{1}{2}$*d.*; and $\frac{3}{7}$ of 6*s.* 6*d.* to the fr. of a *guin.*

(15) 3 min. 37 sec. to the fr. of 11 wks. 5 da.; and $43\frac{4}{5}$ da. to the fr. of $3\frac{1}{2}$ yrs.

(16) 7 fur. 39 po. to the fr. of 2 p^n. $4\frac{1}{2}$ yds.; and $\frac{11}{700}$ mi. to the fr. of $327\frac{1}{4}$ in.

82. *To find the value of a concrete fraction :—*

RULE. Regard the upper number as concrete, the lower as an abstract divisor, and proceed in the ordinary manner of compound division.

Ex. (1) Find the values of $£\frac{5}{8}$ and $\frac{7}{9}$ of a guinea.

$\frac{5}{8}$ of a $£ = \frac{1}{8}$ of $£5 = \frac{1}{8}$ of $100s. = 12s.\ 6d.$ *Ans.*

$\frac{7}{9}$ of a guin. $= \frac{1}{9}$ of 7 guin. $= \frac{1}{9}$ of $147s. = 16s.\ 4d.$ *Ans.*

Ex. (2) Find the value of $£\frac{7}{12} + \frac{4}{9}s. + 1\frac{5}{6}d.$

$\frac{7}{12}$ of a $£ = \frac{1}{12}$ of $140s. = 11s.\ 8d.$; and $\frac{4}{9}s. = \frac{1}{9}$ of $48d. = 5\frac{1}{3}d.$; hence, $11s.\ 8d. + 5\frac{1}{3}d. + 1\frac{5}{6}d. = 17s.\ 3\frac{1}{2}d.$ *Ans.*

Ex. (3) Find the value of $2\frac{9}{10}$ guin. $- £1\frac{5}{6} + \frac{6}{7}s.$

	£	s.	d.
$\frac{9}{10}$ guin. $= \frac{1}{10}$ of $189s. = 18s.\ 10\frac{4}{5}d.$			
$2\frac{9}{10}$ guin. $= £2\ 2s. + 18s.\ 10\frac{4}{5}d.$ =	3	0	$10\frac{4}{5}$.
$\frac{6}{7}s. = \frac{1}{7}$ of $72d.$ =	0	0	$10\frac{2}{7}$.
	£3	1	$9\frac{3}{35}$.
$£\frac{5}{6} = \frac{1}{6}$ of $100s. = 16s.\ 8d.$; hence $£1\frac{5}{6}$ =	1	16	8
Ans.	£1	5	$1\frac{3}{35}$.

Ex. (4) Find the value of $\frac{22}{45}$ cwt. $- (2\frac{13}{20}$ lbs. $+ 10\frac{15}{16}$ oz.)

$\frac{22}{45}$ cwt. $= \frac{88}{45}$ qr. $= 1\frac{43}{45}$ qr. $= 1$ qr. 26 lbs. $12\frac{4}{45}$ oz.

$2\frac{13}{20}$ lbs. $= 2$ lbs. $10\frac{2}{5}$ oz.

$0 \quad 10\frac{15}{16}$

$= 0 \quad 3 \quad 5\frac{27}{80}$

Ans. 1 qr. 23 lbs. $6\frac{541}{720}$ oz.

Ex. 42.

Find the values of—

(1) $£\frac{3}{16}$; $\frac{5}{18}s.$; $\frac{3}{14}$ guin.; $£\frac{5}{6}$. (2) $£\frac{21}{36}$; $\frac{21}{40}$ hf. cr.; $5\frac{6}{11}s.$; $\frac{13}{21}$ moid.

(3) $\frac{7}{32}$ *cwt.*; $\frac{15}{16}$ dwt.; $2\frac{1}{6}$ gall. (4) $\frac{17}{21}$ ton; $\frac{47}{64}$ da.; $11\frac{5}{6}$ sq. yds.

(5) $\frac{11}{18}$ cub. yd.; $\frac{5}{11}$ mi.; $3\frac{8}{9}$ wks.
(6) $\frac{22}{25}$ Eng. ell; $3\frac{2}{9}$ sq. po.; $\frac{13}{20}$ leap yr.
(7) $\frac{5}{8}$ guin. + £$\frac{3}{64}$ + $\frac{15}{16}$s.
(8) $\frac{17}{56}$ hf. guin. + $\frac{17}{36}$ moid. + $17\frac{17}{18}$ hf. crs.
(9) $3\frac{13}{20}$ guin. + $25\frac{14}{33}$s. − $1\frac{8}{15}$ moid.
(10) £$3\frac{11}{54}$ + $\frac{11}{24}$ of ($\frac{14}{15}$s. − $\frac{7}{165}$ guin.)
(11) $\frac{7}{24}$ cwt. − $\frac{1}{64}$ ton + $\frac{1}{12}$ qr.
(12) $1\frac{7}{12}$ mi. − $\frac{3}{8}$ fur. + $\frac{4}{9}$ po. + $1\frac{1}{6}$ yd.
(13) $\frac{187}{487}$ Jul. yr. − $\frac{101}{250}$ min. − $5\frac{16}{35}$ da.
(14) $\frac{9}{28}$ ac. − $\frac{9}{16}$ ($\frac{1}{24}$ ro. − $\frac{4}{25}$ po.) − $\frac{9}{35}$ sq. yd.

DECIMAL FRACTIONS.

83. Decimal Fractions, or Decimals, are fractions whose denominators fit them for continuing the Numeration Table through local values less than unity, as in the example 4 thousands + 7 hundreds + 6 tens + 8 units + 3 tenths + 7 hundredths + 8 thousandths, where the local values decrease in decimal ratio from left to right: thousands ÷ 10 giving hundreds, which ÷ 10 give tens, which ÷ 10 give units, which ÷ 10 give tenths, which ÷ 10 give hundredths; and so forth.

The above mixed number, according to usual practice, is written briefly thus, 4768·378, an elevated point being inserted before the place which is intended for tenths, so that all values on the left of the point are integral, and all on the right of it fractional.

84. It will readily appear, that the fractional expression ·378, which denotes $\frac{3}{10} + \frac{7}{100} + \frac{8}{1000}$, may be read as $\frac{378}{1000}$, for $\frac{3}{10} = \frac{300}{1000}$ and $\frac{7}{100} = \frac{70}{1000}$, and therefore the sum of the three fractions is $\frac{300+70+8}{1000}$, or $\frac{378}{1000}$. In like manner, ·4706 denotes $\frac{4706}{10000}$, and 47·06 denotes $\frac{4706}{100}$.

85. Ciphers at the beginning of a decimal have the effect of lessening the values of the significant figures that

follow: — thus, whereas ·7 denotes 7 tenths, we have ·07=7 hundredths, ·007=7 thousandths, &c. But it is obvious that ciphers on the right of a decimal cannot affect the values of the figures preceding; for ·007 and ·0070 each signify 7 thousandths. Such ciphers, however, are sometimes retained, as in the expression 12·70 francs, where ·70 is significant of the franc being =100 centimes or cents; and they sometimes form a necessary part of what is called circulating decimals, as will be understood hereafter.

86. To express a decimal fraction in the form of a vulgar fraction, we take for the numerator the given figures that follow the point, discarding the point itself, and, for the denominator, 1 with as many annexed ciphers as there are figures in the numerator; thus, $\cdot 275 = \frac{275}{1000}$, $\cdot 0035 = \frac{0035}{10000}$, or $\frac{35}{10000}$, the prefixed ciphers being evidently not necessary in the vulgar form, though they helped to determine the denominator. The fractions here obtained happen to be reducible to lower terms: thus, $\frac{275}{1000} = \frac{11}{40}$, and $\frac{35}{10000} = \frac{7}{2000}$.

87. To multiply a decimal fraction by 10, 100, 1000, &c., we need only shift the decimal point one, two, three, &c., places to the right, that is, as many places as there are ciphers in the multiplier. Thus, $2\cdot7805 \times 10 = 27\cdot805$, for it is evident that in this product each figure has 10 times the value which it previously had: the 2 now means 2 tens, whereas it before denoted 2 units, &c. Thus also, $2\cdot7805 \times 1000 = 2780\cdot5$, where every figure is 1000 times as great in value as before. Similarly, $\cdot02367 \times 10000 = 236\cdot7$, and $\cdot02367 \times 1000000 = \cdot023670 \times 1000000 = 23670$.

88. To divide a decimal fraction by 10, 100, 1000, &c., we need only shift the decimal point one, two, three, &c., places to the left, that is, as many places as there are ciphers in the divisor. Thus, $2\cdot7805 \div 10 = \cdot27805$, where

each figure has become 10 times as little in value as it was before: the 2, which denoted 2 units, now denoting 2 tenths, &c. Thus also, $2{\cdot}7805 \div 100 = 02{\cdot}7805 \div 100 = {\cdot}027805$, where each figure has been lessened in value a hundredfold: the 2, which denoted units, now denoting hundredths of a unit, &c.

Ex. 43.

(1) Express in the vulgar form—

·7, ·309, ·39, 2·51, 15·2, ·365, 3·65, ·09, ·0304, 11·75, ·25, 6·5, ·001, ·0009, 4·08, 20·005, 201·4, ·000075, 144·0000032, 1·23456.

(2) Express in the decimal form—

$\frac{8}{10}$, $\frac{24}{100}$, $\frac{3}{10}$, $\frac{12}{10}$, $\frac{327}{1000}$, $\frac{37}{1000}$, $\frac{156}{10}$, $\frac{3130}{1000}$, $\frac{9185}{100000}$, $\frac{221000}{1000000}$, $\frac{360}{10}$, $\frac{19905}{1000}$, $\frac{17}{10000000}$.

(3) Multiply—

2·3456, separately, by 10 and by 1000.
·475, separately, by 10 and by 100.
31·081, separately, by 1000 and by 100000.
·001, separately, by 10 and by 10000.
·03575, separately, by 100 and by 10000000.

(4) Divide—

6070·2, separately, by 10 and by 1000.
60·702, separately, by 10 and by 1000.
·215, separately, by 100 and by 10000.
3654, separately, by 10 and by 10000.
·001, separately, by 100 and by 1000000.

ADDITION AND SUBTRACTION OF DECIMALS.

89. Rule.—Range the given quantities, one under another, so that the decimal points shall be in one vertical line, and units come under units, tenths under tenths, &c.; *then proceed with* the addition, or subtraction, as in-

whole numbers, treating any blank places on the right as if filled with ciphers, and prefixing the decimal point to the figure which is set down under the column of tenths.

Ex. (1) Add together 359·6, 35·964, 520, 43·7, ·876, 1·01.

359·6	Here the decimals are added as ·600 + ·964
35·964	+ ·700 + ·876 + ·010, which assigns to them
520·	the common denominator 1000, and that will
43·7	therefore be the understood denominator in the
·876	fractional part of the amount. The reason,
1·01	however, of the whole process is evidently that
961·150	on which the addition of simple integers depends, viz. the uniform ratio of the local values of the successive columns, ten of each column being equivalent to one of the next column on the left. The answer is the same as 961·15.

Ex. (2) From 120·31 subtract 1·7086.

120·31	Here the fractional part of the subtrahend is
1·7086	regarded as ·3100, and the subtraction is performed throughout just as with simple integers.
118·6014	

Ex. 44.

Find the value of—

(1) 2·091 + ·5093 + ·37 + 3·0865 + ·9749.
(2) 56·6794 + 5·76 + ·579 + 342·1 + 34·21.
(3) 80·808 + ·80808 + ·0075 + 65 + 9·70005.
(4) 178·8 + 365 + 63·079 + 0·87 + 212·251.
(5) 54·765 − 44·87; and 13·07 − 8·08.
(6) 100·101 − 3·1065; and ·3005 − ·009763.
(7) 8·14 + 38·124 − ·9175 − 16·28 + 46.
(8) 25·000315 − ·00045 + ·2801 − 16 + 21·001.

MULTIPLICATION OF DECIMALS.

90. Rule.—Multiply together the given factors as if they *were integral*, and then divide the product by 1 with

a number of annexed ciphers equal to the joint number of decimals in the factors, that is, take off from the right of the product a number of decimals equal to the joint number of decimals in the factors.

Ex. (1) Multiply 58·75 by 8·7.

58·75
8·7
41125
47000
511·125

Here we first suppose the factors to be 5875 and 87, that is, respectively 100 times and 10 times as great as the given factors; hence, as the product would be 100 × 10 or 1000 times as great as the one required, we divide it by 1000, that is, take off from the right a number of decimal figures equal to the joint number of decimals in the two factors.

Ex. (2) Multiply 2·406 by ·02406.

2·406
·02406
14436
57744
·05788836

In this example we easily obtain 24 times the multiplicand, by multiplying by 4 the product by 6 already obtained. Then, as we multiplied the factors as integers, which supposed the first to be multiplied by 1000 and the second by 100000, we divide the product by 1000 × 100000, or take off eight decimal places from the right.

Ex. 45.

Find, decimally, the value of—

(1) ·59 × ·7; and 5·99 × ·17.
(2) 3·406 × 5·4; and ·379 × 1·84.
(3) 2796 × ·85; and ·4321 × 900.
(4) 487·5 × ·0001996; and ·0365 by ·0064.
(5) ·19734 × 7250; and 1·00378 × ·0378.
(6) 12·3 × 1·23 × ·123; and 189 × ·189 × 1·89.
(7) 3·075 × 80 × ·15; and 895 × ·475 × ·004.
(8) ·0035147 × ·89758 × 125; and ·008 × ·019 × 250000.

DIVISION OF DECIMALS.

91. RULE.—If the given divisor is not integral, multiply it by 10, or 100, or 1000, &c., according as it contains one, two, three, &c., decimal places, and multiply the dividend by the same number; then proceed with the division, inserting the point in the quotient just before dividing the tenths of the dividend.

Ex. (1) Divide 4892·58 by 5.

5) 4892·580
978·516

Here the given divisor is integral; therefore, proceeding at once to divide by 5, we find the 5th part of 4892 units to be 978 units, with 2 units, or 20 tenths, remaining; the next portion of the dividend, therefore, is 20 + 5 tenths, and we accordingly place the decimal point in the quotient before dividing these tenths; then, the 5th part of 25 tenths is 5 tenths; the 5th part of 8 hundredths is 1 hundredth, and 3 hundredths or 30 thousandths remaining, the 5th part of which is 6 thousandths, which completes the quotient.

As ciphers on the right of a decimal make no change in its value, we annexed one cipher in the dividend to carry out the division.

Ex. (2) Divide 48·9258 by 1·65.

48·9258 ÷ 1·65
=4892·58 ÷ 165
165) 4892·580 (29·652
330
1592
1485
1075
990
858
825
330
330

Here the given divisor 1·65 is not integral, but becomes integral if multiplied by 100; therefore, multiplying both it and the dividend by 100, which will not affect the quotient, and which is done by removing the decimal point two places to the right, we have 4892·58 to be divided by 165; that divisor gives 29 for the integral part of the quotient, after which, therefore, a point is inserted, in order that the next quotient figure may indicate the *165th part of 1075*, that is, of 1075 tenths, &c.

Ex. (3) Divide 1·0044, successively, by 12 and 9.

12) 1·0044
9) ·0837
·0093

Here the divisor being integral, we say the 12th part of 10 tenths is 0 tenths, set down 0 with the decimal point prefixed; then, the 12th part of 100 hundredths is 8 hundredths and 4 over; the 9th part of 44 thousandths is 3 thousandths and 8 over, &c. Secondly the 9th part of 0 tenths is 0 tenths, the 9th part of 8 hundredths is 0 hundredths, the 9th part of 83 thousandths is 9 thousandths and 2 over, &c.

Ex. (4) Divide 74·8 by ·275.

$$74{\cdot}8 \div {\cdot}275 = 74800 \div 275 = 272. \textit{ Ans.}$$

In this case, the given dividend and divisor yield a purely integral quotient.

Ex. (5) Divide 3 by 8.

Here we must at the outset suppose decimal ciphers annexed to the dividend, to make the division practicable, 3 ÷ 8 being equal to 3·000 ÷ 8 = ·375. *Ans.*

Ex. (6) Divide ·741 and 631 each by 130.

When, as here, any integral divisor is a multiple of 10, both it and the dividend may first be divided by 10, 100, 1000, &c., according as there are one, two, three, &c., ciphers on the right of the divisor. In the present instance, therefore, we have ·741 ÷ 130 = ·0741 ÷ 13, and 631 ÷ 130 = 63·1 ÷ 13, and the division may be shown as follows:—

13) ·0741
·0057 *Ans.*

13) 63·1000
4.853846$\frac{2}{13}$. *Ans.*

Note.—In the latter of these divisions the quotient would never terminate *decimally*, and we have arbitrarily stopped at the place of millionths, where the remainder 2 being divided by 13 gives $\frac{2}{13}$ of a millionth part of unity. Decimals of this kind, interminate decimals as they are called, will be described more particularly in another article.

Ex. 46.

Find, decimally, the value of:—

(1) 407·281 ÷ 7; and 34·0056 ÷ 9.
(2) 5·8707 ÷ 11; and ·58707 ÷ 18.
(3) 7· ÷ 4; and 1207·8 ÷ 12.
(4) 15 ÷ 16; 1 ÷ 32; and ·15 ÷ 48.
(5) 281·5515 ÷ 8·9; and 352·872 ÷ ·87.
(6) ·014586 ÷ ·748; and ·000273 ÷ ·075.
(7) ·03 ÷ ·0008; 1 ÷ ·0064; and 11 ÷ 256.
(8) 7·03098 ÷ 6700; and 9 ÷ 144000.
(9) 40137 ÷ 13·6; and 12345 ÷ ·823.
(10) 175·68 ÷ 7; ·6594 ÷ 9; and ·00357 ÷ 1·1.
(11) 169 ÷ ·0000728; and ·27 ÷ 1·0465.

92. *To reduce Vulgar Fractions to Equivalent Decimals.*

RULE.—Divide the numerator by the denominator, annexing to the numerator as many decimal ciphers as are wanted to carry on the division.

Note.—This Rule has been anticipated in the preceding article, since the form of a vulgar fraction signifies the division of the numerator by the denominator. But, in connexion with the reduction of vulgar to decimal fractions, there are some peculiarities which yet remain to be pointed out and explained.

93. We have seen that, if the denominator of a vulgar fraction be 1, with any number of annexed ciphers, the numerator will at once supply an equivalent decimal fraction. And it is obvious that a vulgar fraction expressed in lowest terms, but not having 10, 100, 1000, &c., as its denominator, can be converted into a fraction *wholly* decimal only when 1 with some number of annexed ciphers is an exact multiple of its denominator. Thus $\frac{2}{25} = \frac{8}{100} = ·08$; and $\frac{7}{32} = \frac{35}{160} = \frac{175}{800} = \frac{875}{4000} = \frac{4375}{20000} = \frac{21875}{100000} = ·21875$.

Now, since the only factors into which 10 can be broken

up are 2 and 5, it is manifest that only those numbers which are divisible down to unity by using no other divisor than 2 or 5, can be exact measures of a number consisting of 1 with annexed ciphers. The fraction $\frac{27}{80}$, in which 27 and 80 are prime to each other, has a denominator equal to $2\times2\times2\times2\times5$; and these factors, respectively multiplied by 5, 5, 5, 5, and 2, severally become 10; therefore, multiplying both terms of the fraction $\frac{27}{80}$ by $5\times5\times5\times5\times2$, or 1250, we have $\frac{33750}{10000}=3375$. But the fraction $\frac{27}{70}$, which is also in its lowest terms, has for the prime factors of its denominator 2, 5, and 7; and as 7 is not an exact measure of 10, the decimal division of 27 by 70 will never terminate. If we stop at the fourth place of decimals, we shall have $\frac{27}{70}=3857\frac{1}{7}$; if at the seventh place, we shall have $\frac{27}{70}=\cdot3857142\frac{6}{7}$, a mixed decimal very nearly equal to ·3857143; but the quotient will never terminate without a vulgar fraction.

94. Decimals, whether terminating or interminate, are very often used approximately, that is, a portion of the decimal is cut off from the right hand, where the last place retained is deemed low enough for the degree of accuracy required; thus, the decimal equivalent to $\frac{7}{16}$ is $7\cdot0000\div16$, which is nearly $=\cdot4$, more nearly $=\cdot43$, still more nearly $=\cdot437$, and exactly $=\cdot4375$. Now, if perfect accuracy is not required, we may use one of the approximates ·4, ·43, ·437, instead of the perfect value ·4375; if we choose ·4, we sacrifice or neglect something between the 26th and 27th parts of the whole; if ·43, something between the 133rd and 134th parts; if ·437, the 2000th part.

It is important, however, to observe, that the right hand figure, in such partially adopted decimals, should have 1 added to it when the figures discarded begin with a value exceeding 5, as we shall thus more nearly approximate to *the truth.* If, for example, we use only two figures of the *decimal ·4375,* we should add 1 to the 3, making the value

·44, because the two following figures exceed 50; we shall thus take only ·0025 more than the exact value, whereas ·43 is ·0075 less than the exact value.

Ex. 47.

Reduce to decimals

(1) $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{8}$; $\frac{3}{16}$; $\frac{7}{20}$; $\frac{6}{25}$; $\frac{7}{800}$.

(2) $\frac{12}{75}$; $16\frac{71}{125}$; $3\frac{101}{256}$; $\frac{5100}{1088}$; $10\frac{13}{625}$; $\frac{24921}{3250}$.

(3) $3\frac{8}{9}$ of $1\frac{7}{32}$ of $2\frac{5}{14}$; $\frac{35\frac{3}{7}}{59\frac{1}{3}}$ of $\frac{\cdot 35}{3\cdot 3}$ of $\frac{7}{70\frac{5}{11}}$.

Reduce to ten-thousandths

(4) $\frac{27}{320}$; $\frac{5}{13}$; $\frac{2}{21}$; $\frac{4\frac{1}{2}}{8\frac{1}{4}}$ of $\frac{2}{7\cdot 3}$ of $143\frac{3}{8}$.

Reduce to approximate millionths

(5) $\frac{5}{7}$; $\frac{3}{11}$; $\frac{11}{600}$; $\frac{1000}{19}$; $\frac{1}{241}$; $\frac{151}{13}$; $1\frac{18}{25}$; $\frac{2\frac{1}{7}}{5\frac{3}{4}}$.

(6) $\frac{4\frac{4}{7}}{5\frac{5}{9}}$ of $\frac{\cdot 29}{48}$ of $\frac{\cdot 0017689}{\cdot 667}$; $\frac{5\frac{1}{2}}{7\frac{1}{6}}$ of ·54193.

CIRCULATING DECIMALS.

95. *Circulating Decimals*, called also *Recurring* or *Repeating Decimals*, are those in which there is a perpetual recurrence of the same figure, or period of figures. Thus $\frac{5}{9} = \cdot 5555$, &c.; $\frac{5}{6} = \cdot 83333$, &c.; $\frac{4}{11} = \cdot 363636$, &c.; $\frac{82}{675} = \cdot 12148148$, &c.

One or two head points are used to denote a repeating portion of a decimal; one point being placed over a recurring single figure, and one over each of the first and last figures of a recurring period of figures. The above decimals, therefore, will be concisely expressed thus, $\dot{\cdot}5$, $\cdot 8\dot{3}$, $\cdot\dot{3}\dot{6}$, $\cdot 1\dot{2}148\dot{}$.

If the recurring figure or period begins immediately after the decimal point, the decimal is called a *Pure Circulating Decimal*, as $\cdot\dot{5}$, $\cdot\dot{3}\dot{6}$; if otherwise, a *Mixed Circulating Decimal*, as $\cdot 8\dot{3}$, $\cdot 12\dot{1}\dot{4}\dot{8}$.

96. Any interminate decimal that is equivalent to a vulgar fraction is a circulating decimal.* Thus, the fraction $\frac{27}{70}$, being in its lowest terms, and having 7 as a factor in its denominator, is equal to an interminate decimal; and it is evident that if at any step, in the division of 27 by 70, a dividend occurs that has occurred before, the quotient will then repeat a preceding period of figures; for there is always the same figure, viz. a cipher, annexed to a remainder to carry on the division. Now, in dividing by 70, there cannot possibly arise more than 69 remainders all different: and therefore $\frac{27}{70}$ is equal to a circulating decimal of which the recurring portion consists of fewer figures than 70: by actual division we find $\frac{27}{70} = \cdot 3\dot{8}5714\dot{2}$.

Generally, then, any given vulgar fraction, in its lowest terms, whose denominator is not divisible down to unity without using some other factors than 2 or 5, may be reduced to a circulating decimal, having fewer figures in the period than the number which expresses the denominator.

Ex. 48.

Reduce to circulating decimals

(1) $\frac{4}{9}$; $\frac{1}{3}$; $\frac{7}{18}$; $\frac{5}{12}$. (2) $\frac{8}{11}$; $\frac{13}{41}$; $\frac{27}{110}$; $\frac{329}{1350}$.

(3) $\frac{29}{101}$; $\frac{172}{2710}$; $\frac{12077}{111000}$. (4) $\frac{562}{2079}$; $\frac{52337}{245700}$; $\frac{4}{239}$.

97. *To convert Circulating Decimals into their Equivalent Vulgar Fractions.*

RULE.—Under the figures of the given period, as an integral numerator, write a denominator consisting of as

* *Some interminate* decimals, being the values of surd roots, never *assume the circulating* form; as, for example, the square root of 2.

many *nines* as there are figures in the period; then simplify if necessary.

Ex. (1) Reduce $\cdot\dot{1}94\dot{7}$ to a vulgar fraction.

By the Rule, $\cdot\dot{1}94\dot{7} = \frac{1947}{9999} = \frac{649}{3333} = \frac{59}{303}$. *Ans.*

In a mixed circulating decimal, the fraction into which the recurring period is converted will be a fraction of tenths, hundredths, &c., according to the rank of the figure preceding the period.

Ex. (2) Reduce to a vulgar fraction $\cdot 3\dot{8}5714\dot{2}$.

Here, according to the Rule, $\cdot 3\dot{8}5714\dot{2} = \cdot 3\frac{8571428}{9999999} = 3\frac{6}{7}$ *tenths* $= \frac{1}{10}$ of $\frac{27}{7} = \frac{27}{70}$. *Ans.*

To become able readily to ascertain whether a large period has any common measure with the denominator of nines, the student should familiarize himself with the factors of such denominators. Thus, $99 = 9 \times 11$; $999 = 111 \times 9$, $= 37 \times 27$; $9999 = 909 \times 11 = 101 \times 11 \times 9$; $99999 = 41 \times 271 \times 9$; &c. And hence, if a numerator placed over 99 cannot be measured by 3 or 11, or if a numerator placed over 999 cannot be measured by 3 or 37, and so forth, the fraction is already in its lowest terms.

Ex. (3) Reduce to a vulgar fraction $\cdot 2340\dot{7}01\dot{1}$.

By the Rule, $\cdot 2340\dot{7}01\dot{1} = \cdot 234\frac{07011}{99999} = 234\frac{19}{271}$ thousandths $= \frac{1}{1000}$ of $\frac{63433}{271} = \frac{63433}{271000}$. *Ans.*

98. The reason of the Rule for the conversion of circulating decimals to vulgar fractions may be thus shewn:—

1st., Suppose it is required to find a vulgar fraction $= \cdot 6666$, &c.

$$\begin{aligned} \cdot 6666 \text{ \&c.} \times 10 &= 6\cdot 6666 \text{ \&c.} \\ \cdot 6666 \text{ \&c.} \times 1 &= \cdot 6666 \text{ \&c.} \\ \therefore \text{by subtraction } \cdot 6666 \text{ \&c.} \times 9 &= 6\cdot 0000 \\ \text{hence } \cdot 6666 \text{ \&c.} &= \tfrac{6}{9} = \tfrac{2}{3}. \end{aligned}$$

2ndly., Suppose it is required to find a vulgar fraction $= \cdot\dot{8}\dot{6}$.

$$\begin{aligned} \cdot 8686 \text{ \&c.} \times 100 &= 86\cdot 8686 \text{ \&c.} \\ \cdot 8686 \text{ \&c.} \times 1 &= \cdot 8686 \text{ \&c.} \\ \therefore \text{by subtraction } \cdot 8686 \text{ \&c.} \times 99 &= 86\cdot 0000 \\ \textit{hence } \cdot 8686 \text{ \&c.} &= \tfrac{86}{99}. \end{aligned}$$

Ex. 49.

Reduce to vulgar fractions

(1) $\cdot\dot{8}$; $\cdot\dot{2}\dot{4}$; $\cdot\dot{2}\dot{9}\dot{7}$; $\cdot\dot{2}4390\dot{6}$. (2) $\cdot35\dot{2}8\dot{2}$; $\cdot5\dot{7}07\dot{3}$; $\cdot\dot{7}14285$.

(3) $\cdot020\dot{4}87\dot{8}$; $\cdot00123\dot{4}5$. (4) $\cdot17\dot{8}57\dot{1}4\dot{2}$; $\cdot\dot{1}3698\dot{6}3\dot{0}$.

(5) $\cdot120\dot{0}345\dot{9}5$; $\cdot432\dot{0}00\dot{2}3\dot{4}$. (6) $\cdot3146\dot{6}\dot{4}1\dot{3}$; $\cdot00\dot{1}55\dot{9}6\dot{5}$.

99. In addition or subtraction of circulating decimals, when perfect accuracy is required, observe which of the given quantities has its period beginning at the greatest distance from the decimal point, and make all the others have the beginning of theirs at that distance; then express the periods as vulgar fractions, and the quantities will thus be prepared for being added in the usual way.

Ex. Add together $\cdot23\dot{6}\dot{9}$, $\cdot37\dot{5}9\dot{2}$, and $\cdot0687\dot{5}$.

Here the third quantity has the greatest number of non-recurring decimals, and the others may be conformed to it by shifting their head points one place to the right, thus, $\cdot23\dot{6}9\dot{6}$ and $\cdot375\dot{9}25$.

$$\begin{aligned} \cdot23\dot{6}9\dot{6} &= \cdot236\tfrac{32}{33}. \\ \cdot375\dot{9}25 &= \cdot375\tfrac{25}{27}. \\ \cdot0687\dot{5} &= \cdot068\tfrac{25}{38}. \\ &\overline{\cdot681\tfrac{194}{297}}. \quad \textit{Ans.} \end{aligned}$$

For most practical purposes, however, it will be sufficient to add or subtract circulating decimals approximately. Thus, if we desire to have 5 places of decimals in the answer, let us extend the given decimals to 7 places, which will generally ensure the accuracy of the 5th figure if we give to the 7th figure the nearest approximate value. The above example might accordingly be worked as follows:—

$$\begin{aligned} \cdot23\dot{6}\dot{9} &= \cdot2369697 \\ \cdot37\dot{5}9\dot{2} &= \cdot3759259 \\ \cdot0687\dot{5} &= \cdot0687576 \\ &\overline{\cdot6816532} \end{aligned}$$

Ans. ·68165.

100. But, in multiplication and division of circulating

decimals, the given periods should generally be converted into vulgar fractions.

Ex. (1) Multiply $3{\cdot}01464435$ by $\cdot\dot{7}1\dot{7}0$.

$3{\cdot}01464435 = 3{\cdot}0\frac{35}{239} = \frac{1}{10}$ of $30\frac{35}{239} = 3\frac{7}{478}$; also, $\cdot\dot{7}1\dot{7}0 = \frac{2390}{3333}$;

$\therefore \frac{1441}{478} \times \frac{2390}{3333} = \frac{131 \times 5}{303} = \frac{655}{303} = 2\frac{49}{303} = 2{\cdot}1\dot{6}1\dot{7}$. *Ans.*

Ex. (2) Multiply $23{\cdot}95\dot{2}\dot{8}$ by $5{\cdot}5$.

$$\begin{array}{r} 23{\cdot}95\frac{28}{99} \times 5\frac{1}{2} \\ 11 \\ \hline 2)\,263{\cdot}48\frac{1}{9} \\ \hline 131{\cdot}74\frac{1}{18} = 131{\cdot}7405. \end{array}$$

Ans.

Here, the first step in the multiplication is 11 times $\frac{28}{99} = \frac{28}{9} = 3\frac{1}{9}$; set down $\frac{1}{9}$ and carry 3; &c.

Ex. 50.

Find (as far as the 6th place of decimals) the value of

(1) $8{\cdot}30\dot{9} + 7{\cdot}0\dot{3} + 13{\cdot}764\dot{3}7\dot{5} + {\cdot}884510\dot{4}0\dot{3}$.
(2) ${\cdot}75361 + 1{\cdot}09\dot{3}7\dot{5}0\dot{4} + 8{\cdot}9\dot{3}6\dot{4} + {\cdot}7\dot{6}5\dot{7}$.
(3) ${\cdot}849\dot{0}\dot{3} - {\cdot}7\dot{4}\dot{8}$; and $3{\cdot}2915\dot{6}4\dot{8} - {\cdot}3921\dot{2}$.

Find the value of

(4) $3{\cdot}\dot{7} \times {\cdot}\dot{5}$; ${\cdot}\dot{1}\dot{2} \times 8$; $7{\cdot}4\dot{5}\dot{5} \times {\cdot}7\dot{4}\dot{5}\dot{5}$; and $\cdot\dot{1}2\dot{3}\dot{4}5\dot{6}7\dot{8} \times 18{\cdot}\dot{2}$.
(5) ${\cdot}2\dot{3}5\dot{8}70\dot{3} \times {\cdot}\dot{0}1\dot{4}3\dot{5}$; and $2{\cdot}\dot{1}69\dot{0}0\dot{0} \times 5{\cdot}131\dot{2}\dot{1}$.
(6) $24{\cdot}\dot{6} \div 18{\cdot}5$; $51{\cdot}\dot{8}\dot{1} \div 13{\cdot}\dot{0}0\dot{9}$; and $5{\cdot}1642\dot{5} \div 10{\cdot}\dot{4}$.
(7) $57{\cdot}211\frac{3}{7} \div 63{\cdot}20\dot{7}$; and $8{\cdot}\dot{0}0\dot{0}01\dot{2} \div {\cdot}235$.

REDUCTION, ETC., OF CONCRETE DECIMALS.

101. *To reduce a given concrete quantity to an equivalent decimal of another concrete quantity of like kind:—*

General Rule.—Write the former quantity as a dividend, over the latter as a divisor; let both terms be expressed in the same denomination; then simplify as may be required, and reduce to the decimal form.

Ex. (1) Reduce $£\frac{5}{8}$ to the decimal of a guinea.

$$\frac{20s. \times \frac{5}{8}}{21s.} = \frac{20 \times 5}{21 \times 8} = \frac{25}{42} = \cdot\dot{5}952380\dot{} \text{ of a guinea. } \textit{Ans.}$$

Ex. (2) Reduce $3s.\ 1\frac{1}{2}d.$ to the decimal of $6s.\ 8d.$

$$\frac{3s.\ 1{\cdot}5d.}{6s.\ 8d.} = \frac{37{\cdot}5d.}{80d.} = \frac{3{\cdot}75}{8} = \cdot 46875. \textit{ Ans.}$$

Ex. (3) Express $15s.\ 8\frac{3}{4}d.$ as the decimal of £1.

$$\frac{15s.\ 8{\cdot}75d.}{20s.} = \frac{188{\cdot}75d.}{12d. \times 20} = \frac{9{\cdot}4375}{12} = \cdot 7864583\dot{} \text{ of £1. } \textit{Ans.}$$

Note.—When, as in this last example, a compound concrete quantity is to be reduced to a higher denomination, the following method is in most instances a better one.

Special Rule.—Express and annex the right hand quantity as a decimal of the denomination on its left; express and annex what is now the right hand quantity in the same manner, and proceed thus till the whole of the given expression is converted into the single denomination proposed.

According to this Rule, the last example may be worked as follows:—

12) 8·75*d.*
20) 15·729166$\dot{6}$*s.*
£·7864083$\dot{3}$

Beginning with the quantity of lowest denomination, viz. $8\frac{3}{4}d.$, which in the decimal form is 8·75*d.*, we find 8·75*d.* ÷ 12 = ·729 &c. shillings; then, 15·729 &c. shillings ÷ 20 = 1·5729 &c. ÷ 2 = *Ans.*

Ex. (4) Express, decimally, 3 tons 13 cwt. 15 lbs. $11\frac{3}{8}$ oz. in the denomination of cwts.

16) 11·6 oz.
28 { 4) 15·725 lbs.
7) 3·93125
4) 0·5616071$\frac{3}{7}$ qrs.
Ans. 73·1404017$\frac{6}{7}$ cwt.

Here we avoid expanding the number of quarters as a circulating decimal, because we have still to divide by 4. We might easily convert the answer here given into the circulating form, but in many instances an approximate answer, as 73·1404, or 73·140402, or 73·1404018, would be sufficient. The last of these approximations exceeds the *truth by only the 70-millionth* part of a cwt.

Ex. 51.

Reduce

(1) 11*s.* 6*d.*, 16*s.* 10½*d.*, and £1 7*s.* 8½*d.*, each to a decimal expression in the denomination of £.

(2) 15*s.* 11*d.*, £3 13*s.* 1¾*d.*, and 18*s.* 10⅝*d.*, each to the dec. of £1.

(3) 4*s.* 6¼*d.*, and 6*s.* 11⅜*d.*, each to the dec. of a crown.

(4) 2*s.* 7½*d.* to the dec. of £5; and £1 12 3¾*d.* to the dec. of a moidore.

(5) 20*s.* 6⅝*d.* to the dec. of a guinea; and 5¾*d.* to the dec. of 2*s.* 6*d.*

(6) 4*s.* 8*d.* to the dec. of 8*s.* 9*d.*; and 8*s.* 9*d.* to the dec. of 12*s.* 1½*d.*

(7) 13*s.* 3¾*d.* to the dec. of £17 15*s.*; and 10*s.* 9⅕*d.* to the dec. of 2*s.* 2⅝*d.*

(8) £2 13*s.* 4½*d.* to the dec. of £1, and also to the dec. of 2½ guineas.

(9) 4½*d.* to the dec. of £10; and 47½ guin. to the dec. of £47 10*s.*

(10) 9 oz. 11 dwt. to the dec. of 1 lb.; and 33 dwt. 15 grs. to the dec. of 1 oz.

(11) 16 lbs. 9 oz. Av. to the dec. of 1 cwt.; and 3 qrs. 17 lbs. 13 oz. to the dec. of 1 ton.

(12) 10½ lbs. to the dec. of 1 cwt.; and 21 oz. Av. to the dec. of 1 lb. Tr.

(13) 11 cwt. 3 qrs. to the dec. of 12½ tons; and 9 cwt. 75 lbs. to the dec. of 7⅗ cwt.

(14) 54 yds. 2 ft. 6½ in. to the dec. of 1 mile, and also to that of 13 yds. 2¼ ft.

(15) 19 hrs. 15 min. 18 sec. to the dec. of 1 day, and also to that of 22 hrs. 56 min. 2 sec.

(16) 3 ro. 36½ po. to the dec. of 1 acre, and also to that of 1 acre 33 po.

(17) 23 sq. yds. 7 ft. 100 in. to the dec. of 1 sq. yd., and also to that of 1 sq. po. 2½ sq. yds.

(18) 13 bu. 3·7 pks. to the dec. of 1 qu., and also to that of 5 bu. 5⅓ *pks.*

102. *To find the value of a concrete decimal.*

RULE.—Multiply the given decimal by the number of units in the next lower denomination equivalent to one unit of the given denomination, and divide the product by the understood denominator of the given decimal, viz. 1 with annexed ciphers; then reduce the remainder (if any) to the next lower denomination, and divide again by the same divisor, and so on.

Ex. (1) Find the value of ·53875 of a £.

£·53875
20
10·77500*s.*
12
9·30000*d.*

Ans. 10*s.* $9\frac{3}{10}$*d.*

Here we first multiply by 20, to reduce £ to shill., and then divide by 100000, or simply point off five places from the right, which gives 10*s.* and a remainder of 77500*s.* to be divided by 100000; we next multiply 77500*s.* by 12 to obtain pence, and again dividing by 100000, or simply pointing off five places from the right, we have 9·3 pence; so that the required value is 10*s.* 9·3*d.* It will be observed that we have thus reversed the process of reducing 10*s.* $9\frac{3}{10}$*d.* to the decimal of £1.

Ex. (2) Find the value of ·1765625 of £4 13*s.* 4*d.*

£·1765625 × $4\frac{2}{3}$
14
3) 2·4718750
£·8239583$\frac{1}{3}$
20
16·479166$\frac{2}{3}$*s.*
12
5·750000*d.* = $5\frac{3}{4}$*d.*;

The meaning of the given expression is £$4\frac{2}{3}$ × ·1765625 which is found = £·8239583$\frac{1}{3}$; and this decimal of a £ is reduced to shillings thus, 20 times $\frac{1}{3}$ = $6\frac{2}{3}$, to be carried to the product of the other figures, &c.,

therefore the *Ans.* is 16*s.* $5\frac{3}{4}$*d.*

Ex. (3) Find the value of ·3713$\dot{8}$ of £1.

£·3713$\frac{8}{9}$
20
7·42777$\frac{7}{9}$*s.* = 7·42$\frac{7}{9}$*s.*
12
5·13$\frac{1}{3}$*d.* = $5\frac{13\frac{1}{3}}{100}$ = $5\frac{2}{15}$*d.*

Ans. 7*s.* $5\frac{2}{15}$*d.*

Ex. (4) Find the value of $\cdot 59137\frac{1}{11}$ sq. pole.

$\cdot 59137\frac{1}{11}$ sq. po. to be multiplied by $30\frac{1}{4}$, or $\frac{121}{4}$ sq. yds.

$$6\cdot 50508 \times \frac{11}{4} = 17\cdot 88897 \text{ sq. yds.}$$

$$\times 9$$

Ans. 17 sq. yds. 8·00073 sq. ft.

Ex. 52.

Find the value of

(1) ·375 of £1; ·765625 of £1; 1·809375 of £1.
(2) ·095 of £1; ·028125 of £1; 9·6890625 of £1.
(3) ·2345 of £5; $2\cdot 817\frac{5}{8}$ of 15*s.*; 2·375 of a moidore.
(4) ·085 of a hf. crown; ·8513 of a hf. sov.; 9·917 of 6*d.*
(5) 26·5 of 3*s.* 4*d.*; ·0265 of 2*s.* $4\frac{1}{2}$*d.*; 3·65 of 23*s.* 9*d.*
(6) 1·00515 of £3 12*s.* 6*d.*; $\cdot 789\frac{5}{7}$ of £5 10*s.* $7\frac{1}{2}$*d.*
(7) ·8325 of a cwt.; ·08745 of a ton; ·3275 of a lb. Av.
(8) 7·334375 acres; £$10\cdot 917\frac{3}{5}$; 3·1095 yds.
(9) ·985 of an Eng. ell; $7\cdot 78\frac{3}{4}$ of a gall.; ·31694 of an acre.
(10) ·211648 mi. in yards; and £3·675 in guineas.
(11) ·40985 of 3 cwt. 1 qr. 10 lbs.; $4\cdot 1734\frac{1}{6}$ of a lb. Troy.
(12) $\cdot 10\dot{3}\dot{7}$ of £1; $\cdot 0\dot{9}1\dot{6}$ of a crown; $1\cdot 00\dot{5}$ of a guin.
(13) $\cdot 0\dot{4}6\dot{2}9\dot{6}$ of 6 guin.; $3\cdot 4\dot{0}1\dot{1}$ of 14*s.* 8*d.*
(14) $\cdot 8\dot{7}1382\dot{1}$ of £1 10*s.* 9*d.*; $\cdot 7\dot{6}34\dot{0}0\dot{8}$ of 3 miles.

103. *Miscellaneous Examples Worked Out.*

(1) If $\frac{3}{7}$ of a ship be worth £1701, what share of the ship is worth £882?

£1701 = value of $\frac{3}{7}$ of the ship;
∴ £1 = value of $\frac{1}{1701}$ of $\frac{3}{7}$ of it;
∴ £882 = value of $\frac{882}{1701}$ of $\frac{3}{7}$ of it;
$= \frac{126}{567} = \frac{2}{9}$. *Ans.*

(2) What decimal multiplied by the sum of $\frac{1}{8}$, $\frac{4}{15}$, and $\frac{6}{35}$ will produce ·4?

$$\frac{4}{10} \div \left(\frac{5}{15} + \frac{4}{15} + \frac{6}{35}\right) = \frac{2}{5} \div \left(\frac{3}{5} + \frac{6}{35}\right)$$
$$= \frac{2}{5} \times \frac{35}{27} = \frac{14}{27} = \cdot 5\dot{1}8. \quad \textit{Ans.}$$

(3) From 5 acres subtract 1 ac. 3 ro. 39 po. 30 sq. yds. 1 sq. ft. 71 sq. in.

ac.	ro.	po.	yds.	ft.	in.
5	0	0	0	0	0
1	3	39	30	1	71
3	0	0	0	7	73
				6	108
3	0	0	0	0	109

In this example there occurs 31 yards to be subtracted from $30\frac{1}{4}$ yds. which produces a *minus* quantity of $\frac{3}{4}$ of a yard = 6 ft. 108 in.; the proper form of the remainder therefore is 3 ac. 109 in. *Ans.*

(4) If by spending $\frac{3}{11}$ of my money, and also $\frac{4}{7}$ of the remainder, I had only 6*d.* left; what sum had I at first?

$\frac{3}{11}$ of the money being spent, the remainder was $\frac{8}{11}$ of it;
then, $\frac{4}{7}$ of the remainder being spent, $\frac{3}{7}$ of that remainder was left;
and $\frac{3}{7}$ of $\frac{8}{11}$, or $\frac{24}{77}$ of the original sum, is by the question = 6*d.*
$\therefore$ $\frac{1}{77}$ of the original sum = $\frac{1}{4}d.$; or the whole of the original sum = $\frac{77}{4}d. = 19\frac{1}{4}d. = 1s.\ 7\frac{1}{4}d.$ *Ans.*

(5) If by investing all my money in a speculation I gained a sum equal to $\frac{5}{9}$ of the investment, and if then by spending $\frac{3}{8}$ of the whole amount I had 20 guineas remaining; what sum had I at first?

The proceeds were equal to $1\frac{5}{9}$ of the investment, and $\frac{3}{8}$ of that being spent leaves $\frac{5}{8}$ of $1\frac{5}{9}$, or $\frac{35}{36}$, of the investment, which by the question is £21.

$\therefore$ if $\frac{35}{36}$ of the investment = £21,
$\frac{1}{36}$ of it = $\frac{1}{35}$ of £21,
and the whole investment = $\frac{36}{35}$ of £21,
= $\frac{36}{5}$ of £3 = £21 12*s.* *Ans.*

(6) If 23 degrees $20\frac{8}{9}$ min. correspond to $1612\frac{2}{3}$ miles, to how many miles do 30·141 such degrees correspond?

23 deg. $20\frac{8}{9}$ min. : 30·141 deg. :: $1612\frac{2}{3}$ mi.
9 9
210 8 = $210\frac{2}{15}$ deg. 14514
15

3152 deg. : 30·141 deg. × 15 :: 14514 mi.
: 2081 mi. 6 ft. $32\frac{3}{5}$ po. *Ans.*

(7) If 3 men, or 5 women, can do a piece of work in $11\frac{2}{5}$ hours, in what time could 4 men and 6 women do it?

3 men = 5 women, or 1 man = $1\frac{2}{3}$ woman, or 4 men = $6\frac{2}{3}$ women; hence the question is virtually the following—If 5 women take $11\frac{2}{5}$ hours, how many hours would $6\frac{2}{3} + 6$ women require?

$12\frac{2}{3}$ women : 5 wom. :: $11\frac{2}{5}$ hrs. : $4\frac{1}{2}$ hrs. *Ans.*

(8) If 22 men build a wall, $46\frac{3}{8}$ ft. long and $11\frac{1}{7}$ feet high, in 9 days of $8\frac{5}{6}$ hours; how many men, that can severally do in $3\frac{1}{4}$ hours as much work as each of the former can do in $3\frac{1}{3}$ hrs., could build a wall of the same thickness, 60 feet long and $13\frac{1}{5}$ feet high, in $12\frac{1}{10}$ days of 8 hours?

$$\left.\begin{array}{rcl} 46\frac{3}{8} & : & 60 \\ 11\frac{1}{7} & : & 13\frac{1}{5} \\ 12\frac{1}{10} & : & 9 \\ 8 & : & 8\frac{5}{6} \\ 3\frac{1}{3} & : & 3\frac{1}{4} \end{array}\right\} :: 22 \text{ m.}$$

Here the statement being determined according to the usual procedure in the Compound Rule of Three, the fractional quantities are rendered integral by multiplying by the denominator and using that denominator either as a divisor on the same side of the line of separation, or as a multiplier on the other.

$$\frac{22 \text{ m.} \times 12 \times 66 \times 9 \times 8\frac{5}{6} \times 3\frac{1}{4}}{371 \times 11\frac{1}{7} \times 12\frac{1}{10} \times 3\frac{1}{3}}.$$

$$= \frac{\overset{2}{\cancel{22}} \times 3 \times \overset{1}{\cancel{11}} \times 9 \times 53 \times \overset{1}{\cancel{13}} \times \overset{1}{\cancel{3}}}{53 \times \underset{6}{\cancel{78}} \times \overset{1}{\cancel{121}}} = 9 \times 3 = 27 \text{ men. } \textit{Ans.}$$

(9) A and B set out at the same time from their respective homes, which are $21\frac{8}{15}$ miles apart. They travel directly towards each other, and meet in $2\frac{2}{9}$ hrs. If A's average rate of travelling be $4\frac{13}{20}$ miles an hour, what is B's?

Since they travel between them $21\frac{8}{15}$ mi. in $2\frac{2}{9}$ hrs., their united rates per hour must be $21\frac{8}{15}$ mi. $\div 2\frac{2}{9} = \frac{323}{15} \times \frac{9}{20} = \frac{969}{100}$ mi.;

$\therefore \frac{969}{100} - 4\frac{13}{20} = \frac{969}{100} - \frac{465}{100} = \frac{504}{100} = 5\frac{1}{25}$ mi. an hour, B's rate. *Ans.*

(10) If a train running $24{\cdot}722\frac{7}{9}$ miles an hour, were $24\frac{17}{40}$ miles behind another train running $17\frac{2}{3}$ miles an hour, in what time would it overtake the slower train?

The faster train gains on the slower $24{\cdot}722\frac{7}{9} - 17\frac{2}{3}$ miles in 1 hour; and we are to find in what time the faster train will gain $24\frac{17}{40}$ miles, at that rate.

$$\begin{array}{r} 24{\cdot}722\frac{7}{9} \\ 17\frac{2}{3} = 17{\cdot}666\frac{6}{9} \\ \hline 7{\cdot}056\frac{1}{9} \text{ mi. gained in 1 hour;} \end{array}$$

$24\frac{17}{40} = 24{\cdot}425$ mi. to be gained in the time required;

$\therefore 24{\cdot}425 \div 7{\cdot}056\frac{1}{9} = 219825 \div 63505 = 3\frac{6}{13}$ hours. *Ans.*

MISCELLANEOUS EXERCISES.

Ex. 53.

(1) Reduce to lowest terms $\frac{10961}{11639}$ and then divide the result by 9.

(2) Reduce to their decimal forms $\frac{7}{80}$, $\frac{5}{64}$, and $\frac{9}{10}$; subtract the second result from the first, and by the remainder thus formed divide the third. Also prove the result by the subtraction, &c., of the vulgar fractions themselves.

(3) Divide $1111\frac{1}{9}$ by 109 to the fifth place of decimals.

(4) Find the value of $5\frac{16}{51} - \frac{11}{34}$; also of ·217945 of a £ — ·00215 of a guinea.

(5) Calculate $\frac{21}{149}$ of $(\frac{43}{48} + \frac{89}{98} + \frac{25}{112})$.

(6) A pipe of port wine costs 60 guineas in bond; if taken out of bond it will cost £78 16*s.* 9*d.* Compare the amount of duty and other charges with the bonded value.

(7) If $\frac{1}{5}$ of a store of coals be taken away, then $\frac{2}{9}$ of what remains, and then $\frac{1}{7}$ of what still remains, what fraction of the original quantity will be left?

(8) If the value of $65\frac{1}{4}$ gallons of brandy be £73 13*s.* 9*d.*, what are $9\frac{2}{3}$ gallons of it worth?

(9) A guinea is a seventh as much again as the money given for 7 dozen lbs. of sugar. Required the cost per lb.

(10) If an arc of 3·65 inches be the interval of the minute divisions on a clock-dial plate whose diameter is 5·809155 feet, what is the circumference of a circle whose diameter is 113 inches?

Ex. 54.

(1) Reduce $18{\cdot}154\frac{1}{8}$ sq. ft. to the vulgar fraction of a pole.

(2) Find the value of $(2\frac{1}{7}s. - 3\frac{1}{8}d.) \div \frac{2}{5}$ of $\frac{3}{7}$ of $7\frac{1}{2}d.$

(3) Simplify $\frac{3\frac{1}{9} + 4\frac{5}{6}}{21\frac{1}{8} - 1\frac{1}{12}}$; also, find the combined value of $108{\cdot}09375 + {\cdot}15 - 73{\cdot}52 + 20{\cdot}00625$, and verify the result by substituting vulgar for decimal fractions.

(4) If $3\frac{5}{7}$ yds. of cloth cost a guinea, how much of it costs $2\frac{1}{13}s.$?

(5) A person buys $31\frac{1}{5}$ yds. of cloth for $6\frac{1}{4}$ guineas, and $11\frac{3}{8}$ yds. of another sort for $£4\frac{1}{12}$. Compare the prices per yard.

(6) If 31 French mètres correspond to 33 yds. 2 ft. $8\frac{1}{2}$ in., how many mètres correspond to 65·61828 Eng. ells?

(7) By what decimal fraction must 1·00503 be divided, so that $\frac{4}{5}$ of the quotient may be ·859?

(8) The quantity $93\frac{1}{8}$ is the sum of a certain number of ninths, 3 times as many tenths, and 5 times as many twelfths: what are the values of the three aggregate parts of the quantity?

(9) P and Q are two places ten miles apart, from which, respectively, A and B set out to meet each other; A goes 40 yds. in 11 seconds, and B 328 yds. in 81 seconds, and B sets out $3\frac{3}{5}$ seconds after A. In what time will they meet?

(10) Reduce ·4230590̇ and ·0100018084̇ to vulgar fractions in lowest terms.

Ex. 55.

(1) Find the value of $\frac{6\frac{2}{7}}{6\frac{1}{2}}$ of $\left(\frac{67}{144} - \frac{43}{132}\right)$.

(2) Divide 11111·01 by ·5757.

(3) Express $10s.\ 9\frac{3}{5}d.$ as the decimal of £3 16*s.* 8*d.*

(4) Find the value of 10·7965 ac. + 5·075 ro. — 119·75 po.

(5) Find the G. C. M. and also the L. C. M. of $15\frac{15}{32}$ and $29\frac{13}{28}$.

(6) Change $\frac{7}{8\frac{1}{3}}$, $\frac{3\frac{1}{5}}{8\frac{1}{2}}$, and $\frac{2\frac{1}{2}}{8\frac{1}{4}}$ into equivalent fractions, each having 8 for the numerator.

(7) If 28 men do $\frac{5}{8}$ of ·315 of a piece of work in 3·663 days, how many men will do the whole work in 16·28 days?

(8) Compare the value of £6 8 florins 5 cents 3 mils with that of £6 8*s.* $5\frac{3}{4}$*d.*

(9) If 122 French labourers can form a railway cutting, 6014 mètres long, and of $1\frac{7}{13}$ and $1\frac{1}{12}$ mètres in average width and depth, in $13\frac{5}{9}$ days, working $7\frac{5}{8}$ hours a day; in how many days of $8\frac{6}{7}$ hours would 168 English labourers form a cutting, through similar material, 4850 yds. long, $5\frac{1}{3}$ feet wide, and $3\frac{3}{4}$ feet deep; reckoning 31 mètres to be equal to $33\frac{8}{9}$ yards, and 5 English to do as much work, per hour, as 6 French labourers?

(10) Of a sum of money received by A, he has spent the ·7558 part; if he spend the ·44$\dot{6}$0 part of what remains, what part of the whole will then be left?

Ex. 56.

(1) Find (to five places) the decimal quotient of 16 divided by $425\frac{61}{91}$.

(2) Divide $47548\frac{20}{21}$, successively, by 8, $4\frac{1}{3}$, and $3\frac{2}{9}$.

(3) Obtain the simplest form of the ratio of ·0497$\frac{2}{7}$ to ·399$\frac{8}{9}$.

(4) Find the value of $\frac{3}{5}$ of 1·44 of a mile — 1·845 of a furlong + ·007 of ·077 of $3\frac{1}{2}$ miles.

(5) Express 3 ac. 1 ro. 38 po. $27\frac{1}{2}$ yds. as the decimal of 5 ac. 3 ro. 8 po.

(6) If 6·37 lbs. of tea be worth 20·475 shillings, what quantity is worth £5 11*s.* 9*d.*?

(7) Divide 20 ounces of gold among A, B, C, so that A's share may be equal to $\frac{3}{7}$ of B's, and B's $1\frac{3}{10}$ of C's.

(8) A person gave me a certain sum of money, of which I spent 8*d.*, and also $\frac{1}{4}$ of the remainder. He then gave me as much as $\frac{1}{5}$ of what I had left, after which I spent $\frac{1}{4}$ of the amount, and I had now 9 shillings. What sum did I receive at first?

(9) Express 14 yards $0\frac{5}{7}$ feet as the fraction of $\frac{7\frac{2}{7}}{6\frac{1}{4}}$ of $\frac{\frac{1}{18} \text{ of 21 ft. } 1\frac{1}{2} \text{ in.}}{\frac{2}{9}(10\frac{3}{10} - \frac{21}{40})}$

(10) If 32 men of ordinary ability can do a piece of work in ·05$\dot{4}$ *of the time* in which A could do it, how many men of ordinary ability could do it in ·05$\dot{3}$ of the time in which B could do it,

supposing A can do in $8\frac{1}{3}$ days 1·6875 as much work as B can do in 8 days?

Ex. 57.

(1) Multiply $\cdot41\frac{6}{7}$ by 4·13; and divide $\cdot0010\frac{5}{8}$ by ·068.

(2) Find the value of £5·274 − $\frac{11}{12}$ of 5·136 guin. — ·0224 of $1{\cdot}32\frac{1}{7}$ of $\frac{1}{2}$ a crown.

(3) From 2 fur. 13 po. $2\frac{5}{7}$ in. subtract the sum of $\frac{3}{11}$ of a mile and $\frac{5}{42}$ of a furlong.

(4) If ·275 of the price of a yard of cloth exceeds $\frac{2}{9}$ of it by $9\frac{1}{2}d.$, what is the price per yard?

(5) Divide $612\frac{5}{7}$ cwt. into three quantities expressed by equal numbers of tons, cwts., and Troy lbs.

(6) If 6 men, or 10 women, can do a piece of work in $8\frac{1}{2}$ days, how many women along with 7 men could do it in $3\frac{3}{4}$ days?

(7) Find the value of

$$\frac{3\frac{3}{5}}{5\frac{5}{7}} \text{ of } \frac{3 \text{ ac. } 3\frac{1}{6} \text{ ro.}}{516\frac{2}{3} \text{ sq. yds.}} \text{ of } \frac{23\frac{1}{4} \text{ guin.}}{£1{\cdot}716} \text{ of } 16s.\ 8d.$$

(8) A luggage train running $18\frac{2}{5}$ miles an hour, is $34\frac{13}{30}$ miles in advance of an express train running $35\frac{1}{5}$ miles an hour, and will reach the terminus 5 minutes before it. How far from the terminus is the luggage train?

(9) There is a quantity of coals in a shed, and it is increased to $\frac{1}{7}$ as much again: the whole store is then diminished, successively, by $\frac{2}{11}$ of itself, $\frac{3}{10}$ of what remains, and $\frac{1}{4}$ of what still remains; and the quantity now left, valued at $9\frac{2}{3}d.$ a cwt., is worth £32 12*s.* 6*d.* What quantity was in the shed at first?

(10) If $\cdot5\dot{4}\dot{4}0$ of a cwt. cost 1·17 of a shilling, what is the cost of 16·6 tons?

Ex. 58.

(1) Reduce $\frac{7\frac{2}{9}}{7\frac{1}{3}}$ of a yard to the fraction of an Eng. ell; and express $\frac{3}{8}$ of 3 guineas + $\frac{1}{4}$ of $3\frac{1}{2}d.$ − $\frac{2}{7}$ of 5*s.* 6*d.* as the decimal of £2 10*s.* 11*d.*

(2) A cubic mètre corresponds to 1·308 cubic yard, and an imperial gallon contains 277·274 cubic inches; how many gallons contain a cubic mètre?

(3) Divide $£6\frac{17}{28}$ separately by $£8\frac{2}{9}$ and by $8\frac{2}{9}$.

(4) Divide the sum of 4·385 of a guinea and $1·9327\frac{3}{4}$ of a crown by their difference.

(5) If 21 francs 10 cents. be equivalent to $\frac{7}{8}$ of £1, how many francs will correspond to £37 13*s.* 8*d.*?

(6) Find the value of 18·375 cwt. + $·0735\frac{10}{21}$ ton + 1·009375 lbs. + $73\frac{1}{5}$ oz.

(7) A person having a certain sum of money spends first $\frac{11}{21}$ of it, and then $\frac{9}{40}$ of $\frac{10\frac{5}{11}}{1\frac{1}{3} - \frac{2}{15}}$ of $\frac{1}{2\frac{2}{3} + 8\frac{4}{5}}$ of $\frac{15 \text{ yds. } 2·3 \text{ ft.}}{6 \text{ Eng. ells } 1\frac{1}{8} \text{ yd.}}$ of the remainder; and he has now £38 10*s.* left. What had he at first?

(8) A receives a share of a certain sum of money, B one-third of the remainder, C $\frac{5}{6}$ of what then remains, and D all that is then left, which is equal to $\frac{2}{5}$ of A's share. What fraction of the whole does A receive?

(9) If 14 ponies consume $15\frac{3}{4}$ quarters of corn in $63\frac{3}{5}$ days, for what time will £36 13*s.* 4*d.* supply 20 horses with corn, the price being 35*s.* a quarter, and 7 horses being supposed to consume as much per day as 11 ponies?

(10) Simplify $\dfrac{3·01\frac{4}{11} \text{ of } 4·6\dot{4}}{1·34\dot{3}} + \dfrac{6·0\dot{2}9\dot{6}}{5·\dot{7} - 1.9\dot{8}}$

PRACTICE.

104. An *Aliquot Part* of a quantity is an exact measure of that quantity: thus, 7 is an aliquot part of 21, viz. the 3rd part; 2*s.* 6*d.* is an aliquot part of £1, viz. the 8th part.

105. PRACTICE is a convenient way of calculating, by means of Aliquot Parts, the amount of any large number of articles, at a given value of each.

Thus, to obtain the amount of 273 copies of a book at 4*s.* each (which by ordinary calculation is 4*s.* × 273 = 1092*s.* *or £54 12s.*), Practice compares the price 4*s.* with 20*s.*, *that is, with so* many shillings as make 1 of the denomina-

tion above shillings, regarding 4*s.* as $\frac{1}{5}$ of £1, and first valuing 273 copies at £1 each, the amount of which would be £273, it directs to take $\frac{1}{5}$ of that amount, which gives £54 12*s.*

106. When the value of each article is between 1*s.* and 20*s.*, but not an aliquot part of £1, we can resolve that value into aliquot parts, which may be calculated successively; we have then to add the results.

Ex. (1) What is the amount of 273 things at 6*s.* each?

	£	*s.*	*d.*		£	*s.*
	273	0	0	= amount at	1	0
4*s.* = $\frac{1}{5}$ of £1	54	12	0	= amount at	0	4
2*s.* = $\frac{1}{2}$ of 4*s.*	27	6	0	= amount at	0	2
Ans.	£81	18	0	= amount at	£0	6

A better method, however, of treating a price which is an even number of shillings, will be shown hereafter (110).

Ex. (2) Calculate the amount of 365 things at 14*s.* 10½*d.*

	£	*s.*	*d.*		£	*s.*	*d.*
	365	0	0	= cost at	1	0	0
4*s.* = $\frac{1}{5}$ of £1	73	0	0	= cost at	0	4	0
10*s.* = $\frac{1}{2}$ of £1	182	10	0	= cost at	0	10	0
10*d.* = $\frac{1}{12}$ of 10*s.*	15	4	2	= cost at	0	0	10
½*d.* = $\frac{1}{20}$ of 10*d.*	0	15	2½	= cost at	0	0	0½
Ans.	£271	9	4½	= cost at	£0	14	10½

Ex. (3) Calculate 1769 things at £3 17*s.* 4½*d.*

	£	*s.*	*d.*		£	*s.*	*d.*
	1769	0	0	= cost at	1	0	0
			3				
	5307	0	0	= cost at	3	0	0
10*s.* = $\frac{1}{2}$ of £1	884	10	0	= cost at	0	10	0
6*s.* 8*d.* = $\frac{1}{3}$ of £1	589	13	4	= cost at	0	6	8
8*d.* = $\frac{1}{10}$ of 6*s.* 8*d.*	58	19	4	= cost at	0	0	8
½*d.* = $\frac{1}{16}$ of 8*d.*	3	13	8¼	= cost at	0	0	0¼
Ans.	£6843	16	4¼	= cost at	£3	17	4¼

Otherwise.

	£	s.	d.		£	s.	d.
	1769	0	0	= cost at	1	0	0
			3				
	5307	0	0	= cost at	3	0	0
2s. = $\frac{1}{10}$ of £1	176	18	0	= cost at	0	2	0
15s. = $\frac{1}{4}$ of £3	1326	15	0	= cost at	0	15	0
$4\frac{1}{2}$d. = $\frac{1}{40}$ of 15s.	33	3	$4\frac{1}{2}$	= cost at	0	0	$4\frac{1}{2}$
	£6843	16	$4\frac{1}{2}$	= cost at	£3	17	$4\frac{1}{2}$

Ex. (4) Calculate 2709 things at £1 16s. $6\frac{3}{4}$d.

	£	s.	d.		£	s.	d.
	2709	0	0	= cost at	1	0	0
10s. = $\frac{1}{2}$ of £1	1354	10	0	= cost at	0	10	0
5s. = $\frac{1}{2}$ of 10s.	677	5	0	= cost at	0	5	0
1s. 3d. = $\frac{1}{4}$ of 5s.	169	6	3	= cost at	0	1	3
$3\frac{3}{4}$d. = $\frac{1}{4}$ of 1s. 3d.	42	6	$6\frac{3}{4}$	= cost at	0	0	$3\frac{3}{4}$
Ans.	£4952	7	$9\frac{3}{4}$	= cost at	£1	16	$6\frac{3}{4}$

Ex. (5) Calculate 763 things at 8s. $10\frac{7}{8}$d.

	£	s.	d.		£	s.	d.
	763	0	0	= cost at	1	0	0
5s. = $\frac{1}{4}$ of £1	190	15	0	= cost at	0	5	0
3s. 4d. = $\frac{1}{6}$ of £1	127	3	4	= cost at	0	3	4
5d. = $\frac{1}{8}$ of 3s. 4d.	15	17	11	= cost at	0	0	5
$1\frac{1}{4}$d. = $\frac{1}{4}$ of 5d.	3	19	$5\frac{3}{4}$	= cost at	0	0	$1\frac{2}{8}$
$\frac{5}{8}$d. = $\frac{1}{2}$ of $1\frac{1}{4}$d.	1	19	$8\frac{7}{8}$	= cost at	0	0	$10\frac{7}{8}$
Ans.	£339	15	$5\frac{5}{8}$	= cost at	£0	8	$10\frac{7}{8}$

107. When the quantity to be valued contains a fraction, either convert the fraction into its value as a fraction of £1, or, when that seems inconvenient, reserve the fraction to be valued by itself at the given price, and add its value along with the several values of the whole number. Thus, in the following example, $\frac{7}{8}$ of £1 being readily known to be 17s. 6d., we have begun with the full value of $678\frac{7}{8}$ at

£1, but we have also shown the more generally convenient method.

Ex. (6) Find the value of $678\frac{7}{8}$ ounces at £3 17*s.* 9*d.* per oz.

	£	*s.*	*d.*		£	*s.*	*d.*	
	678	17	6	Or,	678	0	0	
			3				3	
	2036	12	6		2034	0	0	= cost at £3
15*s.* = $\frac{1}{4}$ of £1	509	3	$1\frac{1}{2}$		508	10	0	= cost at 15*s.*
2*s.* 6*d.* = $\frac{1}{6}$ of 15*s.*	84	17	$2\frac{1}{4}$		84	15	0	= cost at 2*s.* 6*d.*
3*d.* = $\frac{1}{10}$ of 2*s.* 6*d.*	8	9	$8\frac{5}{8}$		8	9	6	= cost at 3*d.*
Ans.	£2639	2	$6\frac{3}{8}$		3	8	$0\frac{3}{8}$	= $\frac{7}{8}$ of £3 17*s.* 9*d.*
					£2639	2	$6\frac{3}{8}$.	*Ans.*

108. When the quantity to be valued is compound, and contains numbers lower in denomination than the unit whose value is given, we can sometimes, as in the following example, easily reckon and use the full value of the given quantity at £1 : but most frequently we have to begin by multiplying the given value by the number of units to which it is assigned, and then to take aliquot parts for the remainder of the quantity. We present both methods, and it will be observed that in the first we have considered 2qrs. 14lbs. = $2\frac{1}{2}$qrs. = $\frac{2\frac{1}{2}}{4}$ or $\frac{5}{8}$ cwt., and $26\frac{5}{8}$ cwt. at £1 per cwt. = £$26\frac{5}{8}$ = £26 12*s.* 6*d.*

Ex. (7) Calculate 26 cwt. 2 qrs. 14 lbs. at £1 13*s.* 9*d.* per cwt.

	£	*s.*	*d.*		£	*s.*	*d.*
	26	12	6.	Or,	1	13	9
10*s.* = $\frac{1}{2}$ of £1	13	6	3				26
3*s.* 4*d.* = $\frac{1}{3}$ of 10*s.*	4	8	9		43	17	6
5*d.* = $\frac{1}{8}$ of 3*s.* 4*d.*	0	11	$1\frac{1}{8}$	2 qrs. = $\frac{1}{2}$ of cwt.	0	16	$10\frac{1}{2}$
	£44	18	$7\frac{1}{8}$	14 lbs. = $\frac{1}{4}$ of 2 qrs.	0	4	$2\frac{5}{8}$
				Ans.	£44	18	$7\frac{1}{8}$

Ex. (8) Calculate 29 yds. 2ft. 5 in. at 6*s.* 5¼*d.* a yard; and also at 26*s.* 8*d.* a yd.

	£	*s.*	*d.*
	0	6	5¼
			29
	9	6	8¼
1 ft. = ⅓ of 1 yd.	0	2	1¾
1 ft. = ⅓ of 1 yd.	0	2	1¾
4 in. = ⅓ of 1 ft.	0	0	8⅞
1 in. = ¼ of 4 in.	0	0	2⅞
1st Ans.	£9	11	10⅝

The calculation at the first price, 6*s.* 5¼*d.*, is most easily done by the second method; but the calculation at the second price £1 6*s.* 8*d.* is more easily done by the first method, thus;

	£	*s.*	*d.*
	29	16	1⅓
6*s.* 8*d.* = ⅓ of £1	9	18	8⅔
2nd Ans.	£39	14	9⅔

Ex. (9) Calculate a rate of 7½*d.* in the £ on an assessment of £53 18*s.* 6*d.* and also the value of ·318275 of 15*s.* 3½*d.*

If the rate were 20*s.* in the £ it would amount to £53 18*s.* 6*d.*

	£	*s.*	*d.*
6*d.* = 1/40 of £1	1	6	11 11/20
1½*d.* = ¼ of 6*d.*	0	6	8 71/80
Ans.	£1	13	8 7/16

Value at 1*s.* = ·318275*s.*

	15
	4·774125*s.*
3*d.* = ¼ of 1*s.*	·079569
½*d.* = ⅙ of 3*d.*	·013261
	4·866955*s.*

Ans. 4*s.* 10·40346*d.*

109. For the calculation of prices under 1 shilling, the method of Practice is, generally, not more expeditious than that of multiplying by the full price as given, or as the fraction of 1 shilling. Hence, for the price 6¼*d.*, multiply by 6¼ and divide by 12, for 8*d.* take ⅔, for 7½*d.* take ⅝, for 9*d.* take ¾, &c., to obtain the amount in shillings.

110. When the price is an even number of shillings under 20*s.*, if we convert it into the decimal of £1, and use it as a multiplier, the integral part of the product will be the £'s of the required amount, and the tenths, if any, will be half the number of shillings.

Ex. (10) Calculate 273 articles at 6*s.*, at 8*s.*, and at 14*s.*

273 at 6*s.*	273 at 8*s.*	273 at 14*s.*
·3	·4	·7
81·9	99·2	191·1
Answers. £81 18*s.*	£99 4*s.*	£191 2*s.*

111. When the price is exactly one aliquot part short of £1, we may calculate for that aliquot part, and subtract, as in the following example.

Ex. (11) Calculate 247 articles at 17*s.* 6*d.*

17*s.* 6*d.* is 2*s.* 6*d.*, or £$\frac{1}{8}$, less than £1.

8) £247	0*s.*	0*d.*	= cost at £1	0	0
30	17	6	= cost at	0	2	6
Ans. 216	2	6	= cost at	0	17	6

Ex. 59.

Find the value of

		£	*s.*	*d.*			£	*s.*	*d.*
(1) 325	things at	0	6	8	(2) 365	things at	0	12	6
(3) 439	„	0	8	4	(4) 539	„	1	9	0
(5) 867	„	2	15	0	(6) 768	„	0	13	4
(7) 1085	„	0	18	0	(8) 597	„	1	11	8
(9) 2347	„	0	16	8	(10) 1768	„	6	12	9
(11) 547	„	6	17	10	(12) 2174	„	0	14	8
(13) 3065	„	0	7	$8\frac{1}{2}$	(14) 889	„	0	12	$10\frac{1}{2}$
(15) 969	„	1	11	$6\frac{3}{4}$	(16) 2345	„	3	13	6
(17) 1811	„	2	8	9	(18) 2714	„	5	1	$7\frac{1}{2}$
(19) 765	„	0	17	$4\frac{4}{9}$	(20) 239	„	0	15	$7\frac{2}{5}$
(21) 4733	„	0	11	$4\frac{5}{8}$	(22) 1694	„	0	6	$7\frac{11}{12}$
(23) 447	„	17	17	0	(24) 459	„	13	14	1
(25) 3029	„	0	8	8	(26) 1495	„	4	17	4
(27) $1234\frac{3}{8}$	„	0	18	4	(28) $947\frac{5}{6}$	„	0	9	$11\frac{1}{4}$
(29) $677\frac{2}{9}$	„	1	13	$7\frac{1}{4}$	(30) $2173\frac{6}{7}$	„	2	8	$5\frac{1}{3}$
(31) 30·145	„	0	19	$6\frac{3}{4}$	(32) 1·775	„	4	14	10

(33) 24 cwt. 3 qrs. 21 lbs. at £1 16*s.* $4\frac{1}{2}$*d.* per cwt.

(34) 17 cwt. 1 qr. 20 lbs. at £2 7*s.* $8\frac{3}{4}$*d.* per cwt.

(35) 54 yds. 2 ft. 11 in. at £0 13*s.* $5\frac{1}{2}$*d.* per yard.

(36) 80 ac. 3 ro. 16 po. at £3 18*s.* 6*d.* per acre.

(37) 29 *qu.* 5 bu. 3 pks. at £0 7*s.* $1\frac{1}{2}$*d.* per bushel.

(38) 10 sq. yds. 7½ sq. ft. at £0 12*s.* 8½*d.* per sq. yd.
(39) £245 15*s.* 9½*d.* at £0 13*s.* 6½*d.* per £.
(40) 1 lb. 6 oz. 13 dwt. 10 grs. at £0 5*s.* 3¼*d.* per ounce.

RATES PER CENT.

112. The expression PER CENT. means *by the hundred*; and a *Rate per Cent.* is a ratio denoting so many hundredths.

Thus, if 3*d.* is gained, or lost, on every shilling of outlay or investment, the gain or loss is to the outlay as 3 to 12, or it is $=\frac{3}{12}$, or $\frac{1}{4}$, or $\frac{25}{100}$, of the outlay; and the last of these fractions, in which the comparative gain or loss is represented in 100*ths*, is the *percentage* form of the comparative gain or loss: it may be expressed either by $\frac{25}{100}$, or by ·25, or by the phrase 25 *per cent.*

It must be remembered, then, that the phrases 5 per cent., 3 per cent., 10½ per cent., &c., denote ratios in which 5, 3, 10½, &c., are the first terms, and 100 is uniformly the 2nd term; or, denote fractions in which 5, 3, 10½, &c., are the numerators, and 100 is the common denominator.

113. Every expression denoting a ratio can, by the following Rule, have its two terms so altered (if necessary) that 100 shall be the 2nd term.

To reduce a given rate or ratio to an equivalent per cent.

RULE.—Express the given ratio fractionally, and then reduce the fraction to 100ths, by multiplying it by 100.

Ex. (1) A merchant loses £8 on an outlay of £40; how much does he lose per cent.?

The loss is to the outlay as £8 to £40;

$\frac{£8}{£40} = \frac{1}{5}$, therefore $\frac{1}{5} \times 100 = 20$ hundredths, or 20 per cent. *loss. Ans.*

Ex. (2) A person buys at 6*s.* 3*d.* a lb., and sells at 6*s.* 6*d.*; what is his gain per cent.?

The gain is 3*d.* on 6*s.* 3*d.*, or the gain is to the outlay as 3*d.* to 75*d.*;

$$\therefore \tfrac{3}{75} \times 100 = 4 \text{ p. c. gain. } \textit{Ans.}$$

Ex. (3) Ten years ago the population of a town was 6750; it is now 8400; what has been the increase per cent.?

The increase, 1650, is $\frac{1650}{6750}$ of the original number;

hence, $\frac{1650}{6750} \times 100 = 24\frac{4}{9}$ per cent.—*Ans.*

Ex. (4) Standard silver is composed of 37 parts of pure silver, and 3 parts of copper. How much per cent. of the whole is each of the components?

The pure silver is to the whole as 37 to 40, and the copper is to the whole as 3 to 40.

$$\left.\begin{array}{l} \text{Therefore } \frac{37}{40} \times 100 = 92\frac{1}{2} \text{ per cent. pure silver} \\ \frac{3}{40} \times 100 = 7\frac{1}{2} \text{ per cent. copper} \\ \hline 100 \end{array}\right\} \textit{Ans.}$$

114. *To find the absolute value of a given rate per cent. on any given quantity.*

RULE.—Take so many 100ths of the given quantity as are expressed by the rate.

Ex. (1) An agent sells for his employer goods to the amount of £2375, and charges 3 per cent. commission for his trouble. Find the value of his commission.

$$3 \text{ per cent. on } £2375 = \tfrac{3}{100} \text{ of } £2375$$

$$\begin{array}{r} 3 \\ \hline £71{\cdot}25, \text{ or } £71\ 5s. \textit{ Ans.} \\ 20 \\ \hline 5{\cdot}00s. \end{array}$$

Ex. (2) If in a company of 60 persons, composed of English, Scotch, and Irish, 40 per cent. of the whole are English, and $36\frac{2}{3}$ per cent. are Scotch; how many persons are Irish?

The English and Scotch together constitute $40 + 36\frac{2}{3} = 76\frac{2}{3}$ per cent. of the whole; therefore the Irish $= 100 - 76\frac{2}{3} = 23\frac{1}{3}$ per cent. of the whole;

$$23\tfrac{1}{3} \text{ per cent. on } 60 = \frac{23\frac{1}{3}}{100} \text{ of } 60 = \tfrac{7}{30} \text{ of } 60 = 14 \text{ Irish. Ans.}$$

Ex. 60.

(1) Reduce the ratio 7 : 16 to the per centage form

(2) How much per cent. is 2*d.* on a shilling?

(3) What gain on 12*s.* 6*d.* corresponds to a gain of 20 per cent.?

(4) How much per cent. is 7*d.* in the £?

(5) Calculate 4 per cent. on £13 15*s.*

(6) Calculate $\frac{1}{8}$ per cent. on £25.

(7) What is the value of $12\frac{1}{2}$ per cent. on 2 cwt. 3 qrs. 12 lbs.?

(8) Required the amount of $\frac{1}{4}$ per cent. commission on £3765 13*s.* 7*d.*

(9) A man married at the age of 32, and died at the age of 52; how much per cent. of his whole life was spent before marriage?

(10) A draper buys silk at 5*s.* $9\frac{1}{2}$*d.* per yard, and sells it at 6*s.* 9*d.*; what does he gain per cent.?

(11) How many persons constitute 15 per cent. of a population of 12060? and what is the nearest number of persons corresponding to 18 per cent. of the result?

(12) I bought a piece of land for £744, and sold it at a profit of $102\frac{1}{2}$ per cent.; what did I receive for it?

(13) Currants are bought at $6\frac{1}{4}$*d.* a lb.; at what price must they be then sold to gain 12 per cent.?

(14) If 1 in every $23\frac{1}{3}$ counters is red, and the rest are white, how much per cent. of the whole are the white ones?

(15) A draper bought 312 yds. of long cloth for £12 16*s.*, and sold it at the rate of 2*s.* 8*d.* for every three yards; what was his gain per cent.?

(16) If the components of a metallic mass be 244·55 ounces of copper, and 120·45 ounces of zinc, how much per cent. of the mass is copper, and how much per cent. of the weight of the zinc is equal to the weight of the copper?

(17) I bought 7 tons 15 cwt. of iron for £65, but lost $3\frac{1}{8}$ per cent. in selling it. At what price per ton did I sell it?

(18) If wheat, bought at the rate of £25 for 9 quarters, be sold at the rate of £34 15*s.* for 12 quarters, what is gained per cent.?

(19) I bought oranges at the rate of 14 for $11\frac{1}{2}$*d.*; how many *should I sell for* 1*s.* $5\frac{1}{4}$*d.* that my gain may be 5 per cent.?

(20) How much per cent. of £529 1*s.* 4*d.* is equal to 6·702 per cent. of £740?

(21) I bought 168 gallons of brandy at 23*s.* 9*d.*, and have sold 159 gallons of it at 27*s.* 6*d.* What selling price for the remainder will make my gain 15 per cent. on the whole?

(22) A draper buys a number of dresses at £2 15*s.*, and sells $\frac{2}{5}$ of the number at £3 10*s.*, and the rest at £2 12*s.* 6*d.* What does he gain per cent. on the whole?

115. *To find the quantity on which a given quantity is a given per cent.*

RULE.—Express the given per cent. fractionally, and reduce the fraction to one whose numerator is the given quantity. The new denominator thus found will be the quantity required.

Ex. (1) On what quantity will the value of $7\frac{1}{2}$ per cent. be 6 ounces?

$\frac{7\frac{1}{2}}{100} = \frac{15}{200} = \frac{3}{40}$, to be changed to a fraction whose numerator is 6 oz., hence 3 : 40 :: 6 oz. : 80 oz., the required denominator, or the quantity on which 6 oz. correspond to $7\frac{1}{2}$ on 100; therefore 80 oz. is the *Ans.*

Ex. (2) A library consists of English and foreign works: the number of English works is 4270, which is $76\frac{1}{4}$ per cent. of the whole number of works. How many works does the library contain?

$\frac{76\frac{1}{4}}{100} = \frac{305}{400} = \frac{61}{80}$ of the whole number = 4270 works;

therefore 61 : 80 :: 4270 : 5600, the whole number. *Ans.*

Ex. (3) What sum by an increase of 6 per cent. will amount to £$132\frac{1}{2}$?

A quantity increased by $\frac{6}{100}$ of itself will amount to $1\frac{6}{100}$ or $\frac{106}{100}$ of itself: therefore the question is—On what quantity will the value of 106 per cent. be £$132\frac{1}{2}$? Accordingly, as in the 1st example, we have $\frac{106}{100}$ to be changed to a fraction whose numerator is £$132\frac{1}{2}$; hence 106 : 100 :: £$132\frac{1}{2}$: £125. *Ans.*

Ex. (4) The nett proceeds of a sale of sugar amounted to £690,

being at a loss of $5\frac{1}{2}$ per cent. on the prime cost. What was the prime cost?

A decrease of $\frac{5\frac{1}{2}}{100}$ on a quantity reduces the quantity to $\frac{94\frac{1}{2}}{100}$ of itself: therefore the question is—On what quantity will the value of $94\frac{1}{2}$ per cent. be 630? Accordingly, as in the 1st example, we have $\frac{94\frac{1}{2}}{100}$ to be changed to a fraction whose numerator is £630, and hence

$$94\tfrac{1}{2} : 100 :: £630 : £666\ 13s.\ 4d.\ \textit{Ans.}$$

Ex. (5) A person has goods worth £1300; on what amount must he effect insurance, at $2\frac{1}{2}$ per cent. premium, that, in case of the goods being accidentally lost, he may recover both the value of the goods and the premium?

The inquiry here proposed is equivalent to this—What sum diminished by a payment of $2\frac{1}{2}$ per cent. will leave £1300? or, On what quantity will the value of $97\frac{1}{2}$ per cent. be £1300? Hence $97\frac{1}{2} : 100 :: £1300 : £1333\frac{1}{3}$. *Ans.*

Ex. (6) A grocer pays £40 for a quantity of coffee, and sells part of it at a profit of $3\frac{1}{4}$ per cent., and the remainder at a profit of 5 per cent. Compare the selling prices per lb.

The selling prices per lb. are, respectively, $\frac{103\frac{1}{4}}{100}$ and $\frac{105}{100}$ of the prime cost. per lb., and are therefore as $103\frac{1}{4} : 105$, or as 413 : 420, or as 59 : 60. *Ans.*

Ex. (7) If I sell coffee at the rate of 3 lbs. for 3*s.* 4*d.*, I shall lose $1\frac{2}{3}$ per cent.; at what price per lb. must I sell it to gain $3\frac{1}{4}$ per cent.?

3*s.* 4*d.* $\div$ 3 = $13\frac{1}{3}$*d.* per lb., the price at which there would be a loss of $1\frac{1}{3}$ per cent., or which would be $98\frac{1}{3}$ hundredths of the prime cost; and we are here to find the price that would be $103\frac{1}{4}$ hundredths of the prime cost;

$$\text{hence, } 98\tfrac{1}{3} : 103\tfrac{1}{4} :: 13\tfrac{1}{3}d. : \frac{103\frac{1}{4}}{98\frac{1}{3}} \text{ of } 13\tfrac{1}{3}d.$$

$$= \frac{40}{3} \times \frac{413 \times 3}{295 \times 4} = \frac{10d. \times 7}{5} = 1s.\ 2d.\ \textit{Ans.}$$

Ex. 61.

(1) Of what sum is 35*s.* the value of $6\frac{1}{4}$ per cent. ?

(2) If a loss of $1\frac{1}{2}$ per cent. on a pipe of wine would be 15*s.* 9*d.*, what is the prime cost ?

(3) If by spending £10 3*s.* $7\frac{1}{2}d.$ I part with 5 per cent. of my money, what sum had I at first ?

(4) On what sum is a loss of 3 guineas a loss of $3\frac{1}{2}$ per cent. ?

(5) My commission at $2\frac{1}{2}$ per cent. on the nett proceeds of an account-sales is £8 17*s.* $11\frac{1}{2}d.$; find the amount of nett proceeds.

(6) A trader paid for insurance of goods £39 18*s.*, being at the rate of $2\frac{1}{2}$ guineas per cent. [*i.e.* per £100] on the supposed value of the goods. What was the supposed value ?

(7) What number increased by 4 per cent. will amount to 39 ?

(8) What number diminished by 9 per cent. is reduced to $34\frac{1}{8}$?

(9) By selling goods for £50 2*s.* I gained 20 per cent.; what did they cost me ?

(10) If I should gain $\frac{5}{6}$ per cent. by selling 5 dozen of Moselle wine for £13 12*s.* 3*d.*, what is the prime cost per dozen ?

(11) By selling cloth at 13*s.* 10*d.* per yard, the gain would be $3\frac{3}{4}$ per cent.; what would be the gain or loss per cent. by selling it at 13*s.* 6*d.* ?

(12) By selling wine at 52*s.* I should lose $2\frac{1}{2}$ per cent.; what should I gain or lose per cent. by selling it at 53*s.* 6*d.* ?

(13) The population of a town in 1861 was 13167, being $4\frac{1}{2}$ per cent. more than that in 1851; what would have been the number in 1861, if there had been a decrease of $4\frac{1}{2}$ per cent. ?

(14) If a music seller took £28 12*s.* for a pianoforte, his profit would be 10 per cent.; but he wants $12\frac{1}{2}$ per cent. profit. What price does he ask ?

(15) A's share of a sum of money is $\frac{1}{5}$, and is 8 per cent. more than B's, and 8 per cent. less than C's; what are B's and C's shares ?

(16) If by selling 37 yds. of cloth for 6 guineas I should lose $3\frac{3}{4}$ per cent., how many yards should I sell for 40*s.* to gain $2\frac{7}{9}$ per cent. ?

(17) What sum must be insured at $4\frac{1}{8}$ per cent., on a cargo of goods worth £980, that, in case of loss, the owner may recover the value of the goods and the premium?

(18) What amount of premium, at $2\frac{1}{2}$ guineas per cent., must be paid for insurance of property worth £1520, so that, in case of loss, the owner may receive the value of the property and the premium?

INTEREST OF MONEY.

116. INTEREST is payment, at a certain rate per cent. per annum, for the loan or use of money.

The money lent is called the Principal, and the sum of Principal and Interest is called the Amount.

117. If interest is reckoned only on the sum originally lent, it is called *Simple Interest.*

If interest, when it becomes due, is made an additional loan, progressively increasing the value of the interest, it is called *Compound Interest.*

SIMPLE INTEREST.

118. *To find the interest of a given principal at any proposed rate per cent. per annum.*

RULE.—Multiply the yearly rate per cent. by the number of years, and then take as many hundredths of the given principal as that product expresses.

Note.—We have here only a particular application of (114).

Ex. (1) Find the simple interest on £380, at 5 per cent. per annum, for 3 years.

5 per cent. per annum for 3 years is 3 times 5 per cent.;

hence $\frac{5 \times 3}{100}$ of £380 = $\frac{3}{20}$ of £380 = £57. *Ans.*

Ex. (2) Find the simple interest on £11 13*s.* 6*d.*, for 4 years, at $3\frac{1}{2}$ per cent. per annum.

$3\frac{1}{2}$ per cent. per annum for 4 years is 4 times $3\frac{1}{2}$ per cent.;

hence $\frac{3\frac{1}{2} \times 4}{100}$ or $\frac{14}{100}$ of £1157 13*s.* 6*d.* = £162 1*s.* $5\frac{22}{25}$*d.*

Ex. (3) Find the simple interest on £849 17*s.* 6*d.*, for 10 months, at $4\frac{1}{2}$ per cent. per annum.

As the rate of interest is annual, the time must be taken in years; therefore we take 10 months = $\frac{10}{12}$ or $\frac{5}{6}$ of a year; then the absolute per cent. is $\frac{5}{6}$ of $4\frac{1}{2}$ = $3\frac{3}{4}$ per cent., and we may calculate $3\frac{3}{4}$ hundredths of the given principal in either of the two following ways, the first having the fraction $\frac{3\frac{3}{4}}{100}$ simplified as = $\frac{3}{80}$.

£	s.	d.
849	17	6
		3
80) 2549	12	6
Ans. £31	17	$4\frac{7}{8}$

or

£	s.	d.
849	17	6
		$3\frac{3}{4}$
4) 2549	12	6
637	8	$1\frac{1}{2}$
31,87	0	$7\frac{1}{2}$
20		
17,40		
12		

Ans. £31 17*s.* 4·875*d.*

Ex. (4) Find the amount, by simple interest, of £612 13*s.* 8*d.*, for 1 year 9 mths., at $4\frac{3}{8}$ per cent. per annum.

Here the absolute rate is $4\frac{3}{8} \times 1\frac{3}{4} = \frac{245}{32}$ per. cent.
and $\frac{245}{3200}$, = $\frac{49}{640}$ of the given principal, will be the interest,
or $1\frac{49}{640}$ of the given principal will be the amount.

£	s.	d.
612	13	8
		7
4288	15	8
		7
640 { 8) 30021	9	8
(80) 3752	13	$8\frac{1}{2}$
46	18	$2\frac{9}{160}$

Ans. £659 11 $10\frac{9}{160}$

Ex. (5) The sum of £342 10*s.* 9*d.* is borrowed, at 4 per cent. per annum, on 3rd January 1866; what amount of principal and interest will repay the loan on the 27th of June following?

In reckoning the number of days from date to date, we must exclude the 3rd of January, as the borrower will not have had the use of the money for one day till the 4th of January. The number of days, therefore, will be $28 + 28 + 31 + 30 + 31 + 27 = 175$ days, which in the denomination of years is $\frac{175}{365} = \frac{35}{73}$ of a year.

Accordingly the absolute per cent. is $\frac{35}{73}$ of 4 per cent. and $\frac{35}{73}$ of $\frac{4}{100}$, or $\frac{7}{365}$ of the given principal, will be the interest, or $1\frac{7}{365}$ of the given principal will be the amount. Hence the calculation will be as follows:—

					or,		
Princ.	£342	10*s.*	9*d.*			£342·5375	
			7			7	
365)	2397	15	3		365)	2397·7625	
Int.	6	11	$4\frac{223}{365}$			6·56921	
Ans. Amt.	£349	2	$1\frac{223}{365}$			£349·10671	
						£349 2*s.* 1·61*d.* nearly.	

Ex. (6) Find the simple interest on £472 10*s.* 6*d.*, at $3\frac{1}{2}$ per cent. per annum, for 2 years 231 days.

Here we may first reduce the given time to 961 days; then, $\frac{961}{365}$ of $\frac{3\frac{1}{2}}{100} = \frac{6727}{73000}$, the fraction to be taken of the given principal.

$\frac{1}{1000}$ of given princ. = £·472525

6727

73) 3178·675675

£43·543502 = £43 10*s.* 10·44*d.* *Ans.*

Ex. 62.

Find the simple interest on

(1) £5 at 5 per cent. for 2 years.

(2) £325 at 4 per cent. for 3 years.

(3) £350 at $3\frac{1}{2}$ per cent. for 1 year.

(4) £292 10*s.* at 3 per cent. for 3 years.

(5) £174 5*s.* 5*d.* at $2\frac{1}{2}$ per cent. for 4 years.

Find the amount by simple interest of

(6) £1000 at $2\frac{1}{3}$ per cent. for 5 years.

(7) £782 18*s.* 4*d.* at 5 per cent. for $2\frac{1}{2}$ years.
(8) £63 17*s.* 2*d.* at $3\frac{3}{8}$ per cent. for one year.
Find the simple interest and the amount of
(9) £1240 at $4\frac{1}{7}$ per cent. for $\frac{1}{2}$ a year.
(10) £178 8*s.* 9*d.* at 4 per cent. for 3 months.
(11) £365 7*s.* 6*d.* at $2\frac{1}{4}$ per cent. for 8 months.
(12) £51 16*s.* 4*d.* at 5 per cent. for 3 years and 6 months.
(13) £742 12*s.* 6*d.* at $2\frac{1}{2}$ per cent. for 1 year 5 months.
(14) £10375 at 3 per cent. for 5 years 10 months.
(15) £987 14*s.* 7*d.* at $3\frac{1}{3}$ per cent. for 3 years 4 months.
Find the simple interest on
(16) £320 at $3\frac{1}{2}$ per cent. for 30 days.
(17) £230 10*s.* 6*d.* at $5\frac{1}{4}$ per cent. for 146 days.
(18) £1730 at 4 per cent. for 84 days.
(19) £503 14*s.* at $6\frac{1}{2}$ per cent. from 10th March to 10th July.
(20) £168 18*s.* 3*d.* at $4\frac{1}{8}$ per cent. from 16th April to 2nd Oct.
(21) £4891 at $3\frac{1}{4}$ per cent., from 16th January 1865 to 13th April 1865.
Find the amount, by simple interest, of
(22) £270 15*s.* 8*d.* at 5 per cent. from 23rd Oct. 1866, to 7th Sept. following.
(23) £489 5*s.* 11*d.* at 4·35 per cent. for 1 year 219 days.
(24) £5326 13*s.* 6*d.* at 2·7 per cent. for 2 years 270 days.

119. In some questions of Interest, the interest, or the amount, instead of being required, is a given quantity, and either the principal, or the rate, or the time, is the required quantity. Such questions are sometimes best worked by proportion statement.

Ex. (1) At what rate per cent. per annum, for 3 years, will the interest on £380 be £57?

Here the interest of £380 is $\frac{57}{380}$ of its principal, and therefore the corresponding interest on 100 must be $\frac{57}{380}$ of 100 = $\frac{3}{20}$ of 100 = 15 = the three years' interest on 100; hence, 15 ÷ 3 = 5 per cent. per annum. *Ans.*

Ex. (2) In what time will £849 17*s.* 6*d.* amount to £881 14*s.* $10\frac{7}{8}d.$ at $4\frac{1}{2}$ per cent. per annum?

Here the interest on £849 17*s.* 6*d.* is £31 17*s.* $4\frac{7}{8}d.$; whence

the interest on 100 may be found by proportion statement as follows:—

£849 17*s.* 6*d.* : £31 17*s.* $4\frac{7}{8}$*d.* :: 100 : $3\frac{3}{4}$ per cent.

which is $\frac{3\frac{3}{4}}{4\frac{1}{2}}$ of the annual rate, and shows the time to be $\frac{15}{18}$ or $\frac{5}{6}$, of a year = 10 months. *Ans.*

Ex. (3) What principal, at 5 per cent. per annum, will yield £57 simple interest in 3 years?

The given per cent. is in all 3 times 5, or 15, per cent.; and we ask—On what sum is £57 the value of 15 per cent.?

15 : 100 :: £57 : £380. *Ans.*

Ex. (4) What principal, at 5 per cent. per annum, will amount to £437 by simple interest in 3 years?

This is equivalent to the question—What quantity increased by 15 per cent. will amount to 437?

$1\frac{15}{100}$ or $\frac{115}{100}$ of the required quantity = £437; therefore

115 : 100 :: £437 : £380. *Ans.*

Ex. 63.

(1) At what rate per cent. per annum, for 6 years, will the simple interest on £390 be £117?

(2) In what time will the simple interest on £715 12*s.* 6*d.*, at $2\frac{1}{2}$ per cent. per annum, be £53 13*s.* $5\frac{1}{4}$*d.*

(3) In what time will the simple interest on £530, at 4 per cent. per annum, be £12 7*s.* 4*d.*?

(4) At what rate will the amount of £76 0*s.* 10*d.*, by simple interest, for 142 days, be £77 3*s.* $0\frac{1}{4}$*d.*?

(5) Of what principal will £92 16*s.* 3*d.* be the simple interest in $2\frac{1}{2}$ years, at $2\frac{1}{4}$ per cent. per annum?

(6) What principal, at $8\frac{1}{9}$ per cent. per annum, will produce £17 3*s.* simple interest in 105 days?

(7) In what time will the simple interest on £269 16*s.*, at $4\frac{1}{2}$ per cent. per annum, be 18*s.* $9\frac{1}{2}$*d.*?

(8) What principal will in 3 years amount to £1355 9*s.* 4*d.*, at $3\frac{1}{2}$ per cent. per annum, simple interest?

(9) What principal would amount to £8 10*s.* $8\frac{1}{5}$*d.* simple interest in 12 months, at $4\frac{1}{2}$ per cent. per annum?

(10) What principal would amount to £893 2*s*. $7\frac{23}{25}$*d*. in 292 days, at $3\frac{1}{4}$ per cent. per annum simple interest?

(11) In what time would any sum of money double itself, by simple interest, at the rate of $2\frac{1}{4}$ per cent. per annum?

(12) What principal would amount to £1 4*s*. $8\frac{7}{8}$*d*. in 153 days, at $2\frac{1}{3}$ per cent. per annum, simple interest?

COMPOUND INTEREST.

120. In *Compound Interest*, as we have already stated (117), the interest, as it arises, becomes an additional loan, which is to bear interest for the succeeding period. Accordingly, when the interest is payable yearly, we may add the 1st year's interest to the principal, calculate the 2nd year's interest on that new principal, and thence obtain the 2nd amount as the principal on which interest is to be calculated for the next year, and so forth. The final amount, if diminished by the original principal, will, of course, show the whole sum of interest that has accrued.

Ex. (1) Find the compound interest on £1650, for 3 years, at 5 per cent. per annum.

Here each year's interest is $\frac{5}{100}$ or $\frac{1}{20}$ of the principal for that year.

	£	*s*.	*d*.		
20)	1650	0	0	=	1st Principal.
	82	10	0	=	Interest for 1st year.
20)	1732	10	0	=	2nd Principal.
	86	12	6	=	Interest for 2nd year.
20)	1819	2	6	=	3rd Principal.
	90	19	$1\frac{1}{2}$	=	Interest for 3rd year.
	1910	1	$7\frac{1}{2}$	=	Amount in 3 years.
	1650	0	0		
Ans.	£260	1	$7\frac{1}{2}$		Interest in 3 years.

121. The above solution is according to the method best adapted to the particular example. But we will here introduce another form of solution, which is generally the most expedient. Let it be observed that the amount at the end of the 1st year is $1\frac{5}{100}$, or $\frac{105}{100}$, or 1·05 of the original principal; that the amount at the end of the 2nd year is 1·05 of the 1st year's amount, &c., so that the amount at the end of the 3rd year is equal to £1650 × 1·05 × 1·05 × 1·05 = £1650 × 1·57625 = £1910·08125.

122. It is evident that solutions of this kind will always involve products of a number multiplied repeatedly into itself. We will here, therefore, introduce some observations respecting such products.

The continual product of any number multiplied into itself is called a power of that number. It is called the 2nd, 3rd, 4th, &c., power of that number, according as the number is used twice, thrice, or four times, &c., as a factor; and the number itself, in relation to these powers, is called the 1st power, or the root of any other power. Thus 3 × 3, or 9, is the second power of 3; 3 × 3 × 3 × 3 × 3, or 27 × 9, or 243, is the 5th power of 3, &c.; and the number 3 itself is the 1st power of 3, or it is the 2nd root of 9, the 5th root of 243, and so forth. Powers of a number are briefly indicated by small figures raised a little on its right; thus, 3^2 signifies 3 raised to the 2nd power, and is = 9; $3^3 = 27$; $3^4 = 81$, &c.

123. We may now give the following rule for finding the amount of any given principal at compound interest.

RULE.—Add the rate per cent. per annum to 100, and divide the amount decimally by 100, which will give the amount of unity for 1 year. Then raise that amount to the power denoted by the given number of years, and multiply the principal by it.

Ex. (2) Find the compound interest of £276 8*s.* 4*d.* for 4 years at 3½ per cent. per annum.

$(100 + 3\frac{1}{2}) \div 100 = 1{\cdot}035$; and $1{\cdot}035^4 = 1{\cdot}147523$ nearly.

	276
	6·885138
	80·32661
	229·5046
6*s.* 8*d.* = ⅓ of £1	·382508
1*s.* 8*d.* = ¼ of 6*s.* 8*d.*	·095627
	317·194483
	276·416667
Int.	£40·777816 *Ans.*

Note.—When, as here, the interest only is wanted, then, instead of first taking 1·147523 times the given principal, which produces the amount of principal and interest, we may at once multiply the principal by ·147523, thus :—

	·147523
	276
	40·716348
6*s.* 8*d.* = ⅓ of £1	·049174
1*s.* 8*d.* = ¼ of 6*s.* 8*d.*	·012294
	£40·777816

Ex. 64.

Find the amount at compound interest of

(1) £475 at 3 per cent. for 2 years.
(2) £1250 at 4 per cent. for 3 years.
(3) £680 at 5 per cent. for 4 years.
(4) £7000 at 2½ per cent. for 3 years.
(5) £80 16*s.* 8*d.* at 5 per cent. for 4 years.
(6) £1781 13*s.* 4*d.* at 3½ per cent. for 2 years.

Find the compound interest on

(7) £293 13*s.* 6*d.* at 5 per cent. for 5 years.
(8) £1365 9*s.* 2*d.* at 4½ per cent. for 3 years.
(9) £567 at 2¼ per cent. for 4 years.
(10) £2763 8*s.* 4*d.* at 3⅓ per cent. for 6 years.

124. When compound interest is payable half-yearly, it is the same as interest at half the given rate for twice the given number of years.

Ex. (1) Find the amount, at compound interest, of £750, at $6\frac{2}{3}$ per cent. per annum, for $2\frac{1}{2}$ years, when the interest is payable half-yearly.

The amount is required for 5 half years at $3\frac{1}{3}$ per cent. per half-year; and it is $= £750 \times (1{\cdot}03\frac{1}{3})^5 = £750 \times \left(\frac{3{\cdot}1}{3}\right)^5$ $= £750 \times 286{\cdot}20151 \div (9 \times 9 \times 3) = £883{\cdot}61577$ nearly, or £883 12*s.* 3·785*d.* *Ans.*

125. When the principal, instead of being one of the quantities given, is the quantity required, the mode of finding it will appear, by the following example, to be easily derived from the Rule for obtaining the amount.

Ex. (2) What principal, at compound interest, will, in 4 years, amount to £317 3*s.* $10\frac{27}{40}$, at $3\frac{1}{3}$ per cent. per annum?

Here we know that (as in *Ex.* 2, p. 133) the principal multiplied by $1{\cdot}035^4$, or by 1·147523, will produce £317·19448; therefore the principal $= £317194480 \div 1147523 = £276$ 8*s.* 4*d.* *Ans.*

Note 1.—When the rate, or the time, instead of being one of the given quantities, is the required quantity, the question in most instances is inconvenient for common arithmetic.

Note 2.—It has been seen that the amount, at compound interest, of a given principal is found by raising the given amount of £1 for one year to the power denoted by the number of years, and then multiplying the result by the principal. In consistency with this, the amount of £1 by compound interest at $10\frac{1}{4}$ per cent. per annum for half a year, will be that power of $1{\cdot}10\frac{1}{4}$, which is denoted by $\frac{1}{2}$, a form of indicating what is called the square root of a quantity. Now, the square root of 1·1025, viz., the number which when multiplied by itself produces 1·1025, is 1·05, so that £105 is the amount of £100 for half a year by

compound interest at $10\frac{1}{4}$ per cent. per annum, and is less than £$105\frac{1}{8}$, the amount by simple interest. It will suffice here to have thus referred to the peculiarity, that, strictly considered, compound is less than simple interest for fractional parts of a year.

Ex. 65.

(1) Find the compound interest, for 2 years, on £1340, at 3 per cent. per annum, payable half-yearly.

(2) Find the amount, at compound interest, of £630, for $1\frac{1}{2}$ year, at 4 per cent. per annum, interest payable quarterly.

(3) What sum, improved by compound interest at $3\frac{3}{4}$ per cent. per annum, will in 3 years amount to £491 7*s.* 7*d.*?

(4) What principal will yield £127 18*s.* $6\frac{7}{40}d.$, compound interest, in 4 years, at $2\frac{1}{2}$ per cent. per annum?

(5) What principal will produce £254 8*s.* 9*d.* compound interest in 7 years, at 5 per cent. per annum?

TRUE DISCOUNT.

126. DISCOUNT is an allowance deducted for the payment of money before it is due.

If the cash price at which I can afford to sell goods be £25, and if A. B. give me an order for such goods, and I do not expect to receive payment from him till the end of 3 months, I ought to rate the goods, in the bill or invoice, at a higher price than £25. Supposing the worth of the use of money to be 4 per cent. per annum, I ought to add a $\frac{1}{4}$ of a year's interest, or 1 per cent., to the £25, thus making the invoice price £25 5*s.*, payable in 3 months. If, however, he pays me cash on delivery, I ought to deduct 5*s.* from the invoice price, and accept payment at £25.

According to the supposed value of credit, £25 paid at present is exactly worth £25 5*s.* payable 3 months hence;

or £25 is the *Present Worth* of £25 5*s.*, and 5*s.* is the *Discount* on £25 5*s.*

It has been seen, then, that Discount, such as we are now considering, and which is often called *True Discount*, is the interest of the present worth of a given sum due at a future time. What among bankers and merchants is called Discount will be explained afterwards.

127. *To find the sum of discount that should be allowed for present payment of a given sum of money due at a future time.*

RULE.—The interest on 100 for the given time, being the discount on the amount of 100 for the given time, express that interest as a fraction of the amount, and calculate that fraction of the given future payment.

To find the present worth of a given sum of money due at a future time.

RULE.—As 100 is the present worth of the amount of 100 for the given time, express that present worth as a fraction of the amount, and calculate that fraction of the given future payment.

Note.—This Rule has been already exemplified in (119), its object being to find what principal, at a given rate per cent. per annum simple interest, will amount to a given sum in a given time.

Ex. (1) Find the discount on £3 11*s.* 9*d.*, payable 8 months hence, reckoning the use of money to be $7\frac{1}{2}$ per cent. per annum.

Here the interest on 100 for 8 months, or $\frac{2}{3}$ of a year, is $\frac{2}{3}$ of $7\frac{1}{2}$ $= 5$; therefore 5 is the discount on 105, or the discount is $\frac{5}{105}$ or $\frac{1}{21}$ of the future payment. Hence $\frac{1}{21}$ of £3 11*s.* 9*d.* $=$ 3*s.* 5*d.* *Ans.*

Ex. (2) What is the present worth of £126 14*s.* due $2\frac{1}{2}$ years hence, interest at $5\frac{1}{4}$ per cent. per annum?

The interest on 100 for $2\frac{1}{2}$ years is $5\frac{1}{4} \times 2\frac{1}{2} = 13\frac{1}{8}$; therefore 100 is the present worth of $113\frac{1}{8}$, or the present worth is $\frac{100}{113\frac{1}{8}}$, or $\frac{160}{181}$ of the future payment. Hence $\frac{160}{181}$ of £126 14*s.* $=$ £112. *Ans.*

Ex. (3) Find the present worth, and the discount, of £19 18*s.* for 63 days, at 5 per cent. per annum.

Interest on 100 = $\frac{63}{365}$ of 5 = $\frac{63}{73}$; therefore 100 is the present worth of $100\frac{63}{73}$, or 7300 is the present worth of 7363. Hence, $\frac{7300}{7363}$ of £19 18*s.* = £19 14*s.* $7\frac{5}{37}$*d.* the present worth. *Ans.* And that sum subtracted from £19 18*s.* gives 3*s.* $4\frac{32}{37}$*d.* the discount. *Ans.*

Ex. (4) A merchant buys silk at 5*s.* $7\frac{1}{4}$*d.* a yard, on 4 months' credit; he has it on hand a month, and then sells the whole at 5*s.* 11*d.* a yard, on 2 months' credit; what is his gain per cent. at the time of selling, if interest be reckoned at $3\frac{1}{2}$ per cent. per annum?

At the time of selling, the prime cost is 5*s.* $7\frac{1}{4}$*d.* payable in 3 months, the selling price is 5*s.* 11*d.* payable in 2 months; and we have to find the present worths of these prices. Therefore, as 3 months = $\frac{1}{4}$ of a year, and 2 months = $\frac{1}{6}$ of a year, and $\frac{1}{4}$ and $\frac{1}{6}$ of $3\frac{1}{2}$ are respectively $\frac{7}{8}$ and $\frac{7}{12}$ = the interests on 100, the present worth of the buying price is $\frac{100}{100\frac{7}{8}}$ or $\frac{800}{807}$ of 5*s.* $7\frac{1}{4}$*d.* = $66\frac{2}{3}$*d.*, and that of the selling price is $\frac{100}{100\frac{7}{12}}$ or $\frac{1200}{1207}$ of 5*s.* 11*d.* = $70\frac{10}{17}$*d.* The gain, therefore, is $70\frac{10}{17} - 66\frac{2}{3}$, or $3\frac{47}{51}$*d.* on $66\frac{2}{3}$*d.*, or 200*d.* on 3400*d.*, or 1*d.* on 17*d.* Hence $\frac{1}{17}$ of 100 = $5\frac{15}{17}$ per cent. *Ans.*

Ex. (5) Bought 33 dozen of wine at 43*s.* a dozen, on 1 month's credit, and sold it forthwith, on 4 months' credit, at such a price as made my immediate gain $16\frac{2}{3}$ per cent. What was the credit selling price, reckoning interest at 4 per cent. per annum?

Interest on 100 for 1 month = $\frac{1}{12}$ of 4 = $\frac{1}{3}$; ditto for 4 months = $\frac{1}{3}$ of 4 = $1\frac{1}{3}$.

Present worth of buying price = $\frac{100}{100\frac{1}{3}}$ or $\frac{300}{301}$ of 43*s.* = $42\frac{6}{7}$*s.*, which at an increase of $16\frac{2}{3}$ per cent. becomes $\frac{116\frac{2}{3}}{100}$ or $\frac{7}{6}$ of $42\frac{6}{7}$*s.* = 50*s.* the present worth of the credit selling price; so that 50*s.* increased by 4 months' interest will be the credit selling price, =

$$50s. \times \frac{101\frac{1}{3}}{100} = \frac{50s. \times 304}{300} = 50s.\ 8d.\ \textit{Ans.}$$

Ex. 66.

(1) Find the present worth of £583, due 12 months hence, interest at 6 per cent. per annum.

(2) What sum of discount will reduce £1150, due 7 months hence, to its present value, interest at $3\frac{1}{8}$ per cent. per annum?

(3) Find the discount on £98 19*s.* 3*d.* for $1\frac{3}{4}$ year, interest at 5 per cent. per annum.

(4) What is the present worth of £675 2*s.* 6*d.*, payable 3 years hence, interest at $4\frac{1}{4}$ per cent.?

(5) Find the present worth of £474 12*s.* 6*d.*, due 6 months hence, interest at $3\frac{1}{4}$ per cent. per annum.

(6) Required the discount on £243 15*s.* 9*d.*, due 4 months hence, at $2\frac{1}{2}$ per cent. per annum.

(7) Required the present worth of £8700, due 5 months hence, interest at $2\frac{7}{8}$ per cent. per annum.

(8) What is the discount for present payment of a debt of £555, due 17 months hence, interest at $4\frac{1}{2}$ per cent. per annum?

(9) What is the present worth of £357, due 296 days hence, interest at $3\frac{1}{2}$ per cent. per annum?

(10) Reduce £114 10*s.*, due 25 Jan. 1866, to its cash value on 28 Dec. 1865, interest at 5 per cent. per annum.

(11) How long hence is £26 16*s.* payable, if £25 12*s.* be its present cash value, at $6\frac{1}{4}$ per cent. per annum?

(12) If a coal factor buy coals at 19*s.* a ton, on 4 months' credit, and sell them at once at 21*s.* 6*d.*, on 1 month's credit, what is his immediate gain per cent., interest at 4 per cent. per annum?

(13) A merchant buys coffee at 1*s.* 5*d.* a lb., on 44 days' credit, and sells it forthwith at 1*s.* $6\frac{1}{4}$*d.* cash, what is his immediate gain per cent., interest at 5 per cent. per annum?

(14) Bought wine, for cash, at 54*s.* a dozen, and sold it, 2 months afterwards, at 61*s.* 6*d.* a dozen, on 6 months' credit. What, at the time of selling, was my trade profit per cent., reckoning simple interest at 5 per cent. per annum?

(15) I bought silk at 4*s.* 9*d.*, on 7 months' credit, and sold it forthwith at 5*s.* 2*d.*, making thereby an immediate profit of $8\frac{1}{3}$ per cent. What term of credit did I give, interest being reckoned at 5 per cent. per annum?

MERCANTILE DISCOUNT.

128. *Mercantile Discount* is a deduction of so much per cent., rated on a sum not yet due, or regarded as not yet due, in order to reduce the sum to a conventional cash value.

In *True* Discount, as has been seen, the allowance is rated on the true present worth, not on the future debt.

129. Mercantile Discount, though erroneous in principle, is generally employed in business, as it is easily computed, and differs very little from true discount, at those short periods for which discount is usually required. The distinction between conventional and true discount may be exemplified as follows:—

Suppose that when the use of money is worth 5 per cent. per annum, I incur a debt of £21, payable 12 months hence, and that I wish to pay off the debt now, there would be allowed to me, by the usual practice, 5 per cent. off £21, which is £1 1*s.*, and by a present payment of £19 19*s.* I should discharge the debt. But it will be seen that £19 19*s.* improved at 5 per cent. simple interest, for 1 year, would amount, not to £21, but to £20 18*s.* 11$\frac{1}{2}$*d.*, so that £19 19*s.* paid at present is not equivalent to £21 due a year hence. The true present worth of the £21 is £20, and the true discount is £1, viz. the interest of the £20.

130. Mercantile exceeds true discount by the interest on the true discount. For mercantile discount is equivalent to the interest on the future value, and true discount is the interest on the present value; and the difference of these two interests is the interest on the true discount.

131. Bills of Exchange, which are written engagements by which persons bind themselves to pay money to the

holders of them, at a specified future time, have their present worth reckoned according to mercantile discount.

A banker, or a bill broker, is said to discount a bill, when he buys it of the holder at the conventional present value of the sum named in it. Of course he will not discount it, unless he considers the credit of the names connected with it to be satisfactory.

132. When the term of a bill is specified in months, as is very commonly the case, calendar months are to be understood; so that a bill dated 14th March, at 3 months, is due on the 14th June following: but it is then only *nominally* due, for the date at which a bill is *legally* due in England is 3 days beyond the date at which it is nominally due, 3 *days of grace*, as they are called, being allowed, unless a bill is made payable on demand.

But a bill that would be nominally due on the 31st of a month, if that month contained 31 days, would, if the month contained only 30 days, be considered nominally due on the 30th. Also, a bill that falls due on a Sunday must be paid on the previous Saturday. Hence, a bill at 3 months, dated 31st August 1865, became nominally due on Nov. 30th, and therefore actually due on Dec. 3rd, which, however, happened to be a Sunday, therefore the bill was payable on Dec. 2nd.

Ex. (1) A bookseller professes to supply new books for cash, at a discount of $16\frac{2}{3}$ per cent. on the publishing price: What cash does he charge for a book published at a shilling? and what for one published at half a guinea?

$\frac{16\frac{2}{3}}{100} = \frac{50}{300} = \frac{1}{6}$; hence $\frac{1}{6}$ of $12d. = 2d.$ discount on $1s.$; also, $\frac{1}{6}$ of $10s.\ 6d. = 1s.\ 9d.$ discount on $10s.\ 6d.$; hence the cash selling prices are $10d.$ and $8s.\ 9d.$ *Ans.*

Ex. (2) A bill for £350, dated April 15th, at 4 months, is discounted July 3rd, at 4 per cent. per annum. Required the cash proceeds.

Bill drawn April 15th, at 4 months, falls due Aug. 18th; discounted July 3rd, which is 28 + 18 = 46 days before due:—

Contents of bill	£350	0*s.*	0*d.*
46 days' interest	1	15	3
Cash proceeds	£348	4	9 *Ans.*

Ex. (3) A bill for £605 was drawn on March 21st, at 6 months' date. What cash did it realise on the 25th June, on which day it was discounted with A. B. at 5 per cent.? and what did A. B. gain by the transaction?

Bill drawn March 21st, at 6 months, due Sept. 24th; on June 25th the bill has 91 days to run:—

Contents of bill	£605	10*s.*	6*d.*
91 days' interest	7	10	11½
Cash proceeds	597	19	6½ *Ans.*
True pres. worth	598	1	4½ nearly.
A. B.'s gain	£0	1	10 *Ans.*

Ex. 67.

(1) A person buys goods, amounting per invoice to £428 17*s.* 6*d.*, and is allowed 5 per cent. discount for present payment. What amount is paid for the goods?

(2) Find the cash proceeds of a bill of £723 13*s.* 4*d.*, dated June 12th, at 2 months, and discounted June 26th at 4½ per cent.?

(3) Find the sum of discount charged by a banker on Nov. 1st, for cashing a bill of £232 15*s.* drawn Aug. 18th, at 3 months: interest at 5 per cent.

(4) Find the cash proceeds of a bill of £1125, dated Dec. 30th, 1865, at 5 months, and discounted with M. N. on Jan. 15th, 1866, at 6 per cent.

(5) What will be the cash proceeds of a bill for £2000, drawn March 31st, at 1 month, if it be discounted with a banker on April 6th at 5 per cent.; and what will be the banker's gain by the transaction?

SIMPLE DISTRIBUTIVE PROPORTION.

133. *To divide a given quantity into aggregate parts that shall be proportioned, one to another successively, as certain given numbers are.*

RULE.—Express each of the given numbers as a fraction of their entire sum, and calculate that fraction of the given quantity.

Ex. (1) Divide £799 into three parts, having the proportion of 7, 4 and 6.

Here we have $7 + 4 + 6 = 17$; so that the three parts are respectively $\frac{7}{17}$, $\frac{4}{17}$, and $\frac{6}{17}$ of the whole. Therefore as $\frac{1}{17}$ of £799 = £47, the three parts required are £47 × 7, £47 × 4, £47 × 6, or £329, £188, £282. *Ans.*

It should be borne in mind that, when we speak of three quantities being in the proportion of 7, 4, and 6, we mean that the three quantities divided, respectively, by 7, 4, and 6, give equal results. Hence the method employed in the solution of the following example:—

Ex. (2) Divide £799 into three parts, so that 7 times the first, 4 times the second, and 6 times the third may be equal products.

Here we should consider, that the required parts multiplied, respectively, by 7, 6, and 4, must produce the same as if they were divided by $\frac{1}{7}$, $\frac{1}{4}$ and $\frac{1}{6}$; so that the required parts are as $\frac{1}{7}$, $\frac{1}{4}$, and $\frac{1}{6}$, or as 12, 21, and 14. Hence, taking $\frac{12}{47}$, $\frac{21}{47}$ and $\frac{14}{47}$ of £799, we obtain £204, £357, and £238. *Ans.*

Ex. (3) Two merchants, A and B, form a joint capital, A's investment being £1100, and B's £1400. At the end of a year they divide between them a profit of £325. What share of the profit should each receive?

The profit should be divided in the proportion of the stocks, that is, as 11 to 14, the sum of which is £25; therefore A should have $\frac{11}{25}$ of £325 = £143, and B. $\frac{14}{25}$ of £325 = £210. *Ans.*

In Problems under this Rule, the proportions according to which a quantity is to be divided are often given indirectly, and the

student should endeavour to acquire readiness in reducing them to a direct form. Thus, if A is given $= \frac{4}{9}$ of B, B = 3 times C, and C = half as much again as D, we can represent A as $\frac{4}{9}$, B as 1, C as $\frac{1}{3}$, D as $\frac{1}{3} \div 1\frac{1}{2} = \frac{2}{9}$, and then multiplying these direct proportions by 9, to get rid of fractions, we have for A, B, C, D, the direct integral proportions, 4, 9, 3, 2.

Ex. (4) Divide the number 7987 into four parts, such that the first part may be two-thirds as much again as the second, the second one-fourth less than the third, and the third $2\frac{1}{2}$ per cent. more than the fourth.

Here, we may represent the 4th part by 100; then the 3rd will be $102\frac{1}{2}$, the 2nd will be $\frac{3}{4}$ of $102\frac{1}{2} = 76\frac{7}{8}$, and the first will be $1\frac{2}{3}$ of $76\frac{7}{8} = 128\frac{1}{8}$; hence, removing fractions by multiplying by 8, we have 1025, 615, 820, 800, as the direct proportions; or striking out 5 from each, we shall have the easier numbers 205, 123, 164, 160.

Or, we might have begun by taking $1\frac{2}{3}$ to represent the 1st part, then 1 would represent the 2nd, $1 \div \frac{3}{4} = 1\frac{1}{3}$ the 3rd, and $\frac{100}{102\frac{1}{2}}$ of $1\frac{1}{3} = \frac{160}{123}$ the 4th; hence, multiplying by 123, to remove fractions, we have 205, 123, 164, 160, as before.

The sum of these proportional parts being 652, the required parts will be, respectively, $\frac{205}{652}$, $\frac{123}{652}$, $\frac{164}{652}$, $\frac{160}{652}$ of 7987, viz. $2511\frac{1}{4}$, $1506\frac{3}{4}$, 2009, 1960. *Ans.*

Ex. 68.

(1) Divide the number 54 into two parts, one of which shall be 5 times the other.

(2) Divide the number 54 into three parts, proportioned as 2, 3, 4.

(3) Divide £4 12*s.* into three sums, that shall be to each other as the fractions $\frac{1}{4}$, $\frac{1}{7}$, $\frac{1}{10}$.

(4) A is $7\frac{1}{2}$ years old, B $10\frac{1}{2}$. Divide half-a-crown between them in the proportion of their ages.

(5) A, B, C, form a joint capital, A putting in £800, B £350, and C £930. What will be each partner's share in a joint loss of £260?

(6) Divide £40 among A, B, C, giving A thrice as much as B, and B one fifth as much as C.

(7) P and Q are two places 21 miles apart. A sets out from P towards Q at $3\frac{3}{4}$ miles an hour; B sets out at the same time from Q towards P, at 5 miles an hour. How far will each have travelled when they meet?

(8) Three coal waggons contain 195 cwt. in such proportions, that 10 times the load in the first, 12 times that in the second, and 15 times that in the third, are equal quantities. What weight does each waggon carry?

(9) A and B are partners in trade, A's share of joint stock being equal to $\frac{5}{7}$ of B's. What should each man receive as his share of a joint gain amounting to £460 16*s.*?

(10) Divide a sovereign among A, B, C, D, so that A may have twice as much as B, B one fifth more than C, and D as much as A and C.

(11) An assemblage of 700 persons consists of 5 men for every 2 children, and 3 children for every 7 women. How many of each?

(12) The joint capital of four partners, A, B, C, D, is £12,600. A's investment is £10 for every £17 of B's, C's is £34 for every £65 of D's, and B's is half as much again as C's. Required the capital of each.

(13) There are three sums of money amounting together to £429 14*s.*; the first is 7 per cent. more than the second, and the second is 3 per cent. less than the third. Find them.

(14) Divide 365 into three parts, such that twice the first, 5 times the second, and 24 per cent. of the third, may be equal quantities.

(15) What area can be divided into three parts, so that the 2nd shall be $= \frac{5}{8}$ of the 1st, the 3rd $= \frac{6}{7}$ of the 2nd, and the 1st $= 6\frac{1}{2}$ square yards more than the 3rd?

COMPOUND DISTRIBUTIVE PROPORTION.

134. In *Compound Distributive Proportion*, required aggregate parts of a given quantity are determined by means of *compound ratios*.

Ex. (1). A, B, C enter into partnership. A invests £1850 for 12 months, B £850 for 9 months, and C £1200 for 8 months. What should be each man's share of a joint gain amounting to £789?

Here, if the times were equal, the gain would have to be divided in the proportion of the sums invested; but as the times are unequal, the several gains should be proportioned to the products of the investments by the respective times, for we shall thereby replace each given capital by one that would gain as much in 1 month.

Thus £1850 for 12 months = £1850 × 12 for 1 month.
£850 for 9 months = £850 × 9 for 1 month.
£1200 for 8 months = £1200 × 8 for 1 month.

Hence, we have to divide £789 in the proportion of 22200, 7650, and 9600, or as 148, 51, and 64, the sum of which is 263.

Therefore $\frac{148}{263}$ of £789=£444 A's gain
$\frac{51}{263}$ of £789=£153 B's gain
$\frac{64}{263}$ of £789=£192 C's gain
} *Ans.*

Note.—In sharing a joint gain it is sometimes the practice for the partners to take out of the total profit interest at 5 per cent. on their respective capitals, and then to divide the remainder of the profit equally among them. On this principle the preceding question would be worked as follows:—

Total profit	£780	0	0
A's int. on £1850 for 12 ms. =	£92	10	0
B's int. on £850 for 9 ms. =	£31	17	6
C's int. on £1200 for 8 ms. =	£40	0	0
	164	7	6
	3)624	12	6
Hence, giving to each person his interest +	£208	4	2

A would have £300 14*s.* 2*d.*, B £240 1*s.* 8*d.*, C £248 4*s.* 2*d.*

Ex. (2). A and B rent a pasture for 6 months at £15 per month. A at first puts in 30 sheep and B 72; but at the end of 2 months A puts in 20 more and B takes out 20, and C is allowed to put in 87. What portion of the rent should each person pay at the end of the 6 months?

A's 30 sheep for 6 months = 180 sheep for 1 month
20 sheep for 4 months = 80 sheep for 1 month
A's whole benefit = 260 sheep for 1 month
B's 20 sheep for 2 months = 40 sheep for 1 month
52 sheep for 6 months = 312 sheep for 1 month
B's whole benefit = 352 sheep for 1 month
C's whole benefit = 87×4 = 348 sheep for 1 month

$260+352+348=960$;

Hence, A should pay $\frac{260}{960}$ of £90 = £24 7*s.* 6*d.*
B should pay $\frac{352}{960}$ of £90 = £33 0*s.* 0*d.*
C should pay $\frac{348}{960}$ of £90 = £32 12*s.* 6*d.*
} *Ans.*
£90 0*s.* 0*d.*

Ex. (3). Find three sums of money, the second of which shall be $\frac{7}{10}$ of the first, and the third $\frac{3}{5}$ of the second, and such that a year's interest being computed on each sum, at the respective rates of 5, 4, and 6 per cent., the interest of the first shall exceed that of the third by £6 4*s.*

The interests are as 1×5, $\frac{7}{10} \times 4$, and $\frac{3}{5}$ of $\frac{7}{10} \times 6$; or as 125, 70, and 63; the sum of which is 258; hence the difference of the 1st and 3rd interests is $= \frac{125-63}{125}$ or $\frac{62}{125}$ of the 1st interest; therefore the 1st interest $= £6\frac{1}{5} \div \frac{62}{125} = £12\frac{1}{2}$, which, since the rate is 5 per cent., is a year's interest on $£12\frac{1}{2} \times \frac{100}{5}$, or £250. The required sums, therefore, are £250, $\frac{7}{10}$ of £250 = £175, and $\frac{3}{5}$ of £175 = £105. *Ans.*

Ex. 69.

(1) Divide £6 17*s.* 6*d.* among 4 men, 5 women, and 7 children, so that for every 13*d.* given to a man a woman shall receive 8*d.*, and for every shilling given to a woman a child shall receive 7*d.*

(2) Divide £423 10*s.* into four sums, the simple interests on which shall be equal, if computed for 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, years, respectively, and at the respective rates of $2\frac{1}{2}$, 3, 4, $4\frac{1}{2}$, per cent. per annum.

(3) Divide £100 into three parts, such that if a year's interest on each be computed at 3, 4, and $2\frac{1}{2}$ per cent., respectively, the interest on the first shall be to that on the second as 7 : 10, and that on the second to that on the third as 6 : 13.

(4) *A and B* rent a pasture for 12 weeks, at £2 16*s.* 8*d.* per

week. A puts in at first 25 oxen, but at the end of 20 days withdraws 6, and 30 days thereafter puts in 8. B puts in at first 18 oxen, and after 21 days 7 more, but 43 days thereafter removes 16. What sum should each person contribute towards the rent?

(5) A and B are partners in a speculation, by which they gain £140. A invested £350 for 9 months, and B £680 for 7 months. What amount of gain belongs to each?

(6) At the beginning of the year, A commences business with a capital of £3000, and at the end of 7 months takes B and C into partnership with him, B bringing £2000 into the business, and C £1580. At the end of the year they divide a profit of £500. What should each receive?

(7) Two bars of gold are melted together; one is 16 carats fine, and weighs 2 lbs. 10 oz.; the other is 18 carats fine, and weighs $3\frac{1}{2}$ lbs. How much pure gold will be contained in a bar of the compound weighing 3 lbs. 2 oz.?

(8) I bought a stock of wine, at a uniform price per gallon, and forthwith had an offer for the whole, from W. and Co., at $5\frac{1}{2}$ per cent. profit. I declined, however, that offer, and sold $\frac{3}{5}$ of the stock at 5 per cent. profit, $\frac{1}{8}$ at 6 per cent. profit, and the remainder at 7 per cent. profit; but I obtained hereby only £2 more than the amount of W. and Co.'s offer. What did the wine cost me?

(9) A invests a certain sum in a speculation for $13\frac{1}{2}$ months, and is joined by B and C for 8 and 10 months, respectively. At the end of the $13\frac{1}{2}$ months they dissolve the partnership, having sustained a loss of £341 10*s.*, of which A bears $£113\frac{19}{34}$, B $£82\frac{6}{17}$, and C $£145\frac{10}{17}$. What portion of the joint capital, £1885, was invested by each person?

(10) A, B, C, are partners in an undertaking by which they clear £273. A's share of that sum is £50 3*s.* $1\frac{1}{2}d.$, and B's £101 13*s.* $5\frac{1}{4}d.$; the respective capitals having been A £1070, B £1205, C £1175. Required the several periods of investment, C's being three months longer than A's.

(11) Suppose that £47 11*s.* is to be divided among a certain number of men, women, and boys; that there are 10 boys for every 3 men, and that the women are 36 per cent. of the whole number of persons; that each boy is to receive 6*d.*, each woman 1*s.*, and each man 2*s.* 6*d.* Find the respective numbers of men, women, and boys.

(12) A merchant paid £66 3*s.* for 100 yards of cloth of three different kinds. For every 5 yards of the first kind he had 3 of the second, and for every 8 of the second he had 7 of the third. Required the respective prices per yard, the first kind being rated half as dear again as the second, and the second a fifth less than the third.

STOCKS.

135. STOCKS are funds borrowed by Industrial Companies, or by Governments, to carry on some enterprise, to meet some national emergency or demand, &c. The term is most commonly applied to Funds constituting national debt.

136. Our Government has at various times raised loans at various rates per cent., £100 being in the first instance the subscription entitling a creditor to the interest on £100 stock. But as the Government may retain the principal for an indefinite time, so long as it pays the interest, the holder of stock when he wishes to discontinue his investment can do so only by transferring, or selling, his right to some other person. Now, such may have been the alteration in the value of money, since the first institution of a particular description of stock, that £100 stock, which at first cost £100, may now be worth less or more than £100. And, indeed, from a variety of causes, the price of £100 stock is continually fluctuating. When other good sources are open to general investment, and offer a higher rate of interest, the value of stock is of course diminished.

The most important portion of our Public Debt is the Three per Cent. Consolidated Annuities, originated in 1751, when several descriptions of stock were consolidated into one, bearing a uniform rate of interest at 3 per cent. per annum. This stock is commonly called the 3 per Cent. Consols; its *prices rule to a great* extent those of other Public Secu-

rities; when, for instance, £100 of 3 per Cent. stock costs £90, the price of $3\frac{1}{2}$ per Cent. stock may be expected to be about 105.

137. When £100 stock costs £100, the stock is said to be *at par*; when the cost exceeds £100, the stock is said to be at a premium; when the cost is less than £100, the stock is said to be at a discount. The term *par* is also used to denote correspondence in prices: thus, $3\frac{1}{2}$ per Cents. at 105 are *on a par*, or *at par*, with 3 per Cents. at 90.

138. A purchase or sale of stock is generally made through a stockbroker, whose charge for transacting the transfer is $\frac{1}{8}$ per cent., or 2*s.* 6*d.* for every £100 of stock transferred; so that when the price of £100 stock is £80, a person wishing to buy stock will have to pay $80\frac{1}{8}$ for every 100; and a person wishing to sell stock will have to receive $79\frac{7}{8}$ for every 100.

139. An important portion of the business of the Stock Exchange is *time bargains*, that is, bargains to deliver stock at a certain price on a certain day, the interest and expectation of the seller being that the price will fall, and of the buyer that it will rise. These bargains are generally made for certain days, called settling days, on which a settlement is usually effected by the losing party simply paying the difference. The price at which stock is sold to be transferred on the next settling day, is called the price *for the account.*

140. The interest on stock is paid half-yearly, and the value of stock will of course be somewhat greater immediately before, than immediately after, payment of the dividends.

141. Sums of stock are often named in cents or hundreds; thus £1600 stock is 16 cents. When cent means *hundred* it is an abbreviation of the Latin *centum*; when it means *hundredth*, it is an abbreviation of the French centime.

Ex. (1). What amount of 3 per Cent. stock can be purchased by investing £1400, when the price is $87\frac{1}{2}$?

Here as £$87\frac{1}{2}$ is the money price of £100 stock, £1400 will purchase as many cents as the number of times it contains £$87\frac{1}{2}$; hence, £$100 \times \frac{1400}{87\frac{1}{2}}$ = £1600. *Ans.*

Ex. (2). What amount of stock in the $3\frac{1}{2}$ per Cents. at $87\frac{1}{8}$ would £820 purchase? and what income would be thence derived?

$\frac{820}{87\frac{1}{8}}$ cents = £$100 \times \frac{820 \times 8}{697}$ = £$\frac{1600}{17}$ = £941 3*s.* $6\frac{6}{17}$*d.* *Ans.*

The income is £3 for every cent; hence,

£$3 \times \frac{820}{87\frac{1}{8}}$ = £$\frac{160}{17}$ = £9 8*s.* $2\frac{14}{17}$*d.* per annum. *Ans.*

Ex. (3). What sum of money will buy £1110 of $2\frac{1}{2}$ per Cent. stock, the price being $69\frac{1}{4}$ and $\frac{1}{8}$ brokerage? and what will be the income?

Here, the price is $69\frac{1}{4} + \frac{1}{8} = 69\frac{3}{8}$; and, therefore, the money to be paid will be £$69\frac{3}{8}$ for every cent in £1110 stock,

or £$69\frac{3}{8} \times \frac{1110}{100}$ = £770 1*s.* 3*d.* *Ans.*

The income is £$2\frac{1}{2}$ for every cent; hence

£$2\frac{1}{2} \times \frac{1110}{100}$ = £27 15*s.* per annum. *Ans.*

Ex. (4). If I sell out £2008 of 3 per Cent. stock, when the price is 73 and brokerage $\frac{1}{8}$, what money shall I receive?

I shall receive $73 - \frac{1}{8}$ or $72\frac{7}{8}$ for every cent;
hence, £$72\frac{7}{8} \times \frac{2008}{100}$ = £1463 6*s.* $7\frac{1}{5}$*d.* *Ans.*

Ex. (5). If I pay £1110 for $2\frac{1}{2}$ per Cent. stock at $69\frac{3}{8}$, and sell out again when the price has risen to $71\frac{1}{4}$, how much shall I gain by the transaction?

I buy $\frac{1110}{69\frac{3}{8}}$ cents, and sell them at a profit of £$1\frac{7}{8}$ on every cent; hence, £$1\frac{7}{8} \times \frac{1110}{69\frac{3}{8}} = \frac{£15 \times 1110}{555}$ = £30. *Ans.*

Ex. (6). When the $3\frac{1}{2}$ per Cents. are at 72, and the $4\frac{1}{2}$ per Cents. at 92, which stock is it best to invest in?

The dividends per £ of investment would be, respectively, $\frac{3\frac{1}{2}}{72}$ and $\frac{4\frac{1}{2}}{92}$, which are as $\frac{7}{144}$ to $\frac{9}{23}$, or as 7×23 to 9×18, or as 161 to 162; hence, the $4\frac{1}{2}$ per Cents. are best.

Ex. (7). A person sells out £1950 of $3\frac{1}{4}$ per Cent. stock at 75; how much 5 per Cent. stock at $112\frac{1}{2}$ can he purchase with the proceeds? and what will be the alteration of income?

$\frac{1950}{100}$ cents sold out at 75 produce £75 × $\frac{1950}{100}$; and this money will buy as many cents of the 2nd stock as the number of times it contains £$112\frac{1}{2}$, viz., $\frac{75}{112\frac{1}{2}} \times \frac{1950}{100}$ cents $= \frac{£1950 \times 75}{112\frac{1}{2}} = £1300$ of 5 per Cent. stock. *Ans.*

Income from 2nd stock = 13 cents at £5	= £65	0*s.*	0*d.*
Income from 1st stock = 19·5 cents at £$3\frac{1}{4}$	= 63	7	6
Ans. Increase of income	£1	12*s.*	6*d.*

Ex. (8). What description of stock at 77 would yield $3\frac{3}{4}$ per cent. interest on investments, after deducting 9*d.* in the £ income-tax?

$3\frac{3}{4}$ per cent. on 77 = £$\frac{231}{80}$, the dividend on £100 stock after deduction of income-tax; therefore, as a deduction of 9*d.* in the £ reduces the gross income to $\frac{231}{240}$ of what it was, the dividend on £100 stock, before deduction of the tax, is = £$\frac{231}{80} \div \frac{231}{240}$ = £8. Hence, a 3 per cent. stock is required. *Ans.*

Ex. (9). On Oct. 28th A sold to B £1600 of 3 per Cent. stock at $89\frac{1}{8}$, for the 8th Nov. account. The price, however, on the settling day had fallen to $88\frac{1}{4}$. What sum of gain was paid to A at the settlement?

$\frac{1600}{100}$ cents at $(89\frac{1}{8} - 88\frac{1}{4})$ = £$\frac{7}{8}$ × 16 = £14. *Ans.*

Here the settlement is equivalent to the following transaction:—B pays to A $89\frac{1}{8} \times \frac{1600}{100}$, or £1426, for the £1600 of stock, and forthwith sells it to A for £$88\frac{1}{4} \times \frac{1600}{100}$ or £1412, thus losing £14.

Ex. 70.

(1) What amount of stock, in the 3 per Cents. at 78, can be bought for £1833?

(2) What sum of money will be required to purchase £2250 of 5 per Cent. stock at $115\frac{1}{2}$?

(3) If I buy £1700 of 6 per Cent. stock at $91\frac{3}{8}$ and $\frac{1}{8}$ brokerage, what does it cost me?

(4) What quantity of stock is purchased, and what income obtained, by an investment of £858 in the $3\frac{3}{4}$ per Cents. at $89\frac{3}{8}$?

(5) If I sell out £3240 of 5 per Cent. stock at 98, and pay $\frac{1}{8}$ brokerage, what money shall I receive?

(6) What income will be derived from an investment of £7600 in the $3\frac{1}{2}$ per Cents. at $85\frac{1}{2}$?

(7) £5166 is invested in the 4 per Cents. at $102\frac{1}{2}$; find the quantity of stock purchased, the income, and the rate per cent. of interest yielded by the investment.

(8) What rate per cent. of interest shall I have for my money if I invest £3500 in $3\frac{1}{2}$ per Cent. stock at $82\frac{1}{4}$?

(9) A person invested £8950 in the purchase of $3\frac{1}{2}$ per Cent. stock at $76\frac{1}{2}$, and on the price rising to $78\frac{5}{8}$ sold out, and invested the proceeds in 4 per Cent. stock at $89\frac{1}{2}$. What amount of stock did he now hold, and what was his income therefrom?

(10) What rate per Cent. of interest shall I have for my money if I invest in the 3 per Cent. Consols at $84\frac{3}{8}$?

(11) What must be the market value of $3\frac{1}{4}$ per Cent. stock, to yield 4 per cent. interest on investments?

(12) A person invests £839 10*s.* in the $4\frac{1}{2}$ per Cents. at 92, and on the price rising to 94 he sells out, and invests the proceeds in the $3\frac{1}{2}$ per Cents. at 73. What is the change in his income?

(13) When £5333$\frac{1}{3}$ of 3 per Cent. stock can be bought for £3760, what is the market price?

(14) When investments in $5\frac{1}{2}$ per Cent. stock yield 5 per cent. interest, what is the market price of the stock?

(15) A holder of £3208$\frac{1}{3}$ stock in the $3\frac{1}{2}$ per Cents. sold out at 87, and invested the proceeds in 4 per Cent. stock at 98; find the change in his income.

(16) A person has £2051 5*s.* in the $3\frac{1}{4}$ per Cents. He now wishes to invest such a sum in $4\frac{1}{4}$ per Cent. stock at 95, that the united income may be £80. Required the sum to be invested?

(17) By investing £1500 in the $3\frac{1}{2}$ per Cents. a person obtained 4·48 per cent. interest on his capital. What was the price of stock? and what the amount of stock purchased?

(18) If £2850 has been invested in the 3 per Cents. at $78\frac{3}{8}$, at what price must the stock be sold out to gain £50?

(19) A has stock in the $3\frac{1}{2}$ per Cents. amounting to £1350, for which B bargains with him at $83\frac{1}{8}$ for the account. On the settling day the price is 84. Which party gains, and how much?

(20) What sum must be invested in $3\frac{1}{2}$ per Cent. stock at 87,

that the income, after deducting 8*d.* in the £ income-tax, may be 200 guineas?

(21) A person invested £4186 in the 4 per Cents. at $100\frac{5}{8}$. He afterwards sold out at 98, and invested the proceeds in $3\frac{1}{4}$ per Cent. stock at 78. How was his income affected by the transfer?

(22) A person by selling out of the 3 per Cents. at 90, and investing the proceeds in railway $4\frac{1}{2}$ per cent. stock, increases his income $33\frac{1}{3}$ per cent. Required the price of the railway stock.

(23) Which is the better investment—railway shares at £150, producing a yearly dividend of £5 10*s.* per share clear of income-tax, or $3\frac{1}{4}$ per Cent. stock at 86, subject to an income-tax of 7*d.* in the £?

(24) A having invested £2941 in the 3 per Cents. at 85, afterwards sells to B $\frac{2}{3}$ of his stock at $86\frac{1}{8}$ for money, and the remainder at $86\frac{3}{4}$ for the account. On the settling day A and B both sell out, and it is observed that A's gain upon the whole is £18 14*s.* 10*d.* more than B's; what was the market price of the stock on the settling day?

AVERAGES.

142. An AVERAGE quantity is one of two or more equal quantities to which a corresponding number of unequal quantities are reduced by addition and division.

143. A *Simple Average* is the quotient of the sum of several quantities divided by the number of quantities.

Thus, the average of the two ages, 6 years and 3 years, is $(6+3)\div 2=4\frac{1}{2}$ years; the average of the three lengths, 5, 2, and 11 yards, is $(5+2+11)\div 3=6$ yards; $6+3$ being $=4\frac{1}{2}+4\frac{1}{2}$; and $5+2+11=6+6+6$. If I receive 4*s.* to-day, 5*s.* to-morrow, and 9*s.* the day following, I in effect receive $(4+5+9)s.\div 3=6s.$ per day.

144. A *Compound Average* is the quotient of the sum of several products divided by the sum of the multipliers.

Thus, the average price per yard of 3 yards at 4*s.*, and

2 yards at 9*s*., is $(4s. \times 3 + 9s. \times 2) \div (3+2) = 30s. \div 5 = 6s$; the meaning of which evidently is, that as I buy 3 yards at 4*s*., and 2 yards at 9*s*., thus buying 5 yards for 30*s*., I in effect buy the whole at 6*s*. a yard.

145. *To find multipliers for two given multiplicands, so that the sum of the products may equal the product of a given intermediate multiplicand by the sum of the multipliers.*

RULE.—Take the difference between the intermediate and each of the first and second multiplicands; if these differences are equal, any two equal numbers will be multipliers as required; but if not, assign the difference of the 1st and intermediate multiplicands as a multiplier for the 2nd; and the difference of the 2nd and intermediate multiplicands as a multiplier for the 1st. These two multipliers, or any other two having the like ratio, will be such as are required.

Ex. (1). A number is divided into two parts, such that 14 being multiplied by the first, and 7 by the second, the sum of the products is equal to 10 multiplied by the entire number. Find the ratio of the two parts.

By the Rule, any multipliers having the ratio of 10 — 7 to 14 — 10, or of 3 to 4, will be such as are required for 1st and 2nd multiplicands, respectively.

The reason of the Rule will appear from the following consideration of this question:—The question implies that 14 times the 1st part added to 7 times the 2nd will be equal to 10 times the 1st added to 10 times the 2nd. Hence 14 – 10 times the 1st balances or equals 10 — 7 times the 2nd, or 4 times the 1st will equal 3 times the 2nd, or the 1st will be $\frac{3}{4}$ of the 2nd, that is, the 1st and 2nd parts will be in the ratio of 3 to 4. *Ans.*

Hence, if the two parts were required to amount together to a certain sum, they would be obtained by dividing that sum into parts having the ratio of 3 to 4. The following example is of this kind:—

Ex. (2). Divide the number 35 into two such parts, that 14 multiplied by the first added to 7 multiplied by the second may *amount to 350.*

Here, the entire number 35 is to be made by the multiplication of its two parts = 350, that is, $\frac{350}{35}$, or 10 times as great.

Having found, therefore, in the last example, the ratio of the two parts to be that of 3 : 4, we now divide 35 in that proportion, taking $\frac{3}{7}$ and $\frac{4}{7}$ of 35; viz. 15 and 20. *Ans.*—which is verified as follows:—

$$\begin{array}{rrr} 14 & \times\ 15 & = 210 \\ 7 & \times\ 20 & = 140 \\ \hline & 35 & 350 \end{array}$$

Ex. (3). How much tea at 3*s.* 3*d.* a lb. must be mixed with 20 lbs. at 4*s.* 3*d.*, that the mixture may be worth 3*s.* 8*d.* a lb.?

Here, we must first find such multipliers for 3*s.* 3*d.* and 4*s.* 3*d.*, that the sum of the products may equal the product of 3*s.* 8*d.* by the sum of the multipliers.

By the Rule, the two multipliers are as 4*s.* 3*d.* − 3*s.* 8*d.* to 3*s.* 8*d.* − 3*s.* 3*d.*, or as 7 to 5. But, by the question, the 2nd multiplier must be 20; therefore the 1st will be $\frac{7}{5}$ of 20 = 28, and the *Ans.* is 28 lbs., which is verified as follows:—

$$\begin{array}{rl} 28 \text{ lbs. at } 3s.\ 3d. & = 91s. \\ 20 \text{ lbs. at } 4s.\ 3d. & = 85s. \\ \hline 48 \text{ lbs. at } 3s.\ 8d. & = 176s. \end{array}$$

Ex. (4). A person has 3 gallons of spirits at 17*s.* a gallon, and 5 gallons at 16*s.*; how many gallons at 23*s.* must be mingled with these, that the compound may be worth a guinea per gallon?

Here, if definite quantities were not required at the first two prices, we might arbitrarily choose any two equal quantities for these prices, to enable us to substitute one average price for both. But the question determines a particular value, viz. (17*s.* × 3 + 16*s.* × 5) ÷ (3 + 5) = 16*s.* 4½*d.* as the average price of the 8 gallons. We are to find, therefore, how many gallons at 23*s.* must be mingled with 8 gallons at 16*s.* 4½*d.*, that the compound may be worth 21*s.* a gallon.

The quantity at 16*s.* 4½*d.* : the quantity at 23*s.* :: 23*s.* − 21*s.* : 21*s.* − 16$\frac{3}{8}$*s.*, or as 2 : 4$\frac{5}{8}$, or 16 : 37; but, the question requires that the first quantity shall be 8 gall.; hence 16 : 37 :: 8 gall. : 18½ gall. *Ans.*, which is verified as follows:—

$$\begin{array}{rl} 3 \text{ gall. at } 17s. & = 51s. \\ 5 \text{ gall. at } 16s. & = 80s. \\ 18\frac{1}{2} \text{ gall. at } 23s. & = 425\frac{1}{2}s. \\ \hline 26\frac{1}{2} \text{ gall. at } 21s. & = 556\frac{1}{2}s. \end{array}$$

Ex. (5). If 150 apples cost 9*s.* 4½*d.*, how many of them must be sold at the rate of 3 for 2½*d.*, leaving the rest to be sold at the rate of 8 for 6½*d.*, that the gain on the whole may be 10 per cent. ?

The selling price of each of the dearer apples is to be $2\frac{1}{2}d. \div 3$, or $\frac{5}{6}d.$, and that of each of the cheaper ones $6\frac{1}{2}d. \div 8$, or $\frac{13}{16}d.$ Also, the average selling price of both is to be $\frac{112\frac{1}{2}d.}{150} \times \frac{110}{100} = \frac{23}{40}d.$ Accordingly, the number of dearer apples will be to that of cheaper ones as $\frac{23}{40} - \frac{13}{16} : \frac{5}{6} - \frac{33}{40}$, or as $\frac{1}{80} : \frac{1}{120}$, or as 3 : 2; hence $\frac{3}{5}$ and $\frac{2}{5}$ of 150 = 90 dearer, and 60 cheaper apples. *Ans.*

Ex. 71.

(1) Find the average of the values 17, 13, 25, 16, 19, and 20; also that of $\frac{2}{15}$ and $1\frac{1}{9}$.

(2) The ages of the children in a family are $3\frac{1}{4}$, $5\frac{1}{2}$, 7, $8\frac{2}{3}$, $10\frac{5}{12}$. What do they average ?

(3) A party of 11 persons, having supped at a tavern, pay the landlord as follows:—3 persons each 4*s.* 10*d.*, 2 each 4*s.* 8*d.*, 2 each 5*s.* 3*d.*, and the rest each 3*s.* 10*d.*, with the exception of one whom the landlord exempted from paying. What was the landlord's average receipt per head ?

(4) What is the average price, per gallon, of 35 gallons at 26*s.* 6*d.*, 15 at 29*s.*, and 55 at 32*s.* 6*d.* ?

(5) A person mixes 1 lb. of his best coffee with 5 lbs. at 1*s.* 4*d.* a lb., and produces a mixture worth 1*s.* 4½*d.* a lb. What is the price of his best coffee ?

(6) If 15 lbs. of sugar are bought at 4½*d.* a lb., and 24 lbs. at 7½*d.*, and if the two quantities are then mixed and sold at 7*d.*, what is gained per cent. ?

(7) The result of a chemical analysis is variously given by different chemists as 11·1, 13·01, 12·5, and 12. How much per cent. below the average of these numbers is the smallest of them ?

(8) What quantities of coffee, respectively worth 1*s.* 6*d.* and 1*s.* 3½*d.* a lb., will make a mixture of 20 lbs. worth 1*s.* 4¼*d.* a lb. ?

(9) A beer merchant wishes to mix 5 guinea ale with 7 guinea ale, so as to produce £6 ale. In what proportions must he mix the two ?

(10) Divide the number 25 into two such parts, that 5 times one and 11 times the other shall together make 206.

(11) How much gold of the fineness of 18 carats must be combined with $10\frac{1}{2}$ ounces of the fineness of 23 carats, that the compound may be 21 carats fine?

(12) A person bought a certain number of yards of silk at 6*s.* 4*d.* a yard, together with 49 yards at 5*s.* 7*d.*, the average cost of the whole being 5*s.* 9*d.* a yard; how many yards were bought at 6*s.* 4*d.*?

(13) Divide the number 100 into two parts, so that $\frac{6}{7}$ of one added to $\frac{7}{8}$ of the other may make 87.

(14) A bullion-dealer gives £8 8*s.* 9*d.* for 36 ounces of old silver, paying 4*s.* $10\frac{1}{2}$*d.* an ounce for part of it, and 4*s.* 6*d.* an ounce for the remainder; how much of each kind does he buy?

(15) A schoolmaster, six months after his appointment to a mixed school, finds he has 10 boys for every 7 he had at first, and 8 girls for every 9 girls, amounting on the whole to 19 scholars for every 15: compare the original numbers of boys and girls.

(16) I bought goods for £825, and sold them in two lots: one at a profit of 12 per cent., and the other at a loss of 10 per cent., the whole of the nett proceeds being £874 10*s.*; what was the prime cost of each lot?

(17) A market-woman bought a certain number of eggs at 3 a penny, and a number, exceeding the former by 5 dozen, at 2 a penny, thus buying the whole at the average price of $5\frac{1}{2}$*d.* a dozen: how many did she buy?

(18) A person has a certain sum invested in a railway enterprise, which pays a 5 per cent. dividend; he has an equal amount invested in another concern paying $7\frac{1}{2}$ per cent., and £3300 in a third which pays $4\frac{1}{4}$ per cent.; and the whole amount of his dividends is just $5\frac{1}{3}$ per cent. on that of his investments: what is the amount of his investments?

(19) A grocer has three different sorts of tea; the second being $\frac{1}{2}$ as dear again as the first, and the third $\frac{1}{4}$ as dear again as the second. If he mixes a certain quantity of the first with twice as much of the second and 1 cwt. of the third, and finds the mixture to be $\frac{1}{5}$ as dear again as the second sort, of how many lbs. does the mixture consist?

(20) A wine-merchant buys two kinds of whisky, on 3 months' *credit, at the* respective rates of 16*s.* 6*d.* and 18*s.* 9*d.* per gallon:

in what proportions should he mingle these, that he may gain 12½ per cent. by selling the mixture forthwith at the rate of 19*s.* 5½*d.* per gallon, ready money, reckoning credit to be worth 5 per cent. per annum?

(21) A's salary was one-third more than B's; but they were each promoted, with increase of salary, B's increase being £13 for £8 of A's, and each had then £168 per annum; required their original salaries.

EQUATION OF PAYMENTS.

146. The EQUATION OF PAYMENTS is the determination of an average time for one payment of an entire sum that is due in partial payments at sundry times.

The ordinary Rule for the equation of payments is founded on mercantile discount, and is as follows:—

RULE.—Multiply each payment by its time, and divide the sum of the products by the sum of the payments; observing to use all the times, as well as the payments, in a uniform denomination.

Ex. (1) I owe A. B. £61 due 4 months hence, and £83 due 9 months hence; find the time at which the debt may be discharged by one payment of the £144.

Here, according to the Rule,

$$\begin{array}{rl} 61 \times 4 = & 244 \\ 83 \times 9 = & 747 \\ \hline 144 \qquad & \overline{|991} \end{array}$$

6·882 months. *Ans.*

To understand the above procedure, observe that if we could find the present worth of £61 due 4 months hence, and that of £83 due 9 months hence, then the sum of those present worths would be the present worth of £144 for the required time; and this implies that the sum of the discounts of the partial payments, for their respective times, should be equal to the discount of the

entire payment for the required time. Now, in the case of mercantile discount, it will be easily seen, that whatever be the rate of interest, the required time will be constant; for the meaning of the above solution is, that whatever be the value of credit—

the int. on £61 for 4 months = int. on £244 for 1 month.
that on £83 for 9 months = int. on £747 for 1 month.
that on £144 for 6·882 mths. = int. on £991 for 1 month.

Note.—The true equation of payments is, of course, founded on true discount; but it often involves laborious calculation, and differs very slightly from the ordinary equation in the generality of cases that occur in practice. The ordinary Rule, therefore, should be usually followed.*

We will here give, however, a solution of the preceding question on the principle of true discount, assuming the value of credit to be 5 per cent. per annum:—

Interest on £100 for 4 months = $\frac{1}{3}$ of 5 = $1\frac{2}{3}$;
Interest on £100 for 9 months = $\frac{3}{4}$ of 5 = $3\frac{3}{4}$;
$101\frac{2}{3}$: $1\frac{2}{3}$ :: £61 : £1 = the true discount on £61;
$103\frac{3}{4}$: $3\frac{3}{4}$ :: £83 : £3 = the true discount on £83;
therefore, £4 = the true discount on £144.

And now we have to find the time for which £4 is the true discount on £144, at 5 per cent. per annum:

£140 : £4 :: £100 : $\frac{20}{7}$, the interest on £100 for the time sought; $\frac{20}{7} \div 5 = \frac{4}{7}$ of a year = 6·857 months, the true equated time; which is less than the answer formerly found by only ·025 or $\frac{1}{40}$ of a month.

Ex. (2) Suppose £1357 to be due 12 months hence, but that $\frac{1}{9}$ of it is paid in 6 months, and $\frac{1}{18}$ in 9 months; what time should be allowed for payment of the remainder?

The instalments are as $\frac{1}{9}$, $\frac{1}{18}$, $\frac{5}{6}$, or as 2, 1, 15; sum = 18.
£18 for 12 months . . = £216 for 1 month.
£2 for 6 mths. = £12 for 1 mth.
£1 for 9 mths. = £9 for 1 mth. = £21 for 1 month.
Remaining payment = £195 for 1 month.
= £15 for 195 ÷ 15, or 13 months. *Ans.*

* The principle of Equation of Payments is discussed in the author's '*Art of Teaching* Arithmetic,' p. 79.

Ex. 72.

Find the equated time for one payment of—

(1) £60 due in 5 months, and £36 in 9 months.

(2) £320 due in 37 days, and £480 in 27 days.

(3) £140 due in 5 weeks, £100 in 11 weeks, and £90 in 13 weeks.

(4) £135 due in 10 months, £81 in 14 months, and £576 in 17 months.

(5) £143 due in 23 days, £99 in 45 days, £513 in 67 days, and £81 in 89 days.

(6) A. owes B £5700, payable 8 months hence. If he pays £1400 of it 4 months hence, and £1000 of it 7 months hence, how many months hence should the remainder be considered payable?

(7) The sum of £657 is due in 14 months. If $\frac{1}{3}$ of the sum be paid in 9 months, and $\frac{1}{5}$ of it in 13 months, in what time ought the remainder to be paid?

(8) Find, (i) according to true discount, at 5 per cent., (ii) according to mercantile discount, the equated time for one payment of £86 due $1\frac{1}{2}$ year hence, and £99 due 2 years hence.

(9) A certain sum of money is due 6 months hence. If $\frac{3}{16}$ of it be paid in 3 months, and $\frac{7}{16}$ in 5 months; in what time, (i) according to true discount at 5 per cent., (ii) according to mercantile discount, ought the remainder to be paid?

EXCHANGE.

147. EXCHANGE is the comparison and conversion of the moneys of different countries.

148. The *Rate of Exchange* expresses how much of the money of one country is equivalent to a fixed sum of the money of another. From a variety of causes, the rate is continually fluctuating; and the current rate at any particular time is called the *Course of Exchange*.

Ex. (1) Reduce 3609·85 florins, Dutch money, to British, at 11 *flor. 45 cents* per £ sterling.

For every 11·45 flor. in 3609·85 flor. there will be an equivalent £1; hence 360985 ÷ .1145 = £315·27074 = £315 5*s.* 5*d.*, nearly. *Ans.*

Ex. (2) Reduce £421 17*s.* 5*d.* sterling to French money: exchange, 24 francs 93 cents per £.

£421·8708$\frac{1}{3}$ × 24·93 = 10517·239875 francs = 10517 francs 24 cents, nearly. *Ans.*

Ex. (3) What amount of French money corresponds to 362 florins 33 kreutzers of Frankfort, when the course of exchange is 9 florins 25 kreutzers for 20 francs, the florin being=60 kreutzers?

Here 20 francs for 9$\frac{5}{12}$ florins gives one franc for $\frac{113}{240}$ florin; hence, 362$\frac{11}{20}$ ÷ $\frac{113}{240}$ = $\frac{87012}{113}$ francs = 770·02 francs, nearly; or 770 francs 2 cents. *Ans.*

149 The *Par of Exchange* is the intrinsic value of the money of one country, in comparison with a fixed sum of the money of another.

150. *Arbitration*, or *Arbitrated Par of Exchange*, is the appreciation of the rate of exchange, as determined from a comparison of the purchase of bullion, coins, bills, &c., in one country, with the sale of them in another.

Ex. (4) What arbitrated par of exchange, in francs per £ sterling, is established between London and Paris, by bills on Naples, bought in London at 39½*d.* per ducat, and sold in Paris at 418 francs per 100 ducats?

39½*d.* = 1 ducat; ∴ 39½*d.* × 100 = 100 ducats;

79*d.* × 50 = 418 francs; hence $1d. = \frac{418}{79 \times 50}$ francs;

therefore, £1 or $240d. = \frac{418 \times 240}{79 \times 50} = 25{\cdot}4$ francs, nearly;

or, 25 francs 40 cents per £. *Ans.*

Ex. (5) A merchant in London owes one in Lisbon £467 12*s.* 3*d.*, for which the Portuguese merchant may draw directly at 68*d.* per milree (= 1000 rees); but, instead of this, he draws through Amsterdam and Paris. The exchange between London and Amsterdam is 11 florins 17½ cents, Dutch, per £ sterling, between Amsterdam and Paris 140 florins for 300 francs, and between Paris

and Lisbon 460 rees for 3 francs; and an expense of $1\frac{1}{2}$ per cent. is incurred in the circuitous course. By how much is the indirect better or worse, for the Portuguese merchant, than a direct remittance?

Here, a direct remittance of 112227*d.* at exchange 68*d.* per milree = 112227 ÷ 68 = 1650·397 milrees.

For the indirect course we have as follows:—

$\frac{460 \text{ rees}}{3}$ given for 1 franc; $\frac{460 \text{ rees} \times 300}{3}$ for 300 francs, or 140 florins; $\frac{460 \text{ rees} \times 300}{3 \times 140}$ for one florin; $\frac{460 \text{ rees} \times 300 \times 11{\cdot}175}{3 \times 140}$ for 11·17$\frac{1}{2}$ florins, or £1; $\frac{460 \text{ rees} \times 300 \times 11{\cdot}175 \times 467{\cdot}6125}{3 \times 140}$ for £467 12*s.* 3*d.* and this result ÷ 1000 rees, and reduced by $1\frac{1}{2}$ per cent. for the expense of the remittance, becomes

$\frac{46 \text{ milr.} \times 11{\cdot}175 \times 467{\cdot}6125 \times 98\frac{1}{2}}{140 \times 100} = 1691{\cdot}218$ milr. by indirect remittance, which is better, for Portugal, than the direct remittance by 40·821 milrees. *Ans.*

Otherwise.—By the *Chain Rule.*

? milrees = £467·6$\frac{1}{8}$	$\frac{197 \text{ m.} \times 46 \times 447 \times 37409}{100 \times 140 \times 8 \times 8 \times 100} =$
when £1 = 11·1$\frac{3}{4}$ fl.	1515331·60026 ÷ (14 × 8 × 8)
140 fl. = 300 fr.	= 1691·218 milr. &c., as before.
3 fr. = ·46 milr.	
100 milr. = 98$\frac{1}{2}$ milr.	

151. In the United States of America, where accounts are kept in dollars and cents, the variation of the rate of exchange with this country is generally indicated by so much per cent. *premium* or *discount* on the par value of the dollar, which is fixed at 4*s.* 6*d.* Thus, if sterling money be said to bear a premium of 9 per cent. in America, the meaning is, that £100 sterling is equivalent to the number of dollars which, at 4*s.* 6*d.* each, would amount to £109.

152. In France, where accounts are kept in francs and centimes, the standard of value is silver; in England it is *gold.* And between two countries thus adopting different

metals as the standard of currency, an invariable par cannot properly be said to exist. Standard gold, however, having with us a uniform value, viz. 77*s.* 10½*d.* an ounce, and the usual price of our silver being 5*s.* 1*d.* per ounce, £1 is thence estimated to correspond to 25·17 francs, which value is the basis of computation in our newspaper reports of the varying price of gold in France; as in the following example from 'The Times' of Nov. 22, 1865:—

'The quotation of gold at Paris is about 1 per mille premium; and the short exchange on London is 25·17½ per £1 sterling. On comparing these rates with the English Mint price of £3 17*s.* 10½*d.* per ounce for standard gold, it appears that gold is about one-tenth per cent. dearer in Paris than in London.'

To understand the above report, it should be known that the expression *short exchange*, or *exchange at short*, refers to bills drawn at short sight, as at three days, and that the result of one-tenth per cent. is arrived at by the following calculation:—

The fixed number 25·17, increased by 1 per mille, would give exchange 25·195 francs for £1, which is compared with the actual rate thus—

25·175 : 25·195 :: 100 : 100·08 nearly—showing that gold is dearer in Paris than in London by about ·08, or nearly ·1, that is, $\frac{1}{10}$, per cent.

153. The following is another example of a customary mode of reporting the exchanges: it is from 'The Times' of Feb. 15, 1865:—

'By advices from Hamburg, the price of gold is 423 per Mark, and the short exchange on London is 13 · 4½ per £1 sterling. Standard gold at the English Mint price is, therefore, about two-tenths per cent. dearer in London than in Hamburg.'

Here the result is arrived at by first multiplying 423

marks by the fixed number ·03132 (*see* subjoined *Note*), which gives 13 marks 4 schill.; then stating as follows—

13 mks. 4 schill. : 13 mks. 4½ schill. :: 100 : 100·23,

which shows that standard gold is dearer in London than in Hamburg by about 2 tenths per cent.

Note.—The Hamburg Mark weight of fine gold weighs 3608 grains troy; and as an ounce of English standard gold contains 440 grains fine gold, and is worth 77*s.* 10½*d.*, the Hamburg Mark weight, at that rate, is worth 7663*d.*, of which £1 (or 240*d.*) is the ·03132 part; hence the employment of ·03132 as a constant number.

Ex. 73.

(1) Exchange 1020 francs 12½ centimes for British money, at 25·32 francs per £1 sterling.

(2) Convert £360 5*s.* 10*d.* sterling into French money, at 25 francs 18 cents per £.

(3) Reduce 2756 Dutch florins to British money: exchange, 11 florins 17¼ cents per £1 sterling.

(4) Reduce 4885 florins 38 cents Dutch money to British: exchange, 11 florins 20½ cents per £ sterling.

(5) Find the value, in British money, of a remittance from Calcutta of 2257 rupees 13⅔ annas, at 1*s.* 11½*d.* per rupee of 16 annas.

(6) How many rupees are equivalent to 1276·8 francs, at the exchange of 40 rupees for 95 francs?

(7) The value of £234 16*s.* 9*d.* is paid with 5946 francs 8½ cents.; what is the rate of exchange in francs per £?

(8) When £347 15*s.* 6*d.* sterling corresponds to 4174 florins 17 cents of Amsterdam, what is the exchange per £?

(9) When 5691 marks 13½ schill. Hambro' correspond to £417 15*s.* sterling, what is the rate of exchange per £?—16 schill. = 1 mark.

(10) What rate of exchange, in pence per silver ruble, is established between London and St. Petersburg, from the purchase of sugar in London amounting to £1270 17*s.*, the nett proceeds in St. Petersburg being 8356·28 rubles?

(11) Find what par of exchange is established between London and Paris, by bills on Madrid bought in London at 47½*d.* per dollar, and sold in Paris at 5 francs per dollar.

(12) How many United States dollars, each 4*s.* 6*d.* at par, will be received for £853 15*s.* sterling, when English money bears a premium of 8 per cent. ?

(13) How much sterling should be given for 6764 American dollars, each 4*s.* 6*d.* at par, if the exchange is at a premium of 10½ per cent. on the sterling money ?

(14) A merchant in London drew a bill, at 3 months, on Marseilles, for £435 10*s.*, to be encashed when due at 25·60 francs per £ sterling; he retained the bill in his own hand, and when it fell due, the rate of exchange was 25·28 francs; how much sterling did he receive ?

(15) If I buy a bill on Hamburg, at 13 marks 13¼ schillings per £ sterling (16 sch.=1 mk.), and send it to Amsterdam, where it is sold at 35 florins 42 cents per 40 marks; and if the proceeds are thence remitted to Paris at 57½ florins per 120 francs, what rate of exchange is thus established between London and Paris ?

(16) Hamburg owes in London 2852 marks 8 schillings: which will be best for London—to draw by direct course at 13 marks 10 schillings per £, or by an indirect course through Paris and Amsterdam; the exchange of Hamburg on Paris being 150 marks for 275 francs, of Paris on Amsterdam 136 florins for 300 francs, and of Amsterdam on London 10·78 florins per £ sterling ?

(17) St. Petersburg owes £736 18*s.* in London, at a time when the exchange is 30¼*d.* per silver ruble. At the same time the course of exchange between St. Petersburg and Paris is at the rate of 3 francs per silver ruble, between Paris and Frankfort 20 francs for 9 florins 25 kreutzers (1 flor.=60 kreutz.), and between Frankfort and London 11 florins 50 kreutzers for £1. Whether will a direct remittance, or a circuitous one through the places mentioned, be best for St. Petersburg; and by how much in Russian money, supposing that an expense of 1½ per cent. will be incurred in the circuitous course ?

(18) Cotton-wool is bought in London at 3⅜*d.* per lb., and sent to Marseilles, the export charges being 10 per cent.: what is the arbitrated selling price, per 50 kilogrammes, at Marseilles, supposing the sale-charges there to be 5½ per cent., 100 kilogrammes =

220½ lbs. avoirdupois, and the course of exchange to be francs 25·60 per £ sterling?

(19) When gold, bought in London at 77*s.* 10½*d.* per ounce, is sold in Paris at about 4 per mille premium, and the short exchange on London is 25·20 francs per £ sterling, in which place is English standard gold dearer, and by how much per cent. ?

(20) When the quotation of gold at Paris is about 3½ per mille premium, what short exchange on London would indicate gold to be 2 tenths per cent. dearer in London than in Paris ?

(21) When English standard gold is as dear in Paris as in London, and the short exchange on London is 25·22½ francs per £ sterling, at how much per mille premium is the quotation of gold at Paris ?

(22) Prove the truth of the result stated in the following Money Market Report, from 'The Times' of March 25, 1865 :—

'By advices from Hamburg, the price of gold is 425 per Mark, and the short exchange on London is 13..6¼ per £ sterling. Standard gold at the English Mint price is, therefore, about 6 tenths per cent. dearer in London than in Hamburg.'

(23) By advices from Hamburg, the price of gold is 426½ per Mark, and the short exchange on London is 13 marks 5¾ schillings per £ sterling : compare the price of standard gold in Hamburg with that in London.

(24) When standard gold is about 4 tenths per cent. dearer in London than in Hamburg, and the short exchange on London is 13 marks 5 schillings per £ sterling ; what is the price of the Hamburg Mark weight in marks Hambro' ?

RECTANGULAR AREAS.

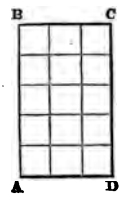
154. The area or square measure of a *Parallelogram*, that is, of a four-sided figure whose opposite sides are parallel, is found by multiplying the length of the figure by its perpendicular breadth.

When a parallelogram is right-angled, it is called a

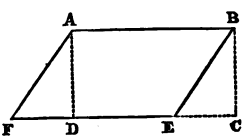
Rectangle; and a rectangle contained by equal sides is called a *Square*.

Ex. (1) What is the area of a rectangular surface 5 feet long and 3 feet broad?

Let the annexed figure represent the rectangular surface; A B, the length, being divided into 5 equal parts representing feet, and B C, the breadth, into 3 such equal parts. Then by drawing, as in the figure, through the points of division, lines parallel to the length and breadth, the whole surface is divided into 15 squares, the side of each being a *lineal* foot, and each of the squares is called a *square* foot. To find, therefore, the square or superficial measure of the given rectangle, we may call the lineal feet in either of the dimensions square feet, and multiply that quantity by the other dimension as an abstract number. For in the whole surface there are 5 rows, each consisting of 3 square feet—that is, 3 square feet $\times 5 = 15$ square feet; or in the whole surface there are 3 rows, each consisting of 5 square feet—that is, 5 square feet $\times 3 = 15$ square feet. *Ans.*



Note.—Every parallelogram is equal in area to a rectangle, whose length is equal to one of two opposite sides of the parallelogram, and breadth equal to the perpendicular distance between these sides. Thus, in the annexed figure, the parallelogram A B E F is equal to the rectangle A B C D, the former being reduced to the latter by cutting off the right-angled triangle A D F from one end of the parallelogram, and making it occupy the space B C E, at the other end.



Ex. (2). How many acres are in a rectangular field whose length is 1525 links, and breadth 425 links?

Area = 1525 square links $\times$ 425 = 648125 square links; which $\div$ 100000 square links (in an acre) = 6·48125 acres = 6 acres 1 rood 37 poles. *Ans.*

Ex. (3) What length of carpet, 2 ft. 11 in. wide, must be bought in order to cover a room-floor 21 ft. long and 16 ft. 3 in. wide?

Here $2\frac{11}{12}$ square feet × the number of feet in the required length of carpet, will be the area of the carpet, and will of course be equal to the area of the floor=21 square feet × $16\frac{1}{4}$; hence, the length of the carpet, in feet, $=21\times16\frac{1}{4}\div2\frac{11}{12}=63\times65\div35=9\times13$ feet; $=3\times13$ yards=39 yards. *Ans.*

Ex. (4) What will be the cost of covering a floor 18 ft. 9 in. by 15 ft. 7 in., with drugget $5\frac{1}{2}$ ft. wide, at 2*s.* 11*d.* a yard?

$5\frac{1}{2}$ square feet × length of drugget, in feet, $=18\frac{3}{4}$ square feet $\times15\frac{7}{12}$; therefore, length of drugget in yards$=\frac{18\frac{3}{4}\times15\frac{7}{12}}{5\frac{1}{2}\times3}$; hence, the cost $=\frac{35d.\times75\times187}{22\times3\times12}=\frac{35d.\times25\times17}{24}=$£2 11*s.* $7\frac{19}{24}$*d.* *Ans.*

Ex. (5) What is the whole area of the walls of a room 23 ft. 6 in. in length, 17 ft. 8 in. in width, and 12 ft. 8 in. in height?

Here the circuit of the room is (23 ft. 6 in. + 17 ft. 8 in.) × 2= 82 ft. 4 in.; and therefore the whole area of the walls is equal to that of a rectangle 82 ft. 4 in. long and 12 ft. 8 in. broad; hence, $82\frac{1}{3}$ sq. ft. $\times12\frac{2}{3}=1042\frac{8}{9}$ sq. ft., or 1042 sq. ft. 128 sq. in. *Ans.*

Ex. (6) A room is $19\frac{1}{2}$ ft. long, 13 ft. 5 in. wide, and 12 ft. high; what will it cost to paper the room with paper $1\frac{1}{2}$ ft. wide, at 9*d.* a yard,—neglecting deductions for door, windows, &c.?

Circuit of room=(19 ft. 6 in. + 13 ft. 5 in.) × 2=65 ft. 10 in.; area of walls = $65\frac{5}{6}$ sq. ft. × 12=$1\frac{1}{2}$ sq. ft. × length of paper in feet; therefore, length of paper in yards$=\frac{65\frac{5}{6}\times12}{1\frac{1}{2}\times3}$; hence, cost of paper $=\frac{9d.\times395\times2\times2}{3\times3}=1580d.=$£6 11*s.* 8*d.* *Ans.*

Ex. 74.

(1) How many square feet are contained in a parallelogram 63 ft. long and $6\frac{1}{4}$ ft. broad?

(2) A rectangular board is 9 ft. 9 in. long, and 2 ft. 7 in. wide: find its area.

(3) How many square yards in a rectangular space $137\frac{1}{2}$ yards long, and 131 ft. broad?

(4) Express, in the denomination of square yards, the area of a surface $2\frac{1}{2}$ feet long and $11\frac{1}{4}$ inches wide.

(5) A field measures 2950 links by 1835 links; how many acres does it contain?

(6) A rectangular field of 2 ac. 1 ro. 22 po. is 250 links in breadth; what is its length?

(7) The side of a square board measures 5 ft. 11 in.; what is its area?

(8) A rectangular surface is 14 ft. 10 in. long, and its area is 12 sq. yds. 5 ft. 104 in.; what is its width?

(9) Find the expense of painting a wall $36\frac{1}{3}$ ft. long and 31 ft. 4 in. high, at 3*s.* 6*d.* the square yard.

(10) How many planks, 11 ft. 8 in. long, and 9 inches wide, would floor a room 10 yards long and 7 yards wide?

(11) What would be the cost of painting a board 5 ft. $7\frac{1}{2}$ in. by 3 ft. $2\frac{1}{2}$ in., at 4*s.* 7*d.* the square yard?

(12) How many square yards of matting would cover a floor, the dimensions of which are 20 ft. 10 in. and 15 ft. $5\frac{1}{7}$ in.?

(13) How many yards of carpet, $2\frac{1}{6}$ feet wide, would cover a floor 20 ft. 7 in. long, and 20 ft. 3 in. wide?

(14) How many yards of carpet, yard wide, would cover a floor $14\frac{1}{6}$ ft. long, and $13\frac{1}{3}$ ft. wide?

(15) Express, in the denomination of square yards, the area of a parallelogram, 10·02 ft. long, and 9·342 in. wide.

(16) How many yards of carpet, $2{\cdot}75\frac{1}{3}$ ft. wide, would cover a square floor 9 yds. $2\frac{1}{2}$ ft. long?

(17) How many square yards of paper would cover the walls of a room $23\frac{1}{2}$ ft. long, 17 ft. wide, $12\frac{1}{2}$ ft. high,—deducting for door, windows, &c., a fifth of the whole area?

(18) How many yards of a paper that is $\frac{1}{2}$ a yard wide would be required for the walls of a room 17 feet long, 15 feet wide, and 13 feet high,—neglecting deductions for windows, &c.?

(19) How many yards of a paper 16½ inches wide would be required for the walls of a room 21 ft. 4 in. long, 17 ft. 5 in. wide, and 13¼ ft. high,—neglecting deductions?

(20) What will be the expense of carpeting a room 23 ft. 4 in. by 18 ft. 9 in., if the cost of carpet, ⅔ of a yard wide, be 6s. 6d. a yard, inclusive of planning and sewing?

(21) Find the cost of papering the walls of a room, whose length is 31 ft. 2 in., breadth 17 ft. 10 in., and height 13 ft. 6 in., with paper ⅞ of a yard wide, at 8d. a yard,—neglecting deductions.

(22) A room is 29 ft. 7 in. long, 21 ft. 4 in. wide, 12 ft. 11 in. high: what will be the expense of papering its walls, with paper 11¾ inches wide, at 7½d. a yard,—deducting for door, windows, &c. a sixth of the whole area?

(23) The length of a room is 27 ft. 8½ in., its breadth 17 ft. 8½ in., and its height 13 ft. 4 in.; the cost of papering its walls is £6 10s., the width of the paper being 20 inches: what is the cost of the paper per yard?

(24) A square room is 18 ft. 3 in. long; and the cost of papering its walls with paper 17 inches wide, at 7¼d. a yard, is £6 12s. 3¾d.: required the height of the room.

RECTANGULAR VOLUMES.

155. *A Rectangular Solid* or *Volume* is the space contained by six rectangular surfaces, and has three dimensions—viz., length, breadth, and height or thickness. When these three dimensions are equal, the solid is called a *cube*.

If we have a rectangular block of wood, 4 feet long, 3 feet wide, and 1 foot thick, the area of each of its largest surfaces is 4 square feet × 3 = 12 square feet; by means of which the block evidently can be divided into 12 equal blocks, each measuring 1 foot in length and breadth and thickness, and therefore forming the cube, called a solid or cubic foot. It will appear, then, that a block 4 feet long,

3 feet wide, and 1 foot thick, contains 4×3 cubic feet$=$ 12 cubic feet; and hence, if the thickness were 2 feet, the content would be 4×3 cubic feet $\times 2 = 24$ cubic feet; and, generally, the content of any rectangular solid is the continual product of its three dimensions, in any uniform denomination.

Ex. (1) What volume of air is contained in a room 16 feet long, 14 feet wide, and 12 feet high?

Here we have 16×14 cubic feet repeated 12 times—that is, 16×14 cubic feet $\times$ 12 $=$ 2688 cubic feet. *Ans.*

Ex. (2) A rectangular cistern is 5 ft. 3 in. long, 3 ft. 6 in. wide, and 2 ft. 8 in. deep; what weight of water will it hold, if a cubic foot of water weigh 1000 ounces avoirdupois?

The content of the cistern is $5\frac{1}{4} \times 3\frac{1}{2}$ cub. ft. $\times 2\frac{2}{3}$; and this multiplied by (1000 oz. $\div$ 16) will give the weight of water in

$$\text{lbs.} = \frac{21 \times 7 \times 8 \times 1000}{16 \times 4 \times 2 \times 3} = \frac{49000}{16} \text{ lbs.} = 1 \text{ ton } 7 \text{ cwt. } 1 \text{ qr. } 10\tfrac{1}{2} \text{ lbs.}$$

Ans.

Ex. (3) A block of stone, 8 ft. 9 in. long, 3 ft. 3 in. broad, and 2 ft. 9 in. thick, is equal in solid content to another rectangular block, each of whose ends is 3 ft. 9 in. square: required the length of the latter.

Here the required length in feet $\times 3\frac{3}{4} \times 3\frac{3}{4}$ is to be equal to $8\frac{3}{4} \times 3\frac{1}{4} \times 2\frac{3}{4}$; therefore the required length will be—

$$= \frac{35 \times 13 \times 11 \times 4 \times 4}{15 \times 15 \times 4 \times 4 \times 4} = \frac{1001}{180} = 5 \text{ ft. } 6\tfrac{11}{15} \text{ in.} \quad \textit{Ans.}$$

Ex. 75.

(1) What is the capacity of a rectangular box, in cubic feet, if the length, breadth, and depth are respectively 38, 30, and 27 inches?

(2) Find the solidity of a piece of timber, 15 ft. 5 in. long, 2 ft. 4 in. broad, and 2 ft. 1 in. thick.

(3) A marble slab is 7 ft. $4\frac{1}{2}$ in. long, 3 ft. 2 in. broad, and $2\frac{1}{4}$ in. thick: compute its weight, at 168 lbs. the cubic foot.

(4) The length of a cistern is 4 ft. 3·7 in., breadth 4 ft. 0·3 in.,

and depth 2 ft. 2·267 in.; find its capacity in imperial gallons, each 277·274 cubic inches.

(5) A rectangular piece of timber, 1 ft. 7 in. long, and 1 ft. 2 in. broad, contains a cubic foot; what is the thickness?

(6) How many rectangular stones, each 8 in. long, $4\frac{1}{4}$ wide, and $3\frac{1}{2}$ thick, would form a cubical pile 19 ft. 10 in. high?

(7) The circuit of a room, that is 3 feet longer than broad, measures 61 feet; and the cost of papering its walls with paper 9 inches wide, at 5*d.* a yard, amounts to £6 7*s.* 1*d.*: how many cubic yards of air does the room contain?

DUODECIMALS.

156. By some artificers, areas and volumes are often computed by the method of DUODECIMALS, according to which square and cubic feet are subdivided, not immediately into inches, but first into *primes*, each one-twelfth of a foot; then into *seconds*, each one-twelfth of a prime; then into *thirds*, each one-twelfth of a second, &c. In square measure the seconds are square inches; and in cubic measure the thirds are cubic inches.

Primes, seconds, &c., lineal, superficial, and cubic, are denoted by accent marks, as in the expression 23 square feet 8′ 10″ 7‴.

Duodecimal multiplication is performed as in the following example:—

Ex. (1) Multiply 7 ft. $4\frac{1}{2}$ in. by 3 ft. $5\frac{5}{6}$ in.

Here the given dimensions are = 7 ft. 4′ 6″ and 3 ft. 5′ 10″. Now, the superficial content of a rectangle 7 ft. 4′ 6″ long and 1 ft. broad would be 7 sq. ft. 4′ 6″, where the 4′ denotes 4 twelfths of a square foot, and the 6″ denotes 6 twelfths of a twelfth of a square foot, or 6 square inches. Accordingly, the breadth being 3 ft. 5′ 10″, the superficial content will be $3 + \frac{5}{12} + \frac{10}{144}$ times 7 sq. ft. 4′ 6″, and the multiplication is performed as follows:—

First, we calculate 3 times the multiplicand thus: 3 times 6 are $18'' = 1'\ 6''$; set down $6''$ and carry $1'$; 3 times $4' = 12'$, and $1'$ carried makes $13'$, = 1 sq. ft. $1'$ &c. Next we calculate $\frac{5}{12}$ of the multiplicand, by multiplying it by 5, and setting down the first result one place to the right of the preceding line; for the 12th of $6''$ is $6'''$, and therefore $\frac{5}{12}$ of $6'' = 5$ times $6'''$; similarly, $\frac{5}{12}$ of $4'$ is $= 5$ times $4''$; and so forth. Then we calculate $\frac{10}{144}$ of the multiplicand, by multiplying it by 10, and setting down the first result one place to the right of the preceding line; for the 144th of $6''$ is $6''''$, and therefore $\frac{10}{144}$ of $6''$ is $= 10$ times $6''''$; and so forth. Lastly, we add the partial products, to obtain the entire product, which denotes 25 sq. ft., 8 twelfths of a square foot, $6\frac{1}{4}$ twelfths of a twelfth of a square foot.

7 sq. ft.	4′	6″		
3	5	10		
22	1	6		
3	0	10	6	
	6	1	9	0
25 sq. ft.	8′	6″	3‴	

To express this result, if required, in square feet and inches, we reduce $8'$ to seconds, that is, to square inches, by multiplying by 12, which gives $96''$, and adding in the $6''\ 3''' = 6\frac{1}{4}''$, we obtain $102\frac{1}{4}$ sq. in.—thus making the entire product equal to 25 sq. ft. $102\frac{1}{4}$ sq. in.

Ex. 76.

Calculate, duodecimally, the areas contained by the following dimensions; and in each case convert the result into ordinary square measure:—

(1) 8 ft. 7 in. by 7 ft. 3 in.; and 11 ft. 5 in. by 6 ft. 8 in.
(2) 8 ft. $5\frac{5}{6}$ in. by 7 ft. 11 in.; and 13 ft. $3\frac{5}{8}$ in. by 13 ft. $3\frac{5}{8}$ in.
(3) 13 ft. $1\frac{7}{12}$ in. by 12 ft. $8\frac{5}{12}$ in.; and 6 ft. $0\frac{3}{4}$ in. by $6\frac{3}{4}$ in.
(4) $3\frac{2}{3}$ in. by $10\frac{1}{4}$ in.; and 2 ft. $11\frac{11}{12}$ in. by $2\frac{1}{4}$ ft.
(5) 16 ft. 7 in. by $7\frac{5}{12}$ in.; and 13 ft. 11 in. by 6 ft. $11\frac{1}{3}$ in.

Calculate, duodecimally, the volumes contained by the following dimensions; and in each case convert the result into ordinary cubic measure:—

(6) 4 ft. 9 in., 3 ft. 6 in., 2 ft. 3 in.
(7) 15 ft. $4\frac{1}{2}$ in., 5 ft. 8 in., $3\frac{1}{4}$ ft.
(8) 21 ft. $0\frac{1}{6}$ in., 4 ft. 7 in., 11 in.

(9) The area of a rectangular surface, 2 ft. 5′ 6″ long, is 5 sq. ft. 8′ 5″ 1‴; what is its width?

(10) Divide 250 sq. ft. 8′ 4″ by 25 sq. ft. 4′.

(11) One of the dimensions of a rectangle is 47 ft. $9\frac{11}{12}$ in., and the area is 41 sq. ft. 10′ 2″ 1‴ 6‴″; find the other dimension.

(12) The capacity of a rectangular vessel is 61 cub. ft. 1′ 6″ $3\frac{1}{2}$‴, and the area of the bottom is 24 sq. ft. 5′ 4″ 11‴; what is its depth?

SQUARE ROOT.

157. The SQUARE ROOT of a number is one of the two equal factors of which that number is the product.

Thus, 7 is the square root of 49, because 7×7 is 49; the number 49 being called the square of 7, because 49 is the area of a square whose side is 7.

158. The square root, or second root, of a number is denoted by prefixing to the number the letter *r* in the old form $\surd$; thus, $\surd 81 = 9$, denotes that the square root of 81 is 9.

The square root of a number is also indicated by the fraction $\frac{1}{2}$ placed above the number, a little to the right; thus, $81^{\frac{1}{2}} = 9$.

159. Most numbers are not exact squares. From 1 to 100 the only exact integral squares are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100; and therefore no numbers ending with 2, 3, 7, or 8, or with an odd number of ciphers, can be an exact square. But we can find approximate roots as near to accuracy as we please.

160. *To extract the square root of a given number.*

RULE.—Place a dot over the units' figure, and a dot also over every second figure to the left of units; and in the case *of decimals,* over every second figure to the right of units.

This will resolve the given number into periods, each consisting of two figures, except that occasionally the left-hand period contains only one.

Determine by inspection the nearest square not greater than the first, or left-hand period; write the root of that square as the first figure of the root required, and write the square itself under the period, subtracting it from the period. On the right of the remainder bring down the next period, and on the left of the number thus formed write, as a partial divisor for all but its right-hand figure, the double of the first figure of the root, and try to determine a quotient figure, as the next figure in the root, so that annexing that quotient figure on the right hand of the partial divisor, to form the complete divisor, that complete divisor may not have been quoted too high for the entire dividend. Then multiply the complete divisor by its quotient figure, subtracting the product from the dividend, and bringing down on the right of the remainder the next period, thus forming a new dividend, for which a trial divisor is to be formed, as before, by doubling the number composing the two figures of the root; and so forth, till the last period has been brought down, and the last figure of the root ascertained.

Ex. (1) Find the square root of 139876.

Here, placing a point over the units figure 6, and one over every second figure from it towards the left, the first period is found to consist of 13, and the highest square not greater than 13 is 9, the square of 3. Accordingly, writing 3 as the first figure of the required root, and subtracting its square from 13, and bringing down the next period, we have 498 as a dividend; then doubling the 3 gives 6, which we use as a trial divisor for 49, the quotient from which is 8. But on annexing 8 to the 6, and multiplying that completed divisor by 8, the result would exceed 498; and thus we see that the

$\sqrt{139876} = 374$. *Ans.*

	9
67	498
	469
744	2976
	2976

second figure of the root cannot be greater than 7, and, on trial, it is found to be not less; for annexing 7 to the 6, and multiplying by 7, gives 469, which subtracted from 498 leaves 29. Then we bring down the next period, 76, forming the final dividend 2976, as a divisor for which, we double the 37; or, which comes to the same thing, add to the preceding divisor, 67, its units figure 7. We now use 74 as a trial divisor for 297, the quotient from which is 4; and that figure, on being annexed to the 74, is found to be allowed by the dividend, since 744 multiplied by 4 gives the final dividend exactly.

It should be observed that the above method of extracting the square root is an abbreviation of the following:—

$$\begin{array}{rrl} & & \sqrt{139876} = 300 + 70 + 4 \\ & 300^2 & = 90000 \\ \hline 2\ ce\ 300 = & 600 & 49876 \\ & 70 & \\ \hline & 670 \times 70 = & 46900 \\ \hline 2\ ce\ 370 = & 740 & 2976 \\ & 4 & \\ \hline & 744 + 4 = & 2976 \end{array}$$

The demonstration of the preceding process is most conveniently shown by Algebra.*

161. The reason of pointing off the figures of the given square into periods will appear from the following considerations. The squares of numbers that consist of one figure occupy from one to two places of figures, those of numbers consisting of two figures occupy from three to four places, those of numbers consisting of three places occupy from five to six places, and so forth: thus,—

* Thus, the square of $x+y$ is equal to $x \times x + (2\,x+y) \times y$, which indicates the process for extracting a root that consists of two digits. Again, $(x+y+z)^2 = x \times x + (2\,x+y) \times y + \{2\,(x+y)+z\} \times z$; which indicates the procedure we have employed in the first example, where $x=300$, $y=70$, $z=4$.

$1^2 = 1$	$10^2 = 1,00$	$100^2 = 1,00,00$
$3^2 = 9$	$30^2 = 9,00$	$300^2 = 9,00,00$
$4^2 = 16$	$40^2 = 16,00$	$400^2 = 16,00,00$
$9^2 = 81$	$99^2 = 98,01$	$999^2 = 99,80,01$

Ex. (2) Extract the square root of 7·338681.

$\sqrt{7\cdot338681} = 2\cdot709$. *Ans.*

	4	
47	333	
	329	
5409	48681	
	48681	

Here we place a dot over the units figure 7, and a dot also over every second figure to the right of the decimal point. The nearest square to 7 being 4, we write 2 units as the first figure in the root, and proceed as in the first example, until we have formed the trial divisor 54, which is not contained in 48, and therefore has 0, the next figure in the root, annexed to it; upon which we bring down the next period 81, and then find that the trial divisor, 540, is contained 9 times in the dividend, 4868 &c.

Ex. (3) Extract the square root of ·35976004.

$\sqrt{\cdot35976004} = \cdot5998$ *Ans.*

	25	
109	1097	
	981	
1189	11660	
	10701	
11988	95904	
	95904	

Here the units figure, at which the pointing should begin, is 0; and from thence the dots will be over every second figure to the right of the decimal point. In this example we have an instance of a remainder, 116, exceeding the trial divisor, 109, which indicates that the next figure of the root will be high.

Ex. (4) Find the square root of 9·635; and also that of 23.

$\sqrt{9\cdot6350} = 3\cdot10403$ nearly. *Ans.*

	9	
61	63	
	61	
6204	25000	
	24816	
62080	1840000	

$\sqrt{23\cdot00} = 4\cdot7958$ &c. *Ans.*

	16	
87	700	
	609	
949	9100	
	8541	
9585	55900	
	47925	
9590	797500	

In these two examples, the given numbers are not exact squares, that is, the extraction will produce neither a terminating nor a circulating decimal; we can therefore only find approximate roots; and, in the first instance, the answer we have obtained is a little more, in the second a little less, than the truth.

Ex. (5) Extract the square roots of ·144 and ·001.

$\sqrt{\cdot 1\dot{4}\dot{4}0} = \cdot 37947$ &c. $\sqrt{\cdot 00\dot{1}0} = \cdot 031623$ nearly.

9	*Ans.*
67 | 540	
469	
749 | 7100	
6741	
7584 | 35900	
30336	
75887 | 556400	

9	*Ans.*
61 | 100	
61	
626 | 3900	
3756	
6322 | 14400	
12644	
6324 | 175600	

162. *To extract the square root of a vulgar fraction.*

RULE.—If the given denominator be an exact square, take the square roots of numerator and denominator separately, and the resulting fraction will express the value of the square root required. If the given denominator be not an exact square, make it so, by multiplying or dividing both terms of the fraction by any one number which will suit the purpose; then take the roots of numerator and denominator separately.

Ex. (6) Find the square roots of $\frac{9}{16}$, $\frac{5}{8}$, and $5\frac{3}{7}$.

Here, since $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$, we have $\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$. *Ans.*

To make the denominators of the two other proposed numbers exact squares, we may take $\frac{5}{8} = \frac{2 \cdot 5}{4}$, and $\frac{38}{7} = \frac{266}{49}$.

$\sqrt{2\cdot50} = 1\cdot581$ &c.

	1	
25	150	
	125	
308	2500	
	2464	
316	3600	

$\sqrt{266} = 16\cdot31$ nearly.

	1	
26	166	
	156	
323	1000	
	969	
326	3100	

Hence, $\sqrt{\frac{5}{8}} = \frac{\sqrt{2\cdot5}}{\sqrt{4}} = \frac{1\cdot581}{2}$ nearly $= \cdot7905$ &c. *Ans.*

$\sqrt{5\frac{3}{7}} = \frac{\sqrt{266}}{\sqrt{49}} = \frac{16\cdot31}{7}$ nearly $= 2\cdot33$ nearly. *Ans.*

Note.—A vulgar fraction may be reduced to the decimal form before extracting its root; and this is often more convenient than the method prescribed in the Rule. To obtain the square roots of $\frac{5}{8}$ and $5\frac{3}{7}$ in this way, we proceed as follows:—

$\sqrt{\frac{5}{8}} = \sqrt{\cdot6250} = \cdot7905$ &c.

	49	
149	1350	
	1341	
1580	90000	

$\sqrt{5\frac{3}{7}} = \sqrt{5\cdot4285}$ &c. $= 2\cdot33$ nearly.

	4	
43	142	
	129	
46	1385	

Ex. 77.

Find the square root of—

(1) 1369; 6241; 77841.
(2) 434281; 739600.
(3) 817216; 1428025.
(4) 3481956; 5322249.
(5) 24502500; 3620·4289.
(6) 9798·822121; 1·048576.
(7) 3481826049; ·96177249.
(8) ·10227204; 8260·9921.
(9) ·00717409; ·0009771876.
(10) 609804810000; ·000801.
(11) 24·681; 1·524.
(12) 127021·55; ·30103.
(13) 39484; ·0103039.
(14) 6; ·6; 26; ·02.
(15) $\frac{2}{5}$; $\frac{4}{5}$; $\frac{1}{3}$; $1\frac{5}{8}$.
(16) $2\frac{1}{12}$; $5\frac{4}{5}$; $55\frac{1}{8}$; $30\frac{1}{4}$.
(17) $1592\frac{1}{3}$; $558\cdot53\dot{4}$; $\cdot5\dot{1}$.
(18) $13\cdot7\frac{2}{7}$; $\cdot0700\frac{5}{9}$; $\cdot3\dot{1}0\dot{0}$.

163. The square root of a number is equal to the products of the square roots of the factors of that number.

thus, $\sqrt{36}=\sqrt{4}\times\sqrt{9}=2\times3=6$; also, $\sqrt{8}=\sqrt{4}\times\sqrt{2}=2\sqrt{2}$. Accordingly, we may put

$\sqrt{24\times56\times21}^{*}=\sqrt{3\times8\times8\times7\times7\times3}=\sqrt{3^2\times8^2\times7^2}$, which is equal to $3\times8\times7$, or 168.

Conversely, the product of any number of square roots equals the square root of the product of the squares; thus, $\sqrt{3}\times\sqrt{7}=\sqrt{3\times7}=\sqrt{21}$; and, accordingly, we may put $\sqrt{5}\times\sqrt{15}\times\sqrt{10}\times\sqrt{2}=\sqrt{5\times5\times3\times5\times2\times2}=5\times2\times\sqrt{15}=10\sqrt{15}$.

164. A fractional expression, containing a denominator under the radical sign, may be simplified by multiplying both its terms by that denominator. Thus,

$$\frac{3}{\sqrt{2}}=\frac{3\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}=\frac{3\sqrt{2}}{2}, \text{ or } \frac{3}{2}\sqrt{2}; \text{ so also,}$$

$$\frac{6\sqrt{7}}{\sqrt{3}}=\frac{6\times\sqrt{7}\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}=\frac{6\times\sqrt{21}}{3}=2\sqrt{21}.$$

Ex. 78.

Simplify the following expressions:—

(1) $\sqrt{49\times81}$; $\sqrt{64\times30\frac{1}{4}}$.

(2) $\sqrt{81\times8\times2}$; $\sqrt{18\times12\times6}$.

(3) $\sqrt{42\times24\times28}$; $\sqrt{12\frac{1}{2}\times18}$.

(4) $\sqrt{56\times40\times35}$; $\sqrt{33\times8\frac{1}{4}}$.

(5) $\sqrt{65}\times\sqrt{45}\times\sqrt{52}$; $\sqrt{13\frac{1}{2}}\times\sqrt{1\frac{1}{2}}$.

(6) $\sqrt{2\frac{1}{2}}\times\sqrt{1\frac{2}{3}}\times\sqrt{1\frac{1}{2}}$; $\sqrt{87\frac{1}{2}}\times\sqrt{\frac{7}{8}}$.

(7) $\sqrt{15\times27}$; $\sqrt{(32\times30\frac{1}{4})}\times\sqrt{22}$.

(8) $\frac{3}{14}$ of $2\sqrt{5}\times\frac{2}{3}$ of $\frac{1}{2}\sqrt{10}$.

(9) $\sqrt{\frac{1}{13}\text{ of }(\frac{49}{156}-\frac{25}{144})}$. (10) $\sqrt{(\frac{3}{95}+\frac{5}{57}+\frac{15}{76})}\times\sqrt{\frac{19}{60}}$.

(11) $\frac{3}{\sqrt{5}}$; $\frac{8}{3\sqrt{6}}$; $\frac{\sqrt{8}}{5\sqrt{2}}$. (12) $\frac{\frac{1}{3}\sqrt{15}}{\frac{1}{2}\sqrt{3}}$; $\frac{\sqrt{4\frac{1}{2}}}{6\sqrt{90}}$; $\frac{5\sqrt{6}}{\sqrt{8}+2\sqrt{3}}$.

---

* A line drawn over the factors or terms of an expression is often *used, as above,* instead of a bracket, and is called a *vinculum*. See p. 85.

165. By means of the square root, we can find the side of a square whose area is given; for, in this instance, length and breadth are equal, and the product of length and breadth will be just the square of either; consequently, the square root of the number expressing the square measure will be the number expressing the side of the square in corresponding lineal measure.

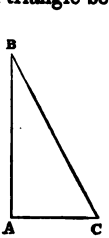
Ex. Find the side of the square whose area is equal to that of a parallelogram 5 ft. 3 in. long and 2 ft. 4 in. broad.

$\sqrt{63 \times 28} = \sqrt{9 \times 7 \times 7 \times 4} = 3 \times 7 \times 2 = 42$ in. $= 3$ ft 6 in. *Ans.*

166. It is usual, in Treatises on Arithmetic, to refer to the following property of the right-angled triangle:—

In any right-angled triangle, the squares on the sides containing the right-angle are together equal to the square on the hypotenuse, or side opposite the right-angle.

Ex. (1) If the perpendicular sides of a right-angled triangle be 30 and 16, what is the hypotenuse, or longest side?



Here, AB = 30, AC = 16; and $30^2 + 16^2 = 900 + 256 = 1156$, the square on BC; therefore, BC $= \sqrt{1156} = 34$. *Ans.*

Ex. (2) Two persons set out from one place at the same time; one of them walks due north, the other due east, and at the end of 6 hours the direct distance between them is 30 miles. If the rate of the former be 3 miles an hour, what is the rate of the latter?

Here AC = 3 mi. $\times$ 6 = 18 mi., and BC = 30 mi.; therefore, $30^2 - 18^2 = 900 - 324 = 576$, the square on AB; hence AB $= \sqrt{576} = 24$ mi.; and $24 \div 6 = 4$ mi. an hour. *Ans.*

Note 1.—We shall form a right-angled triangle if we take for the three sides any lengths bearing the proportion of 3, 4, 5. We shall also form a right-angled triangle, if we take for the three sides any lengths that are, respectively, the sum of two square numbers, the difference of these squares, and twice the product of

their roots; as, for example, $25 + 9$, $25 - 9$, and $5 \times 3 \times 2$, or 34, 16, and 30. See *Ex.* 1.

Note 2.—It is often convenient to treat the difference of the squares of two numbers as equivalent to the product of the sum and difference of the numbers; thus, $13^2 - 8^2$, instead of being put $= 169 - 64 = 105$, may be put $= (13 + 8) \times (13 - 8) = 21 \times 5 = 105$.

Ex. 79.

(1) The surface of a square table is 26 sq. ft. 100 in.; find the length of the side.

(2) A square field contains 1 ac. 1 ro. $13\frac{4}{11}$ po.; required the length of its side in links.

(3) Find the side of a square equal in area to a rectangle whose dimensions are 16 ft. by 12 ft. 3 in.

(4) Find the hypotenuse of a right-angled triangle, the other two sides being 45 and 28 yards.

(5) The hypotenuse of a right-angled triangle is 75 feet, and one of the other sides is 45 feet; find the remaining side.

(6) Required the area of a rectangle the diagonal of which is 4 ft. 5 in., and the length 3 ft. 9 in.

(7) Find the side and area of a square, of which the diagonal is 36 yards.

(8) Find the length of a ladder that will reach from the edge of a horizontal path, 6 ft. $4\frac{2}{3}$ in. wide, to the top of a wall on the other side, 14 feet high.

(9) The surface of a parallelogram, thrice as long as it is wide, is 6 sq. ft. 108 in. What are its dimensions?

(10) The area of a rectangle, twice as long as it is broad, is 1364·1954 sq. yds. Find the diagonal.

(11) The length of a rectangular field is ·0206 of a mile more than its width, and its perimeter is ·1084 of a mile. What is its diagonal?

(12) The diagonal of a cube of ivory is ·53 of an inch longer than the diagonal of one of its faces. What is the length of its edge?

CUBE ROOT.

167. The cube root of a number is one of the three equal factors of which that number is the continued product.

Thus, 7 is the cube root of 343, because $7 \times 7 \times 7 = 343$; the number 343 being called the cube of 7, because 343 is the volume of a cube whose side is 7.

168. The cube root, or 3rd root of a number, is denoted by writing the figure 3 a little above the radical sign on the left, or the fraction $\frac{1}{3}$ a little above the number on the right. Thus, $\sqrt[3]{81}$, or $81^{\frac{1}{3}} = 3$.

169. Most numbers are not exact cubes. From 1 to 1000 the only exact integral cubes are 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000.

170. *To find the cube root of a given number.*

RULE.—Place a dot over the units figure, and a dot also over every third figure to the left of units, and, in the case of decimals, over every third figure to the right of units. This will resolve the given number into periods, each consisting of three figures, except that occasionally the left hand period contains only one or two.

Determine, by inspection, the nearest cube not greater than the first, or left hand period; write the root of that cube as the first figure of the root required, and write the cube itself under the period, subtracting it from the period. On the right of the remainder bring down the next period; at some distance to the left of the number thus formed, write the triple of the first figure of the root, and between them write 300 times the square of the first figure of the root as a partial divisor; then try to determine a quotient figure, as the second figure in the root, so that after annexing that quotient figure on the right hand of the triple of the first radical figure, multiplying the number thus formed

by the second radical figure, and adding the product to the partial divisor to form the complete divisor, that complete divisor may not have been quoted too high for the dividend. Then multiply the complete divisor by its quotient figure, subtracting the product from the dividend, and bringing down on the right of the remainder the next period, if there be another, thus forming a new dividend, for which a trial divisor is to be formed, by writing under the complete divisor, just used, the square of its quotient figure, adding this square number to the two lines above it, and annexing two ciphers on the right of the sum.

If there be any more periods to be brought down, there must be a repetition of the above procedure.

Ex. (1) Find the cube root of 387420489.

$\sqrt[3]{387420489} = 729$. *Ans.*

		343
212	14700	44420
	424	
	15124	30248
	4	
2169	1555200	14172489
	19521	
	1574721	14172489

In this example, placing a point over the units figure, 9, and one over every third figure from it to the left, the first period is found to be 387, and the highest cube not greater than 387 is 343, the cube of 7; accordingly, writing 7 as the first radical figure, and subtracting its cube from 387, and bringing down the next period, we have 44420 as a dividend. Then trebling the 7 gives 21, and 300 times the square of 7 is 14700, which we use as a trial divisor for 44420, the quotient for which would be 3; but, as that quotient would be found too high for the complete divisor, we try 2, and annex it to the 21, making 212, which, multiplied by its last figure, gives 424 to be added to the partial divisor; and the sum or complete divisor, being multiplied by the quotient figure, is found to be *allowed by the* dividend, with a remainder 14172, on the right of

which we bring down the last period for a new dividend. To form the partial divisor for this dividend, we write 4, the square of the second radical figure, under the last complete divisor, adding it with the two lines above it, and annexing to the sum two ciphers. The divisor thus formed suggests that 9 is probably the next quotient figure; trebling, therefore, the 72, annexing 9 to the product, and multiplying the number thus formed by its last figure, we obtain what is to be added to the partial divisor to make the complete divisor; and then multiplying that complete divisor by its quotient figure, the operation is concluded.

It should be observed that the above method of extracting the cube root is an abbreviation of the following:—

$$\sqrt[3]{387420489} = 700 + 20 + 9$$

			343000000
2100		1470000	44420489
20			
2120	× 20 =	42400	
		1512400	30248000
20	× 20 =	400	
2160		1555200	14172489
9			
2169	× 9 =	19521	
		1574721	14172489

The demonstration of the preceding process is most conveniently shown by Algebra.*

171. The reason of pointing off the figures of the given cube into periods will appear from the following considerations. The cubes of numbers that consist of one figure

* Thus, the cube of $x+y$, equal to $x^3+3x^2y+3xy^2+y^3$, may assume the form $x^3+\{3x^2+(3x+y)\cdot y\}\cdot y$, which indicates the procedure we have employed in finding the first two radical figures in the 1st example. Again, the cube of $x+y+z$ is equal to $(x+y)^3+3x^2z+6xyz+3y^2z+3xz^2+3yz^2+z^3$, where the part additional to $(x+y)^3$ may assume the form $[3x^2+2(3x+y)\cdot y+y^2+\{3(x+y)+z\}\cdot z]\cdot z$, which indicates the procedure employed in finding the third or units figure of the root. See Hunter's *Examination Questions* on Colenso's 'Algebra,' Chap. iv. *Qu. 11.*

occupy from one to three places of figures; those of numbers consisting of two figures occupy from four to six places; those of numbers consisting of three places occupy from seven to nine places, and so forth.

Ex. (2) Extract the cube root of 68117·264704.

$\sqrt[3]{68117\cdot264704} = 40\cdot84$. *Ans.*

		64
12	4809	4117264
1208	480000	
	9664	
	489664	3917312
	64	
12244	49939200	199952704
	48976	
	49988176	199952704

Ex. (3) Extract the cube root of ·0000198 to five places of decimals.

$\sqrt[3]{\cdot000019819819}$ &c. = ·02706. *Ans.*

		8
67	1200	11819
	469	
	1669	11683
	49	
	218700	136819
8106	21870000	136819819
	48636	
	21918636	131511816
		5308003

Ex. 80.

Find the cube root of—

(1) 185193; 24389. (2) 753571; 262144000.
(3) 9938375; 12812904. (4) 481890304; 1295029.
(5) 122763473. (6) 6913292625. (7) 11914842304.
(8) 2738124199. (9) 24414·238701. (10) ·002480875.

(11) 73·8448; 47. (12) 213·955; ·0001. (13) ·148624; $30\frac{1}{2}$.
(14) $\frac{1}{8}$ of 7414875. (15) $\frac{27}{64}$; $\frac{189}{875}$; $\frac{5}{9}$. (16) $\frac{8}{27}$ of 2032·296.

172. By means of the cube root we can find the edge of a cube whose volume is given; for the cube root of the number expressing the cubic measure must be the number expressing the edge of the cube in corresponding lineal measure.

Ex. What is the length of the edge of a cubical stone, equal in bulk to a cylindrical block, that is $6\frac{3}{4}$ feet long, and each of whose ends contains $2\frac{1}{4}$ square feet?

Here, the solid content of the cylindrical block is $= 2\frac{1}{4}$ cub. ft. $\times 6\frac{3}{4}$, which is also the content of the cubical stone. Therefore, the length of the edge of the stone will be $= \sqrt[3]{(\frac{9}{4} \times \frac{27}{4})} = \sqrt[3]{(\frac{27}{8} \times \frac{9}{2})} = \frac{3}{2}$ of $\sqrt[3]{4·5} = \frac{3}{2}$ of $1·6509 = 2·476$ ft. *Ans.*

Ex. 81.

(1) Find the edge of a cube, equal in volume to a rectangular solid whose dimensions are 5 ft. 3 in., 4 ft. 8 in., and 3 ft. 6 in.

(2) The cost of a cubical mass of silver, worth 27*s.* 10*d.* per cubic inch, is £18 1*s.* 6*d.* Find the length of its edge.

(3) What is the area of a side of a cube, equal in volume to a block of marble containing 40000 cubic feet?

(4) A cubical cistern will exactly hold a ton of water. How many square feet of lead does it require to line it, the weight of a cubic foot of water being 1000 oz. avoird.?

(5) One cubical vessel holds thrice as much as another: compare the lengths of their edges.

(6) Find the diagonal of a cube containing 120 cubic inches.

173. MISCELLANEOUS EXAMPLES WORKED OUT.

(1) A garrison of 1000 men is victualled for 30 days; after 10 days it is reinforced, and then the provisions are exhausted in 5 days; find the number of men in the reinforcement.

In 10 + 5, or 15 days, 1000 men consume 15000 daily rations; but at first there were $1000 \times 30 = 30000$ rations; therefore $30000 - 15000 = 15000$ daily rations for the reinforcement for 5 days; and hence $15000 \div 5 = 3000$ men. *Ans.*

(2) A and B set out from one place, on a journey of 26 miles, in the same direction. A travels uniformly $7\frac{1}{2}$ miles, and B $8\frac{3}{4}$ miles, per hour; but B is 25 minutes later than A in starting. How far will both of them be from their journey's end when B overtakes A?

25 min. $= \frac{5}{12}$ of an hour; so that A has the start of B by $\frac{5}{12}$ of $7\frac{1}{2}$ miles $= 3\frac{1}{8}$ miles, which B makes up for by travelling $8\frac{3}{4} - 7\frac{1}{2} = 1\frac{1}{4}$ miles per hour more than A; hence, A will be overtaken by B in $3\frac{1}{8} \div 1\frac{1}{4} = 2\frac{1}{2}$ hours, and each will then have travelled $8\frac{3}{4} \times 2\frac{1}{2} = 21\frac{7}{8}$ miles, which is $4\frac{1}{8}$ miles from the end of the journey. *Ans.*

(3) A person contracted to supply the government with mules, at a certain price per head, and imported 20 for delivery at 10 guineas per head below the contract price; but 7 of the mules having died on the passage, he made only £14 profit on the transaction. At what price per head did he supply the government?

Here the total profit expected was £10 10*s.* $\times$ 20 = £210; so that 210 − 14, or £196, was lost by the death of 7 mules; and hence £196 ÷ 7 = £28 per head. *Ans.*

(4) A can do a piece of work in 6 hours, B in 5 hours, C in $4\frac{1}{2}$ hours. A and B begin together the performance of it; but A is called away at the end of 36 minutes, and B at the end of 2 hours: in what time after this could C finish the work?

A does 1 measure of work per hour, and the whole work is 6 such measures; of which B can do $6 \div 5 = 1\frac{1}{5}$ measures per hour, and C $6 \div 4\frac{1}{2} = 1\frac{1}{3}$ measures per hour. A is called away when he has done $\frac{3}{5}$ of 1 measure, and B when he has done $1\frac{1}{5} \times 2$ or $2\frac{2}{5}$ measures, and then there remain $6 - (2\frac{2}{5} + \frac{3}{5})$

or 3 measures to be done by C, which, of course, he can do in $3 \div 1\frac{1}{3}$ or $2\frac{1}{4}$ hours. *Ans.*

(5) At what time does the sun set, when the duration of the day, from sunrise till sunset, is 6 times that of the morning or evening twilight, and the evening twilight is equal to $\frac{3}{14}$ of the time from its termination till daybreak?

Here we have half the day = 3 times twilight, and twilight $= \frac{3}{7}$ of half the night. Therefore, the three periods, from noon to sunset, thence to the beginning of night, and thence to midnight, are as 3, 1, and $\frac{7}{3}$, or as 9, 3, 7; but these three periods make up 12 hrs., so that the time from noon to sunset is $\frac{9}{19}$ of 12 hrs. = 5 hrs. 41 mins. nearly. *Ans.*

(6) My nett income, after paying 7*d.* in the pound for income-tax, was £338 11*s.* $6\frac{3}{4}$*d.*: what was it on a subsequent occasion, when my gross income was 7 per cent. less than before, and the tax was 9*d.* in the pound?

For every £1 of gross income, the nett income was at first 233*d.*, and afterwards 231*d.*

$$\left.\begin{array}{l} 233d. : 231d. \\ 100 \ : \ 93 \end{array}\right\} :: £338\ 11s.\ 6\tfrac{3}{4}d. : £312\ 3s.\ 5\tfrac{77}{80}d. \quad \textit{Ans.}$$

(7) At what time between 4 and 5 o'clock do the hour and minute hands of a clock intercept 13 of the minute divisions?

At 4 o'clock the hour hand (H) is 20 minute divisions, or $\frac{1}{3}$ of the circumference in advance of the minute hand (M); and we are to find in what time after 4 o'clock M will have gained $20 - 13$ minute divisions, or $\frac{7}{60}$ of the circumference, on H; since, therefore, M passes over the whole circumference in the time that H passes over $\frac{1}{12}$ of it, viz., in 1 hour, M gains on H $\frac{11}{12}$ of the circumference in 60 minutes.

$\frac{11}{12}$ of circumf. : $\frac{7}{60}$ of circumf. : : 60 min. : $7\frac{7}{11}$ min. past 4.

But when M has gained $20 + 13$ minute divisions on H, the two hands will again intercept 13 minute divisions; and as $\frac{20 + 13}{60}$ or $\frac{11}{20}$ is a proper fraction, the proposed interval will occur for the second time before 5 o'clock; hence $\frac{11}{12}$ of circumf. : $\frac{11}{20}$ of circumf. : : 60 min. : 36 min. past 4. We have, therefore, two *Answers*, viz., at $7\frac{7}{11}$ min., and at 36 min., past 4.

(8) A block of English oak, $3\frac{1}{2}$ feet long, 2 feet broad, and 1 foot 9 inches thick, weighs 719·8 lbs.; find its specific gravity:—a cubic foot of water weighing 1000 ounces avoirdupois.

The specific gravity of a substance is the number of times that the weight of any given volume of it contains the weight of the same volume of water. Here, then, we have $3\frac{1}{2} \times 2 \times 1\frac{3}{4}$ cubic feet of oak, weighing $719{\cdot}8 \times 16$ oz., which gives $\frac{719{\cdot}8 \times 16}{7 \times 1\frac{3}{4}}$ ounces per cubic foot; and the number of times that quantity contains 1000 oz. is $= \frac{{\cdot}7198 \times 16 \times 4}{7 \times 7} = 2\frac{4}{5}$ nearly. *Ans.*

(9) A lady wishing to give 18*d.* each to some poor women, found she had not money enough by 6*d.*; but she gave each of them 16*d.*, and had 14*d.* remaining. How much money had she?

18*d.* − 16*d.*, or 2*d.* less to each woman, proved to be 6*d.* + 14*d.* or 20*d.* less on the whole; therefore, there must have been 20*d.* ÷ 2*d.* = 10 poor women; and the lady's money amounted to 18*d.* × 10 − 6*d.* = 174*d.* = 14*s.* 6*d.* *Ans.*

(10) A merchant buys cloth at 11*s.* 3*d.* a yard, and sells it so that his gain on every £5 is equal to the sum he receives for $\frac{5}{9}$ of a yard: what does he gain per cent.?

£5 ÷ 11*s.* 3*d.* = $8\frac{8}{9}$ yards bought for £5; and therefore $8\frac{8}{9} - \frac{5}{9} = 8\frac{1}{3}$ yards are sold for £5; or the gain is $\frac{5}{9}$ on $8\frac{1}{3} = 5$ on 75, or 1 on 15, which is equal to $\frac{100}{15}$, or $6\frac{2}{3}$, per cent. *Ans.*

(11) A horse was sold at a loss for 40 guineas; but if it had been sold for 50 guineas, the gain would have been three-fourths of the former loss. Find the real value.

50 − 40 guin. = £10½, the sum of loss and gain, which is to be divided into two parts in the proportion of 3 to 4, thus giving $\frac{4}{7}$ of $10\frac{1}{2} =$ £6 the loss, and $\frac{3}{7}$ of $10\frac{1}{2} =$ £4½ the gain; consequently the original value was 42 + 6, or $52\frac{1}{2} - 4\frac{1}{2} =$ £48. *Ans.*

(12) A person paid 19 guineas for sheep and lambs, receiving 9 more of the latter than of the former; he gave 26*s.* for each sheep, and 15*s.* for each lamb: how many of each did he buy?

If he had bought 2 lambs fewer, which cost him 30*s.*, he would have received the same number of lambs as of sheep, and the whole cost would have been $399s. - 30s. = 369s.$; we have, therefore, to find what number of sheep at 26*s.*, along with the *same* number of lambs at 15*s.*, would amount to 369*s.*; this is evidently $369s. \div (26s. + 15s.) = 9$ of each; hence, he bought 9 sheep and 11 lambs. *Ans.*

(13) A is 12 years older than B; and should they both live 8 years longer, B's age will then be $\frac{7}{10}$ of A's. Required the present age of each person.

A's age 8 years hence will of course still exceed B's by 12; their future ages, therefore, will be two numbers as 10 to 7, and differing by 12; and that difference will be to A's age as 10 - 7 to 10, or 3 to 10; hence 3 : 10 : : 12 : 40, A's future age; hence the present ages are 32 and 20 years. *Ans.*

(14) Two casks, A and B, are each filled with two kinds of whisky, mixed in A in the ratio of 2 to 5, and in B in the ratio of 5 to 7. What quantity must be taken from each, in order to have 11 gallons mixed in the ratio of 7 to 11?

The first kind of whisky constitutes $\frac{2}{7}$ of the contents of A, and $\frac{5}{12}$ of B, and is to form $\frac{7}{18}$ of the 11 gallons. So we are to find how much at $\frac{2}{7}$, and how much at $\frac{5}{12}$, will together make 11 gallons at $\frac{7}{18}$. Therefore, according to the rule in Art. 145, the proportions will be as $\frac{5}{12} - \frac{7}{18}$ to $\frac{7}{18} - \frac{2}{7}$, or as $\frac{1}{36}$ to $\frac{13}{126}$, or $\frac{1}{2}$ to $\frac{13}{7}$, or 7 to 26; hence $\frac{7}{33}$ and $\frac{26}{33}$ of 11 gallons $= 2\frac{1}{3}$ gallons from A, and $8\frac{2}{3}$ gallons from B. *Ans.*

(15) A stream flows at a uniform rate from A to B, passing the intermediate object C, which is 4 miles from B. A boatman can row from A to B in $\frac{5}{13}$ of the time he would take to row from B to A; and he finds that to row from A to B, and return thence to C, takes him just as long as to row from B to A. Suppose his time of rowing from C to A to be $52\frac{1}{2}$ minutes, what is his power of rowing in still water?

Here the boatman's rate of rowing *against* the current is to his rate *with* the current as 5 to 13; and since his rate *with* the current must exceed his rate *against* it by *twice* the rate of the current itself, $(13 - 5) \div 2 = 4$ is the proportional number for the velocity of the current, and $5 \div 4$

or 13 − 4, viz., 9, is the proportional number for the boatman's power of rowing in still water. Now, since his time of rowing from B to A, that is, from B to C and C to A, is also his time of rowing from A to B and B to C, his time from C to A is also his time from A to B, viz. $52\frac{1}{2}$ minutes; and therefore his time from B to A is $\frac{13}{5}$ of $52\frac{1}{2}$ minutes = $136\frac{1}{2}$ minutes; so that he takes $136\frac{1}{2} - 52\frac{1}{2} = 84$ minutes to row from B to C, 4 miles, which is $2\frac{6}{7}$ miles an hour *against* the stream; but his rate against the stream is to his power of rowing in still water as 5 to 9; therefore $\frac{9}{5}$ of $2\frac{6}{7}$ miles = $5\frac{1}{7}$ miles an hour. *Ans.*

(16) Suppose there is a meadow of 20 acres, in which the grass grows uniformly, and that 81 oxen could eat up the whole of the grass in 20 days, and 52 of the oxen could eat up 16 acres of it in 27 days: find (i.) how many acres of it 57 of the oxen could eat up in 12 days, and (ii.) in what time 41 of the oxen could eat up $9\frac{1}{3}$ acres of it.

Here we have given, that the original grass, together with 20 days' growth, of 1 acre, would maintain 81 oxen for 1 day, or 1 ox for 81 days, and that the original grass, together with 27 days' growth, of 1 acre, would maintain 52 oxen for $\frac{27}{16}$ of a day, or 1 ox for $\frac{27 \times 52}{16} = 87\frac{3}{4}$ days.

Hence, for each acre, the original grass + 20 days' growth is to the original grass + 27 days' growth as 81 to $87\frac{3}{4}$, or as 12 to 13, or as $1 : 1\frac{1}{12}$, where we have an excess of 7 days' growth = $\frac{1}{12}$ of (the original grass + 20 days' growth), or 84 days' growth = the original grass + 20 days' growth; or, the original grass = 64 days' growth.

The first question now is—If 64 + 20 days' growth of 20 acres maintain 81 oxen for 20 days, of how many acres will 64 + 12 days' growth maintain 57 oxen for 12 days?

$76 \times 81 \times 20 : 84 \times 57 \times 12 :: 20$ ac. $: 9\frac{1}{3}$ ac. *1st Ans.*

The second question is—If 84 days' growth of 20 acres maintain 81 oxen for 20 days, how many days' growth of $9\frac{1}{3}$ acres will maintain 41 oxen for 1 day?

$81 \times 9\frac{1}{3} \times 20 : 41 \times 20 \times 1 :: 84$ days' growth : $4\frac{5}{8}$ days' growth; thus it appears that in 1 day 41 oxen consume $4\frac{5}{8} - 1$ or $3\frac{5}{8}$ *days'* growth of the original grass, so that they would consume the whole of the grass in $64 \div 3\frac{5}{8} = 18$ days. *2nd Ans.*

EXAMINATION PAPERS.

Paper I.

1. Reduce 54 ac. 29 po. $29\frac{1}{4}$ yds. to square feet.

2. By what must $71\frac{2}{3}$ shillings be multiplied that the product may be equal to £365 10*s.*?

3. How many rupees, valued at 1*s.* $11\frac{3}{4}$*d.* each, are equivalent to 38 napoleons at 15*s.* $11\frac{7}{8}$*d.*?

4. Calculate, by the method of Practice, the amounts of—

(i.) 369 ounces of gold at £3 17*s.* 9*d.* an ounce.

(ii.) 7 cwt. 3 qrs. 18 lbs. at £1 14*s.* 2*d.* per ton.

5. If a piece of work can be done in $8\frac{1}{2}$ hours by 27 men, in what time could 34 men do it?

6. If $45\frac{1}{2}$ quarters of barley would make $46\frac{2}{3}$ quarters of malt, how many quarters of malt may be made from 156 bushels of barley?

7. Find the simple interest on £249 11*s.* 8*d.* for $3\frac{1}{2}$ years, at 5 per cent. per annum.

8. How much will £2670 amount to in 3 years at $4\frac{1}{2}$ per cent. per annum compound interest?

9. A person invests £3745 in $3\frac{1}{4}$ per Cent. stock at $80\frac{1}{4}$. Find his income; the rate per cent. of interest which he receives; the amount of stock which he holds; and what he would gain by selling out at $83\frac{3}{5}$.

10. If ·03577 of a cubic foot costs 3·65 shillings, what quantity costs 10·8 pence?

11. Find the present worth, at $3\frac{1}{2}$ per cent. per annum, of £1450 due $2\frac{1}{2}$ years hence.

12. Find the square and cube roots of 841·5.

13. Simplify $\dfrac{5\frac{1}{4} + \frac{3}{11} \text{ of } 12\frac{5}{6}}{3\frac{1}{2} - (1\frac{2}{3} + \frac{7}{10} + \frac{2}{15})}$.

14. A can do a piece of work in 14 days, B in 16 days; in what time can they do it if they work together?

15. If 22 gallons of colza oil be worth £4 15*s.*, and a dozen lbs. of composite candles be worth 11*s.* 3*d.*, how much oil should be given in exchange for 57 lbs. of candles?

16. A draper has bought 1517 yards of calico, at the rate of 18*s.* 9*d.* for 37 yards; what will be his gain per cent. if he sell 1186¼ yards of it at the rate of 36*s.* 9*d.* for 73 yards, and the remainder at the rate of 28*s.* 10*d.* per 49 yards?

17. Thomson and Mercer traded with a joint capital of £5400. T's capital was invested at the beginning of the year; and M's, which was equal to four-fifths of T's, a short time afterwards; and they agreed to share equally their nett trade profit, after allowance of interest at 5 per cent. per annum on the respective capitals. Accordingly, at the end of the year, T's whole gain was £450, and M's £400. At what time was Mercer's capital paid in?

18. When a quart of sack was one-fourth dearer than a quart of claret, and one-fourth cheaper than a quart of muscadel, a company at a tavern drank 1½ pint more of claret than of muscadel, 3 times as much sack as muscadel, and 5 pints of claret for every 6 quarts of sack. The amount of the reckoning was an angel. What was then the price of a quart of sack?

Paper II.

1. How many men, earning 7*s.* 11*d.* each, can be paid out of £20? and what would be the surplus money?

2. How many articles bought at 1*s.* 4½*d.* must be sold at 1*s.* 9*d.* that the profit may amount to 27*s.*?

3. If 3 men can mow 12¼ acres in 5¼ days, how many acres will 8 men mow in 6 days?

4. How much simple interest would arise on £599 8*s.* 1*d.* from March 12th to July 31st, if the rate were 3½ per cent. per annum?

5. How often could ·2035 be subtracted from 29·37?—and what would the remainder be?

6. If 28 yds. 2 qrs. 2½ nls. of silk be worth £9 0*s.* $9\frac{11}{16}$*d.*, what is the worth of 37⅘ yds.?

7. If I get as many apples for 8*s.* 9*d.* as I get pears for half-a-guinea, how many apples cost as much as 5 dozen pears?

8. What fraction of 7 furl. 3 po. 3 yds. 2½ ft. is 6 furl. 39 po. 5 yds. 6 in.?

9. Simplify $\frac{6\frac{3}{7} + \frac{3}{4} \text{ of } \frac{3}{7} - \frac{7}{45} \text{ of } 2\frac{1}{2}}{4\frac{3}{8} + \frac{6}{13} \text{ of } 9\frac{1}{3} + \frac{1}{8}}$.

10. Convert into vulgar fractions ·987654320 and ·97531.

11. A person sold out £1280 of Russian 4½ per Cent. stock, at 85⅝, and with the proceeds bought Turkish 6 per Cent. stock, at 68½; what change did he make in his income?

12. What principal, bearing simple interest at 4¼ per cent. per annum, would in 4 years amount to £975 9*s.* 9*d.*?

13. What is the difference of profit per cent. between selling for a moidore what was bought for a guinea, and selling for four guineas what was bought for three guineas?

14. Suppose that £10685 is to be divided between A and B in the proportion of the square roots of their ages, 41 and 53 years respectively, what sum will each receive?

15. What length of paper, 25 inches wide, would be necessary to cover the walls of a room that is 17 feet long, 13 feet broad, and 11½ feet high?

16. A rectangular space consists of a lawn, 82·45 feet long, and 52·8 feet wide, and a surrounding area, uniformly 10¾ feet wide, that is to be paved with flagstones each 4·94 feet long and 3·74 feet wide. How many flagstones will be required?

17. A wine merchant bought 196 gallons of ale, at the rate of 8*s.* 6*d.* for 7 gallons, and has sold 180 gallons of it at the rate of 7*s.* 7*d.* for 5 gallons; for what must he sell the remainder per gallon, to gain 25 per cent. on the whole?

18. In buying cloth, and then selling it at 24 per cent. profit, I observe that an outlay equal to $\frac{3}{8}$ of the value of a lb. of tea gains a sum equal to $\frac{2}{9}$ of the value of a lb. of coffee; what decimal fraction of the value of a lb. of tea is gained by an outlay equal to $\frac{5}{6}$ of the value of a lb. of coffee?

Paper III.

1. The cost of 963 tons 12 cwt. of iron rails is £6745 15*s.*; find the corresponding cost of 182½ tons.

2. How much more should be received for 21 apples when they are sold at 5 for 10½*d.* than when they are sold at 7 for 13½*d.*?

3. Divide 725 cwt. 1 qr. 16⅘ lbs. by 65⅞; and then divide the result by 6 cwt. 8⅖ lbs.

4. What principal, bearing simple interest at 4⅓ per cent. per annum, would produce £80 6*s.* 3¾*d.* in 3¼ years?

5. Multiply 165⅙ by 11⅔, and to the product add $\frac{1}{6}+\frac{6}{7}+\frac{20}{21}$.

6. If 13 cwt. 3 qrs. 17 lbs. of sugar cost £29$\frac{4}{7}$, what is the cost of 29·41 cwt. ?

7. Simplify $\frac{10\cdot738 \div \frac{2}{9} \text{ of } (\cdot1 - \cdot009)}{76\frac{8}{17} \text{ of } 63\frac{3}{19} \text{ of } 2\cdot907 \times \cdot0059}$.

8. Calculate, by the method of duodecimals, the area of a rectangle, 17 ft. 3 in. 3 pts. by 14 ft. 8 in. 11 pts.; and then reduce the result to square yards, feet, and inches.

9. By selling flour at 23*s.* 10*d.* a barrel, I should lose 2$\frac{1}{2}$ per cent.; at what must I sell it to gain 1$\frac{1}{4}$ per cent. ?

10. If 14 men, 3 times as expert as boys, do 375 measures of work in 15 days; of what degree of expertness must 8 men be, that they may do 350 measures in 21 days ?

11. Divide £12 16*s.* among A, B, and C, so that A may receive a third as much again as B, and 3 guineas more than C.

12. A by himself can do a piece of work in 27$\frac{1}{2}$ days, B in 32$\frac{1}{2}$, C in 33, D in 39; in what time would it be done by A, B, C, and D, together ?

13. The cost of fencing A's field is to that of fencing B's, as the square root of 7 to twice the square root of 2; if B's field costs 16*s.* 11*d.* more than A's, what is the cost of A's ?

14. Suppose that A buys 13·97 gallons of wine at the rate of £8 10*s.* for 11 gallons, and 6$\frac{3}{4}$ gallons at the rate of 4 guineas for 5 gallons; at what average price must he sell the whole in order to gain 12 per cent. ?

15. A would take 25 hours to do a piece of work that with B's assistance he could do in 14 hours 35 minutes; in what time would B alone do it ?

16. Divide £114 10*s.* among A, B, C, giving A 5 per cent. more than B, and 6 per cent. less than C.

17. If 56 gallons of a mixture, consisting of 4 parts of brandy and 1 of water, be worth £84, how many gallons of another mixture, consisting of 5 parts of the same brandy and 1 of water, should be worth £75 ?

18. I have bought 66 barrels of American flour for £75, cash. At what price per barrel should I sell it, with allowance of 2 months' credit; that my present gain may be 10 per cent. ?—the value of credit being 5 per cent. per annum.

Paper IV.

1. A gives B 237 tons of coals, at 15*s.* 7*d.* a ton, in return for which B gives A 27 tons of iron and £2 8*s.* 3*d.* cash to balance. What is the iron valued at per ton?

2. How many yards of silk would be worth £9 13*s.* 5*d.* at the rate of £11 18*s.* 4*d.* for $35\frac{2}{7}$ yards?

3. How many men, working $9\frac{3}{4}$ hours a day for 17 days, will do as much work as 18 men, working $8\frac{1}{2}$ hours a day for 13 days?

4. Calculate, by the method of Practice, the values of—

(i.) 5701 yards, at 11*s.* $6\frac{3}{4}$*d.* a yard.

(ii.) 7 gall. 3 qts. $1\frac{7}{8}$ pt., at £1 12*s.* 9*d.* a gallon.

5. If $87\frac{1}{2}$ yards of paper, $\frac{3}{4}$ of a yard wide, cover a rectangular area that is $10\frac{1}{2}$ feet broad, how many yds. of paper, yard wide, will cover another rectangular area, of the same length as the former, but only 10 feet broad?

6. Divide a nett profit of £1425 among three partners, A, B, C, in the proportion of their capitals; A's capital being £3400, B's £2890, C's £1870.

7. If 10 boys or 6 men can do a piece of work in $11\frac{1}{2}$ days, in what time should 7 boys along with 5 men be expected to do it?

8. The simple interest on £63 17*s.* 5*d.* for $2\frac{1}{2}$ years is £6 7*s.* $8\frac{9}{10}$*d.* Required the rate per cent. per annum.

9. If barley is increased in bulk $3\frac{1}{2}$ per cent. by being converted into malt, how much malt can be made from $44\frac{4}{5}$ quarters of barley?

10. In 14 hours 56 minutes a certain amount of work can be done by F and G together. F alone could do it in 28 hours; in what time could G alone do it?

11. If in England 4 articles cost 13*s.* 6*d.*, and in India 5 such articles can be bought for 1 rupee 10 annas, the rupee being equal to 16 annas, and 8 rupees worth 15*s.* 9*d.*; how many of the articles in England would cost as much as 40 dozen of them in India?

12. A in 36 minutes can do a piece of work, which B can do in 28 minutes. In what time could C do it, if he had the joint ability of A and B?—and in what time could D do it, if his ability were equal to the excess of B's above A's?

13. A's share of a certain sum of money is to B's, as the cube root of 3 to the cube root of 4. If B's share be £28 13*s.* 4*d.*, what is A's?

14. Find the value of $\frac{\sqrt{8\frac{11}{14}} + 6\frac{20}{21} + \sqrt{8\frac{11}{14}} - 6\frac{20}{21}}{\cdot00011581 \div \cdot0037}$.

15. One set of labourers, 28 in number, can excavate 832 cubic yards in 3 days of $9\frac{3}{4}$ hours; another set, 33 in number, can excavate 1408 cubic yards in 5 days of $10\frac{1}{2}$ hours. Compare the average abilities of the men in the two sets.

16. If barley in being converted into malt decreases $17\frac{1}{2}$ per cent. in weight, how much barley will be required to make 22 stones of malt?

17. The compound interest on £2076 12*s.* 3*d.*, for 3 years, being £242 9*s.* $9\frac{2}{5}$*d.*, what is the rate per cent. per annum?

18. A person invested a certain sum in the 3 per cents. at $77\frac{5}{8}$; on the price rising to $82\frac{1}{2}$ he sold out, and with the proceeds bought 5 per cent. stock at 96, by which means he obtained an addition of £41 10*s.* to his income. Find the sum originally invested.

Paper V.

1. Reduce 2 mi. 7 fur. 35 po. to French mètres, each measuring 39·371 inches.

2. How many cwt. of gunpowder, weighing 927 lbs. per cubic foot, would fill a box 3 ft. 4 in. long, 2 ft. 11 inches wide, and 1 ft. 2 in. deep?

3. By selling cucumbers at 9*d.* a fruiterer loses $\frac{2}{5}$ of the prime cost. Find the prime cost.

4. If I can buy 2 dozen articles for 1*s.* $5\frac{1}{2}$*d.*, and sell 8 of them at the rate of 9*d.* a dozen, and 7 at $10\frac{1}{2}$*d.* a dozen, at what rate per dozen must I sell the remaining 9 articles, that my gain on the whole may be 10 per cent.?

5. In what time would the simple interest on £12410, at $3\frac{2}{3}$ per cent. per annum, amount to £168 6*s.*?

6. If 3 men earn as great an amount of wages in 4 days as 8 boys earn in 9 days, in what time would a boy's wages come to as much as a man earns in a day?

7. Simplify $\frac{\cdot035}{14\frac{5}{6}} \times \frac{2{\cdot}67}{5\frac{4}{9}} \div 1{\cdot}9\dot{2}85714\dot{}$.

8. If, when a goose is half as dear again as a duck and the price of a fowl is half that of a goose, 1 can buy 4 geese and 3 ducks for 2 guineas, what should I pay for 3 geese and 9 fowls?

9. For the sum of £2 5*s.* 6¾*d.* I can buy a number of yards of silk equal to the number of shillings which a yard would cost. What is the price per yard?

10. What would be the whole cost of papering the walls of a room 18 feet long, 15 feet broad, and 13½ feet high, with paper 2¼ feet wide, at 3¼*d.* a yard?

11. The extent of one field is $\frac{£55\ 16s.\ 3d.}{£267\ .\ 6s.\ 3d.}$ of that of another, the total value of the first being $\frac{4\text{ ac. }18\frac{2}{3}\text{ po.}}{14\text{ ac. }3\text{ ro. }6\text{ po.}}$ of that of the second. Compare the average values per acre.

12. A has goods worth £9 12*s.* cash, which he sells to B on 6 months' credit for £10 16*s.*; and B sells goods to A on 4 months' credit for £15 18*s.* 6*d.* If A and B give mutual credit at the same rate of interest, what is the cash value of B's goods?

13. A piece of work can be performed by A in $\frac{5}{7}$ of the time that B would take to do it; and A and B together could do it in $8\frac{1}{6}$ days; in what time could A by himself do it?

14. Divide 1420 acres of land into three farms, so that 5 times the extent of the first may be 9 times that of the second and 12 times that of the third.

15. Suppose that a company of 250 persons have a store of provisions that will last them 14 days, at a uniform daily allowance to each; but that at the end of 2 days 110 persons are added to the company, and at the end of 1 day more 30 are withdrawn from it: in what time will the whole of the provisions have been consumed?

16. Find the present worth, (i.) by simple interest, (ii.) by compound interest, of £4384 1*s.* 9*d.*, due 3 years hence, at 3½ per cent. per annum.

17. I bought a number of sheep at 24*s.* a head, and losing 7 of them, I sold the rest at 30*s.* a head, and gained £12. How many did I buy?

18. A person invested a certain sum in 3½ per Cent. stock at 85, and a second sum, £1400 less than the first, in 4½ per Cent. stock at 108; and by the whole income thus obtained he had $4\frac{1}{8}$ per cent. for his money. What were the sums invested?

Paper VI.

1. How much of an article that sells at £1 17*s.* 9¼*d.* a ton would amount to £1 5*s.* 10⅝*d.*?

2. Find, by the method of Practice, the values of—

(i.) 4570¾ articles at £5 14*s.* 10½*d.* each.

(ii.) 8 qrs. 7 bu. 3½ pks. at £2 16*s.* 7*d.* per quarter.

3. How many English ells would cost £150·08, at the rate of ·39 of a yard for ·3484 of a guinea?

4. What length of plank, $\frac{7}{8}$ of an inch thick, and 9 inches wide, would be equal to 1 solid foot?

5. Simplify $\dfrac{3 \times \sqrt{(3-\frac{1}{5})} \div \sqrt{(16-\frac{1}{4})}}{2 \times \sqrt{(3+\frac{1}{5})} \times \sqrt{(1+\frac{1}{8})}}$.

6. A merchant in London incurred cost of repairing a ship at Amsterdam to the amount of 313 florins 27 cents., course of exchange 11 florins 53 cents. per £ sterling; what did he save by delaying to remit payment till the exchange rose to 11 florins 68 cents.?

7. Twenty-two sovereigns should weigh 113 dwts.; but a parcel of light sovereigns, having been weighed and counted, was found to contain 6 more than was supposed from the weight; and it appeared that 63 of these coins weighed the same as 61 true sovereigns. How many were there altogether?

8. A cistern is fed by a spout that could fill it in 1 hour 20 minutes; but a leak at the bottom of the cistern makes the actual time of filling to be 22$\frac{6}{7}$ minutes longer. If the supply is stopped when the cistern is $\frac{5}{6}$ full, in what time will it be emptied?

9. The values of two jewels are as $\sqrt{5^3}$ to $\sqrt{8^3}$. If their joint value be £488 19*s.* 8*d.*, what is the worth of each?

10. A boatman can row 7¾ miles an hour with the tide, and 3¼ miles an hour against it. What is his power of rowing in still water?

11. A man's income arises by uniform monthly earnings. What fraction of his income does he save, if he spends in every 3 months a number of crowns equal to the number of guineas that he earns in 10 months?

12. One of two sorts of almonds is 10*d.* a lb. cheaper than the other; and $\frac{3}{8}$ of the sum paid for 3 lbs. of the finer sort is $\frac{5}{12}$ of

what is paid for $3\frac{3}{5}$ lbs. of the inferior. What are the prices per lb.?

13. I gained $5\frac{3}{4}$ per cent. by the sale of a quantity of sheathing copper; but if I had received a halfpenny a lb. less, I should have lost $\frac{1}{8}$ per cent. At what price did I sell it per lb.?

14. A is 11 years of age, B 31. How long ago was A's age $\frac{2}{7}$ of B's?

15. L can do a piece of work in $3\frac{3}{4}$ hours, M in $2\frac{1}{2}$ hours, N in $3\frac{1}{8}$ hours. In what time will it be finished by the three, if M begin half an hour after L, and N $22\frac{1}{2}$ minutes after M?

16. I find that by selling 102 quarters of corn at a profit of $6\frac{1}{2}$ per cent., I should receive just 3 shillings less than if I sold 65 quarters at a profit of 6 per cent., and the remainder at a profit of $7\frac{1}{2}$ per cent. What did the corn cost me per quarter?

17. A offers to purchase of me 132 quarters of corn at an advance of 10 per cent. on the cost price; but he requests a certain term of credit. B offers cash, at an advance of 9 per cent. on the cost price of 128 quarters, and of 12 per cent. on that of the remaining 4 quarters. Now, if the use of money be reckoned worth 5 per cent. per annum, the two offers are equivalent. What term of credit does A request?

18. Suppose that in a meadow of $29\frac{1}{4}$ acres the grass grows uniformly, and that 66 oxen could eat up the whole of the grass in 26 days, and 34 oxen could eat up 20 acres of it in 40 days; how many oxen could eat up 15 acres of it in $15\frac{1}{2}$ days?

Paper VII.

1. An ounce of British coinage gold is worth £3 17*s.* $10\frac{1}{2}$*d.*; how much gold is coined into 267 sovereigns?

2. How many rods are equal to 7 miles 6 fur. 7 chains $8\frac{1}{4}$ yds.?

3. Find the length of the side of a square court-yard, the cost of paving which, at 3*s.* 8*d.* per square yard, amounts to £10 6*s.* 3*d.*

4. Find, in lowest terms, the vulgar fraction that is equal to $\cdot 10605 \div \cdot 0254370$.

5. A post is a fourth of its length in the mud, a third of its length in the water, and 10 feet above the water; what is its length?

6. If the uniform wages of 63 men for $13\frac{1}{2}$ days amount to 81

days' interest of £20000 at 4½ per cent. per annum, in what time would the wages of 28 of the men amount to 85 days' interest of £6000 at 4 per cent. per annum?

7. Suppose that 2 horses do the same work as 3 mules: how many horses along with 7 mules will be required to draw 257 cwt. 3 qrs. 12 lbs. the same distance that 6 horses and 5 mules can draw 9½ tons?

8. If it costs £5 4*s.* to supply 28 men, 42 women, and 50 boys with bread for a week, supposing a man to consume twice as much as a boy, and a woman one-fourth less than a man; how much will it cost to supply 22 men, 52 women, and 60 boys for 10 days?

9. Calculate the compound interest on £347 15*s.* for 3 years, at 5 per cent. per annum, payable half-yearly.

10. Suppose that £350 will be due as follows:—£90 due 50 days hence, £150 due 70 days hence, and the remainder 80 days hence; what is the equated time, according to mercantile discount, for a single payment of the £350?

11. A invests £2090 in the 3 per Cents. at 86¼, and B £3080 in the 3½ per Cents. at 100⅝; what will be the difference of their incomes after deducting 10*d.* in the £ income-tax?

12. Suppose that a sum of money, which, if put out at simple interest for 5 months at 4½ per cent., would amount to £14022 9*s.*, is invested by me in the 3 per Cents. at 78, and that shortly afterwards I sell out at 81½, and invest the proceeds in 4 per Cent. stock at 108, what would now be my income?

13. A, B, and C, enter into partnership, A's investment being ·24 of the joint capital, and B's ·32. At the end of 3 months, A adds £420 to his capital, and B withdraws £100, and C's investment is then $\frac{11}{29}$ of the joint capital. Find the original capital of each.

14. At what rate per cent. per annum would the compound interest of a sum of money, for 3 years, be the same as its simple interest at 4½ per cent. per annum, for the same period?

15. A wine merchant has 25 gallons of spirits, worth 27*s.* a gallon, consisting of a mixture of two kinds respectively worth 21*s.* and 36*s.* How many gallons of each kind were mingled together?

16. A grocer has 229 lbs. of mustard in 1 lb. canisters, which

cost him a guinea per dozen; for what must he sell the whole, so as to gain on every £100 of outlay the selling price of 8 dozen?

17. A piece of work can be done by A alone in 6 hours, by B in 5 hours, by C in $4\frac{1}{2}$ hours. If A, B, and C, begin it together, and A continues till it is finished, but B leaves off 2 hrs. 10 min. before its completion, and C $1\frac{1}{2}$ hours before the completion, in what time is the work done?

18. A merchant in St. Petersburg owes a merchant in London 8000 rubles, at a time when the exchange between London and St. Petersburg is 38*d.* per ruble. How much more or less advantageous for the London merchant will it be, if, instead of drawing directly on St. Petersburg, he draws through Paris, Amsterdam, Hamburg, and Vienna, the exchange between Vienna and St. Petersburg being 156 kreutzers (=1 florin 56 kreutzers) per ruble, between Vienna and Hamburg 69 florins, 68 kreutzers, for 100 marks Hambro', between Amsterdam and Paris 48 florins for 100 francs, and between Paris and London 25·55 francs per £ sterling?

Paper VIII.

1. Reduce 98437112 square inches to acres, &c.

2. Find the value of $\sqrt{1{\cdot}980677} + \sqrt{{\cdot}0173}$.

3. Calculate, by the method of Practice, £158 13*s.* $7\frac{1}{2}$*d.* at 9*s.* $10\frac{1}{2}$*d.* per £.

4. The 3 per Cents. are at $87\frac{1}{8}$, and the $3\frac{1}{2}$ per Cents. at $101\frac{5}{8}$; in which of the stocks is it more advantageous to invest?

5. What sum, at $3\frac{1}{7}$ per cent. per annum, simple interest, will amount to £1297 6*s.* 1·58*d.* in 245 days?

6. A person's nett income, after paying 9*d.* in the £ income-tax, is 385 guineas; what would it be if the tax were 7*d.* in the £?

7. What decimal fraction of a shilling is equal to

$$\frac{3\frac{2}{3}}{17\frac{1}{2}} \text{ of } \frac{\cdot 32000}{279{\cdot}5} \text{ of } \frac{\text{£}29\ 11s.\ 6d.}{\text{£}199\ 15s.} \text{ of } \frac{63 \text{ qrs. } 1 \text{ bu. } 1 \text{ pk.}}{110 \text{ qrs. } 1 \text{ pk.}} \text{ of 374 guineas?}$$

8. Find a fraction equivalent to $\frac{5}{9}$, and having the denominator greater than the numerator by 68.

9. At what time between 6 and 7 o'clock are the hour and minute hands of a watch coincident?

10. A accomplishes a journey of a certain number of miles, by

walking a certain number of hours a day, for a certain number of days. B, walking $\frac{5}{7}$ of A's number of hours a day, for $\frac{3}{5}$ of his number of days, accomplishes a journey $\frac{5}{6}$ as long. Express in simple form the comparison of their rates of travelling.

11. By selling a piece of silk, containing 37 yards, at 7*s.* 5*d.* a yard, a mercer gains $11\frac{1}{4}$ per cent.; and by selling another piece, containing 54 yards, at 6*s.* 1*d.* a yard, he gains $9\frac{1}{2}$ per cent. Compare the prime costs per yard.

12. When the $3\frac{1}{2}$ per Cents. are at $94\frac{1}{2}$, what should be the value of £1675 in the $4\frac{1}{2}$ per Cents.?

13. If 44 gallons of ale, worth 8*s.* 6*d.* a cask, be 8 men's allowance for 9 days, what price per cask would make $93\frac{1}{2}$ gallons a corresponding allowance to 15 men for 10 days?

14. A on horseback, and B on foot, travel from one place in one direction. A, who rides $8\frac{1}{2}$ miles an hour, starts half-an-hour after B, and comes up with him in 21 minutes. What is B's rate of walking?

15. The walls of a room 21 ft. 7 in. long, 18 ft. 11 in. broad, and 13 ft. 4 in. high, are to be covered with paper $2\frac{3}{5}$ feet wide, deducting for doors, windows, &c., a fifth of the whole surface. Suppose the paper costs 10*s.* 10*d.* per piece of 12 yards, and that, whatever portion of a piece may be required beyond an exact number of pieces, not less than half a piece can be purchased, how much will be paid for paper more than the room requires?

16. A person invests a sum in a speculation, by which he loses 10 per cent.; and he then invests the remainder in another speculation, by which he loses 12 per cent.; but by a third investment with what now remains, he recovers exactly the previous losses. Required his gain per cent. in the last instance.

17. A person invests £5524, part in the 3 per Cents. at $79\frac{3}{4}$, and the remainder in the $3\frac{1}{4}$ per Cents. at 88, and his joint income from the two investments is £206. What were the sums invested?

18. If 90 oxen would eat up a field of grass in 10 days, and 35 of the oxen would do the same in 36 days, the grass growing uniformly; in what time would 24 of the oxen do it?

Paper IX.

1. A sovereign should weigh $123\frac{171}{623}$ grains: What number of Troy lbs. of standard gold is coined into 934 sovereigns and 1 half-sovereign?

2. The worth of an ounce of standard gold is £3 17*s.* $10\frac{1}{2}d.$; that of an ounce of pure gold is £4 4*s.* $11\frac{5}{11}d.$ Express the former value as a fraction of the latter in lowest terms.

3. How much of waking life, in the course of 35 years, will a person who rises at 5 in the morning gain over one who rises at half-past 7, supposing both to go to bed at 10 P.M., and reckoning the year at $365\frac{1}{4}$ days?

4. Find, by the method of Practice,

(i) the weight of 383 bags of saltpetre, at 1 cwt. 2 qrs. 15 lbs. per bag.

(ii) the value of 625·795 cwt. saltpetre, at £1 12*s.* $8\frac{1}{2}d.$ per cwt.

5. How much sterling money, augmented by a charge of $3\frac{1}{2}$ per cent. for commission, would amount to the value of 20059 dollars 48 cents, American, the exchange being at 4·80 dollars per £?

6. How many florins per £ sterling will be the arbitrated exchange between London and Vienna, if bills on Paris, bought in London at 25 frs. $54\frac{1}{2}$ cts. per £ sterling, are sold in Vienna at 100 florins for 262 francs?

7. The true discount on £286 12*s.* 3*d.* for 3 months, at a certain rate of interest, is £3 10*s.* $9\frac{2}{5}d.$; what is the corresponding discount on £325 10*s.* 7*d.* for 5 months?

8. A watch that gains $2\frac{1}{2}$ minutes in the twelve hours indicates right time at 45 minutes past 8 P.M.; what hour did it indicate at 2 o'clock P.M. the same day?

9. A person buys 392 bags of Bengal saltpetre, each weighing 1 cwt. 2 qrs. 16 lbs., at 31*s.* 3*d.* per cwt.; and he sells 224 bags at 33*s.* 9*d.* per cwt., and 105 bags at 32*s.* 2*d.* How must he sell the remaining quantity, that his gain on the whole may be 6 per cent?

10. The diagonal of a square measures $2740\frac{3}{4}$ feet; find the length of its side.

11. A and B travel from London to Windsor, 22 miles, A's rate

being $6\frac{2}{9}$ miles, and B's $3\frac{7}{8}$ miles per hour. C travels from Windsor to London at the rate of $5\frac{3}{8}$ miles an hour. If A and C set out when B has been gone an hour and a half, in what time after that will A overtake B, and how long will C have travelled when he meets B?

12. If the solid content of a cube be 2·197 inches, what is its diagonal? And if the diagonal of a cube be 2·2795 feet, what is the solid content?

13. A cistern has two supplying pipes, A and B, the former of which could fill it in 51 minutes; and it can be emptied by a tap, C, in 76 minutes. The cistern is empty, and A and B are turned on and run together for $8\frac{1}{2}$ minutes; A is then turned off and C is turned on; and 16 minutes afterwards B is shut and A reopened; and when A has run with C for $13\frac{3}{5}$ minutes, C alone runs for 36 minutes more, and then the cistern is again empty. In what time could B alone fill the cistern?

14. A's age is $\frac{9}{16}$ of B's; $8\frac{1}{2}$ years ago, it was $\frac{4}{9}$ of B's. Required the age of each.

15. A cistern 3 ft. $8\frac{3}{8}$ in. long, 2 ft. 2 in. wide, and 2 ft. $1\frac{1}{4}$ in. deep, is filled in 13 minutes by a pipe which supplies $80\frac{4}{5}$ lbs. of water per minute; in what time will a pipe that supplies 99 lbs. per minute fill a cistern 7 ft. 6 in. long, 5 ft. 11 in. wide, and 2 ft. 11 in. deep, there being at the bottom of this cistern a leak that could empty it in 8 hrs. $57\frac{3}{5}$ min.?

16. Suppose 575 ounces of light gold coin to be melted into three bars, and the loss in melting to be ·06 per cent.; that the weight of the first bar exceeds that of the third by 151 oz. $4\frac{1}{2}$ dwt.; and that the second weighs half as much again as the third. Compare the weights of the three bars.

17. Two watches, one gaining 3 seconds, the other $4\frac{1}{2}$ seconds, per hour, are set to right time at 2 o'clock P.M. After 5 o'clock P.M., but before half past 5, the same day, the hands of the latter are observed to intercept 12 of the minute divisions; what is then the interval between the hands of the former? and what is the right time of day?

18. Suppose there is a meadow in which the grass is of uniform quality and growth, and that either 19 oxen in 8 days or 16 oxen in 10 days could eat up all the grass; what number of oxen, diminished by the removal of 20 at the end of 3 days, could eat it up in 15 days?

Paper X.

1. How many guineas used to be coined out of 24 lbs. of standard gold, corresponding to the present coinage of 1869 sovereigns from 40 lbs. of standard gold?

2. If 18206·25 florins be the nett proceeds of the sale of coffee in Amsterdam, that was bought in London for £1532 10*s.* 3*d.*; what rate of exchange is thereby established between London and Amsterdam?

3. If at the Mint 2530 silver coins can be manufactured at one press in an hour, and if 2 presses are employed, at that rate, for coining half crowns, 3 for florins, and 3 for shillings, the presses being at work $10\frac{1}{2}$ hours a day for 140 days, what will be the total value of the pieces coined?

4. Given ·95184, the square root of the sum of the squares of four quantities:—the first three quantities being ·4, ·6, and ·04, what is the fourth?

5. From a certain number 464 is subtracted, the remainder is divided by 8, the quotient is multiplied by 5, and 4 is added to the product; then, the square root of the sum being extracted, and $\frac{7}{8}$ of that root cubed, the result is $521660\frac{1}{8}$. Find the original number.

6. The French metre = 10 decimetres = 39·371 inches:—

(i) If 56 metres 7 decim. of cloth cost 264 francs 57 cents, what is the corresponding cost per yard in sterling money —reckoning 8 francs worth 6*s.* 3*d.*?

(ii) The French metre corresponds to the ten-millionth part of the distance of the pole of the earth from the equator: what is the circumference of the earth in miles?

7. Simplify $\frac{15\frac{2}{3} - 4\frac{5}{8}}{3\frac{1}{9} + 2\frac{1}{11}}$ of 1 acre + $\frac{7}{11}$ of $\frac{£2\frac{1}{7} - \frac{3}{5} \text{ of } 4s.\ 6d.}{\frac{3}{7} \text{ of } 3s.\ 3\frac{2}{3}d. + 2s.\ 10\frac{1}{2}d.}$ of 3 roods 12 poles.

8. Sixty-six shillings weigh 1 lb. Troy, and 22 sovereigns weigh 113 dwts. What weight of standard silver is equal in value to a bar of standard gold weighing 5 lbs. 3 oz. 14 dwts.?

9. Given 1 sovereign = 10 florins = 100 cents = 1000 mils.:— If the simple interest on £3530 10*s.*, for $2\frac{1}{2}$ years, be equal to

278 sovs. 9 flor. 9½ mils., find, and express in the supposed decimal coinage, the sum on which the interest for 2⅓ years, at the same rate per annum, will be £150 18s. 5¼d.

10. The shares in a certain undertaking pay 16s. 8d. each, being at the rate of 5 per cent. What interest per cent. will a person have for his money who buys them at $\frac{5}{8}$ premium?

11. A and B were partners in trade, A's share being three-fourths. They dissolved partnership at a time when the value of their stock of goods was £2564, and the debts due to them amounted to £3658 6s. 8d. A agreed to take the risk of recovering the debts for an allowance of 12 per cent., and B took off the goods at 5 per cent. discount. How did accounts then stand between them?

12. A person sells out £1050 of 3½ per Cent. stock at 95, and invests the proceeds in a speculation, for 15 months, which clears 8 per cent. per annum. With the amount he now buys 3 per Cent. stock, and finds his income £6 better than at first. At what price were the 3 per Cents. bought?

13. A boat's crew can row 15 miles with the tide in the time that they can row 7 miles against it. If they could row 5¼ miles an hour in still water, what is the velocity of the tide?

14. The whole cost of constructing a railway was £1350000, of which $\frac{4}{5}$ was held in shares, the remainder having been borrowed on mortgage at 5 per cent. The receipts in one year amounted to £100000, of which 38 per cent. was devoted to pay the working expenses of the line, 8 per cent. to a reserve fund, and the remainder to pay the interest of the borrowed money and a dividend to the shareholders; how much per cent. had they on their investments?

15. Divide £600 into two parts, such that the annual interest on the first part at 3½ per cent., added to that on the second at 6½ per cent., may equal that on the whole sum at 5¼ per cent.

16. The length of our year being taken as 365¼ days, what length of year has the planet Mars, whose mean distance from the sun is about 1·524 as great as that of the Earth? – the cubes of the mean distances of the planets from the sun being as the squares of their times of revolution round the sun.

17. What quantity of gold containing 36½ dwts. of silver in the *lb.* must be melted with 5 lbs. 3 oz. containing 25 dwts. of silver in *the lb.*, that the compound may contain 13⅓ per cent. of silver?

18. A quantity of brandy and rum amounted, per invoice, to £15 19*s.*, the brandy at 78*s.* a dozen, the rum at 34*s.*; but the order had been mistaken: the quantity of brandy should have been that of rum, and *vice versâ*, and thus the amount should have been £14 17*s.* How many dozen of each were ordered?

Paper XL

1. Reduce 14113420 square inches to acres, roods, &c.

2. Reduce 2170 oz. Troy to lbs. Avoirdupois.

3. Calculate, by the method of Practice, 7 mi. 6 fur. 120 yds. at £4 15*s.* 6*d.* per mile.

4. At what rate per cent. per annum, simple interest, would £3246 amount to £3646 in $3\frac{1}{2}$ years?

5. What principal will amount to £3592, in $3\frac{1}{2}$ years, at $3\frac{1}{2}$ per cent. per annum, simple interest?

6. If by selling 2·53 ounces for 12*s.* 10*d.* I should gain 5 per cent., how many ounces should I sell for 17*s.* 9*d.* to gain $6\frac{1}{2}$ per cent.?

7. Express 2 cub. ft. 234 in. as the fraction of

$$\frac{\frac{3}{5}\text{ of }(\frac{7}{8}-\frac{1}{9})-\frac{5}{54}}{\frac{7}{8}-\frac{6}{7}\text{ of }(\frac{5}{6}-\frac{4}{5})}\text{ of a cubic yard.}$$

8. When the $3\frac{1}{2}$ per Cents. are at $97\frac{1}{2}$, what is the simple interest on £156 for 18 months?

9. What amount of premium at $2\frac{1}{4}$ per cent. must be paid for insurance of goods worth £805, that in the event of loss the owner may receive the value of the goods and the premium?

10. Reduce to vulgar fractions ·85636 and ·856̇3685.

11. Find the exact value of $\cdot 2468\frac{2}{7}$ of £24 6*s.* $8\frac{2}{7}$*d.*

12. Find the compound interest on £4000 in 4 years, at 4 per cent. per annum, interest payable half-yearly.

13. How much stock in the $3\frac{1}{2}$ per Cents. must be sold out at $92\frac{3}{4}$, in order to pay a debt of £583 13*s.* 3*d.* three months before it is due, according to mercantile discount at $4\frac{1}{2}$ per cent.?

14. When the French 3 per Cents. are at 69 francs 30 centimes, what, in sterling money, would be the income derived from an investment of £675 8 florins 4 cents.?

15. The specific gravity of pure gold is 19·25, and that of copper

is 8·9: Required the specific gravity of standard gold, which is a mixture of 22 parts pure gold and 2 parts copper.

16. With a certain quantity of spirits, bought at 15*s.* a gallon, and 4 times as much at 13*s.*, a wine merchant mingles 6 gallons at 19*s.*; and by selling the mixture at 18*s.* 8*d.* a gallon, he gains $33\frac{1}{3}$ per cent. Required the quantities at the first two prices.

17. It is found that 14 men and 10 boys can do a piece of work in 3 days, working 9 hrs. 25 min. per day, and that 3 men can do as much per hour as 7 boys: How many boys, along with 12 women and 5 girls, could do it in 8 days, working 7 hours 48 min. per day, if 6 men can do the work of 13 women, and 4 women that of 9 girls?

18. Assuming that twenty-two sovereigns weigh 113 dwts., and that standard gold is $18\frac{3}{8}$ times as heavy as water, of which a cubic foot weighs 1000 ounces Avoirdupois:—what would be the length of the edge of a solid cube of standard gold worth £100?

PAPER XII.

1. Multiply 2 ac. 39 po. 30 yds. 2 ft. 16 in. by $5\frac{1}{2}$.

2. How often could 1·234 be subtracted from 450·4465, and what would be the value of the remainder?

3. Express £1 12*s.* $7\frac{1}{2}$*d.* as the decimal of £43 9 florins 7 cents.

4. Calculate, by the method of Practice, $13\frac{3}{4}$ per cent. on £1056 12*s.* 8*d.*

5. What would be gained per cent. by buying coals at 25*s.* 6*d.* a ton, and selling them at 2*s.* 9*d.* per sack of 2 cwt.?

6. A piece of work can be executed by A alone in $7\frac{1}{2}$ hours, or by B alone in $8\frac{1}{4}$ hours: If they do it together, and their joint remuneration is to be half-a-guinea, what sum ought each to receive?

7. Three partners, A, B, C, agree to divide a joint profit of £328 10*s.* according to their respective capitals and periods of investment: A's capital, which has been invested 12 months, is £1120, B's is £1230, C's £1260; and their shares of the gain are, respectively, £115 4*s.*, £110 14*s.*, and £102 12*s.* How long were the capitals of B and C invested?

8. A school has an endowment of £120 per annum, derived from

an investment of £3135 in the 3 per Cent. Consols. What was the price of Consols when the investment was made?

9. Compare the rates per cent. of the interest obtained by investing in $3\frac{1}{2}$ per Cent. stock at $82\frac{1}{2}$, and $4\frac{1}{2}$ per Cent. stock at 105.

10. When the 3 per Cents. are at $83\frac{1}{4}$, in what time will the simple interest on £346 17*s.* 6*d.* be £31 5*s.*?

11. What principal, increasing annually by simple interest, at the rate of $2\frac{1}{2}d.$ on every 3*s.* $7\frac{1}{2}d.$, would in $3\cdot\dot{9}5\dot{4}$ years amount to £63 9*s.*?

12. Simplify

$$\frac{4 \text{ ac. } 2 \text{ ro. } 39 \text{ po.}}{7\cdot48 \text{ ac.}} \times \frac{1\cdot19}{3\sqrt{2}} \times \frac{15\sqrt{2\cdot5}}{4} \times \frac{\sqrt{(6\frac{1}{2} \times 5\frac{1}{2} - 4\frac{1}{2})}}{6\frac{1}{2} \times 5\frac{1}{2} + 4\frac{1}{2}}.$$

13. A commences business with a capital of £2235. At the end of 5 months he takes B into partnership, with a capital of £3210; and 3 months afterwards he admits C, a third partner, who invests £1980. At the end of 2 years from the commencement of the business there is a joint gain of £1400, from which the partners are remunerated with interest, at 5 per cent. per annum, on their respective advances, and with equal shares of the remaining trade profit. Required the total gain of each.

14. Given, that $\frac{1}{3\frac{2}{3} + 1\frac{5}{12}}$ of $\sqrt{(3\frac{2}{3} - 1\frac{5}{12})}$ of a sum of money, together with $\frac{3}{7}$ of the remainder, would buy 6 yards of silk at 5*s.* 8*d.* a yard. Find the sum.

15. If 4 tons of iron, or $37\frac{1}{2}$ tons of coal, be an equivalent in barter for 165 deal planks, of uniform thickness, each 12 ft. long and 9 in. wide; how many tons of coal, along with 2 tons of iron, would be an equivalent for 240 planks, each $10\frac{1}{2}$ ft. long, $8\frac{1}{4}$ in. wide, and the same in thickness, and in price per cubic foot, as the former?

16. On mixing 63 pints of sulphuric acid, at 1·82 specific gravity, with 24 pints of water, it is found that 1 pint is lost by their mutual penetration. Find the specific gravity of the compound.

17. A wine merchant bought a quantity of spirits at the rate of £17 for 18 gallons; and by selling 74 gallons at the rate of £15 for 14 gallons, and the remainder at £20 for 21 gallons, he gained on the whole $12\frac{1}{2}$ per cent. How many gallons did he buy?

18. A invested £2125 in the 3 per Cents. at 76½, and B £6760 8*s.* 4*d.* in the 3½ per Cents. at 88½. On the respective stocks rising to 78¾ and 90, A sold out, and with the proceeds bought part of B's stock; shortly after which B sold out the remainder of his stock at 94, and invested the whole of his proceeds in the 3 per Cents. By these transfers B's income was made just as much less than before, as A's was made greater. At what price did B purchase the 3 per Cents.?

174. The preceding Examination Papers are designed to test the student's knowledge and skill in arithmetic, to the full extent of what is taught in the present treatise; and they accordingly include questions of much greater difficulty than any that could reasonably be proposed in Papers set at the ordinary public Examinations. It may be expedient, therefore, to append here a Paper of the latter kind, accompanied with *model* solutions, showing the young student distinctly the manner in which the actual work in the Examination Room may be satisfactorily performed.

APPENDIX (A).

SPECIMEN, WITH MODEL SOLUTIONS, OF A CIVIL SERVICE EXAMINATION PAPER.

Set to all Candidates that were required to show a Knowledge of 'Arithmetic, including Vulgar and Decimal Fractions.'— From *Appendix to Third Report of the Civil Service Commission*, p. 300.

(Time allowed, 3½ hours.)

Reduction.

1. In 86754 oz. (avoirdupois) how many tons, &c. ?
2. Reduce 6 miles 5 furlongs 7 poles 2 yards to feet.

16 { 4)86754
4)21688..2

28 { 4)5422 lbs. 2 oz.
7)1355..2

4)193 qrs. 18 lbs.

20)48 cwt. 1 qr.

Ans. 2 t. 8 cwt. 1 qr. 18 lb. 2 oz.

6 mi. 5 fur. 7 po. 2 yds.
8
53 fur.
40
2127 po.
5½
1063½
10637
11700½ yds.
3
Ans. 35101½ ft.

Proportion.

8. Find the income-tax on £8313 5*s.*, at 7*d.* in the £.

£1 : £8313¼ :: 7*d.* : £242 9*s.* 4¾*d.* *Ans.*

7

12)58192¾*d.*

20)4849*s.* 4¾*d.*

Ans. £242 9*s.* 4¾*d.*

4. If a bar of gold weighing 7 lbs. 1 oz. 14 dwt. is worth £257 2*s.*, what is that per oz.?

7 lbs. 1 oz. 14 dwt. : 20 dwt. :: £257 2*s.* : £3. *Ans.*

12

85 oz.

20

1714 dwt.

20

1714)£5142

£3

Practice.

5. Find the cost of 14 cwt. 3 qrs. 19 lbs. at £5 16*s.* 8*d.* per cwt.

£5 16*s.* 8*d.*

14

					cwt.	qrs.	lbs.
	81	13	4	= value of	14	0	0
2 qrs. = ½ of 1 cwt.	2	18	4	= „	0	2	0
1 qr. = ½ of 2 qrs.	1	9	2	= „	0	1	0
16 lbs. = ⅛ of 1 cwt.	0	16	8	= „	0	0	16
2 lbs. = ⅛ of 16 lbs.	0	2	1	= „	0	0	2
1 lb. = ½ of 2 lbs.	0	1	0½	= „	0	0	1
Ans.	£87	0	7½,	value of	14	3	19

6. If a person's estate is worth £1384 16*s.* per annum, and the rent charges upon it amount to 14*s.* 9½*d.* in the £, what is his net annual income?

Income per £ gross = 20*s.* − 14*s.* 9½*d.* = 5*s.* 2½*d.*

£1384 16*s.* at 5*s.* 2½*d.* in the £.

2*s.* 6*d.* = ⅛ of £1	173	2	0	= net income at	2*s.*	6*d.* in the £
2*s.* 6*d.* = ⅛ of £1	173	2	0	= „	2	6
0*s.* 2½*d.* = 1/12 of 2*s.* 6*d.*	14	8	6	= „	0	2½

Ans. £360 12*s.* 6*d.*, net income at 5*s.* 2½*d.*

Interest.

7. Find the simple interest on £1248 12*s.*, at $3\frac{1}{8}$ per cent., for 3 years.

$3\frac{1}{8}$ per cent. per ann. for 3 yrs. $= \frac{3\frac{1}{8} \times 3}{100} = \frac{25 \times 3}{8 \times 100} = \frac{3}{32}$;

£1248 12*s.*

3

32 { 4)3745 16

8) 936 9

Ans. £117 1*s.* $1\frac{1}{4}d.$

Otherwise.

£1248 12*s.* 0*d.*

$\frac{4}{32} = \frac{1}{8}$ | 156 1 6

$\frac{1}{32} = \frac{1}{4}$ | 39 0 $4\frac{1}{2}$

Ans. £117 1*s.* $1\frac{1}{2}d.$

8. How much will £8000 amount to in 4 years, at $3\frac{1}{4}$ per cent., compound interest?

Amt. of £1 for 1 year $= £1 \times \frac{103\frac{1}{4}}{100} = £1 \times \frac{413}{400}$;

do. do. for 4 years $= £1 \times (\frac{413}{400})^4$;

hence $8000 \times \left(\frac{413}{400}\right)^4 = \frac{413^4}{3200000} = \frac{290937{\cdot}83761}{32}$.

413 × 413

1239

1652

170569 × 170569

1193983

9551864

1535121

$413^4 = 29093783761$

32 { 8)£290937·83761

4) 36367·22970

£9091·80742

20

16·14840*s.*

12

1·78080*d.*

Ans. £9091 16*s.* 1·78*d.*

Fractions.

9. Add together $3\frac{5}{12}$, $3\frac{1}{6}$, $2\frac{7}{16}$, $2\frac{3}{4}$.

$\frac{5}{12} + \frac{1}{6} + \frac{7}{16} + \frac{3}{4} = \frac{20 + 8 + 21 + 36}{48} = \frac{85}{48} = 1\frac{37}{48}$.

$3 + 3 + 2 + 2 = 10$

Ans. $11\frac{37}{48}$.

10. Subtract $8\frac{3}{7}$ from $18\frac{2}{3}$.

$18\frac{2}{3} - 8\frac{3}{7} = 10 + \frac{2}{3} - \frac{3}{7} = 10 + \frac{14}{21} - \frac{9}{21} = 10\frac{5}{21}$. Ans.

11. Multiply $11\frac{5}{9}$ by $7\frac{7}{8}$.

$11\frac{5}{9} \times 7\frac{7}{8} = \frac{104}{9} \times \frac{63}{8} = 13 \times 7 = 91$. *Ans.*

12. Divide $18\frac{2}{7}$ by $5\frac{1}{7}$.

$18\frac{2}{7} \div 5\frac{1}{7} = 128 \div 36 = 32 \div 9 = 3\frac{5}{9}$. *Ans.*

Decimals.

13. Add together 140·17, ·6432, ·07042, and 3·12.

14. Subtract 83·450392 from 1210·3.

140·17	
0·6432	
0·07042	1210·3
3·12	83·450392
144·00362 *Ans.*	1126·849608 *Ans.*

15. Multiply 80·46 by ·00392.

16. Divide 37·52 by 2871·3, to 4 places of decimals.

80·46 = 8046 ÷ 100	37·52 ÷ 2871·3 =
·00392 = 392 ÷ 100000	375·2 ÷ 28713
16092	28713)375·20(·0131 nearly. *A*
72414	28713
24138	88070
3154032 ÷ 10000000	86139
= ·3154032. *Ans.*	19310

17. Divide 507·97 by ·0023, to 4 places of decimals.

18. Reduce $7\frac{1}{2}$ guineas to the decimal of £1000.

507·97 ÷ ·0023	20*s.* = $\frac{1}{1000}$ of £1(
= 5079700 ÷ 23	1*s.* = $\frac{1}{20000}$ of d
23)5079700·0	$21s. \times 7\frac{1}{2} = \frac{21 \times 7\frac{1}{2}}{20000}$ of do.
Ans. 220856·5217	$\frac{21 \times 15}{40000} = \frac{63}{8000}$ = ·007875. *A*

Miscellaneous.

19. Add together $\frac{5}{13}$ of £1, $\frac{3}{16}$ of 6*s.* 8*d.*, $\frac{1}{40}$ of a crown, and $\frac{9}{13}$ of a penny.

$240d. \times \frac{5}{13} = \frac{1200}{13}d.$; which added to $\frac{9}{13}d. = \frac{1209}{13}d. = 93d.$;
also, $80d. \times \frac{3}{16} = 15d.$; and $60d. \times \frac{1}{40} = 1\frac{1}{2}d.$;
hence, $93d. + 15d. + 1\frac{1}{2}d. = 109\frac{1}{2}d. = 9s.\ 1\frac{1}{2}d.$ *Ans.*

20. If $\frac{2}{57}$ of $3\frac{1}{2}$ of $7\frac{3}{5}$ of $\frac{4}{7}$ of 36 lbs. of sugar cost 12*s.* $6\frac{1}{2}d.$, how much will 17 tons 17 cwt. cost?

36 lbs. $\times \frac{4}{7} \times 7\frac{3}{5} \times 3\frac{1}{2} \times \frac{2}{57}$: 21 cwt. $\times$ 17 :: 12*s.* $6\frac{1}{2}d.$
or, 36 lbs. $\times \frac{4}{7} \times \frac{38}{5} \times \frac{7}{2} \times \frac{2}{57}$: 112 lbs. $\times$ 21 $\times$ 17 :: $\frac{301}{2}d.$

$$\frac{301d.}{2} \times \frac{\overset{7}{\cancel{112}} \times \overset{7}{\cancel{21}} \times 17}{1} \times \frac{\overset{3}{\cancel{7}} \times 5 \times \cancel{2} \times 5 \times \overset{1}{\cancel{7}}}{\underset{4}{\cancel{36}} \times \cancel{4} \times \underset{2}{\cancel{38}} \times \cancel{7} \times \cancel{2}} = \frac{301d. \times 7 \times 7 \times 17 \times 5}{4}$$

$= 1253665d. \div 4 = 313416\frac{1}{4}d. = £1305\ 18s.\ 0\frac{1}{4}d.$ *Ans.*

21. If 12 men can dig a trench 15 yards long and 4 broad in 3 days of 12 hours each, in how many days of 9 hours each can 8 men dig a trench 20 yards long and 8 broad?

$$\left.\begin{array}{l} 8 \text{ m.} : 12 \text{ m.} \\ 15 \text{ yds.} : 20 \text{ yds.} \\ 4 \text{ yds.} : 8 \text{ yds.} \\ 9 \text{ hrs.} : 12 \text{ hrs.} \end{array}\right\} :: 3 \text{ da.} : \frac{3 \text{ da.} \times 12 \times 20 \times 12}{15 \times 4 \times 9} = \frac{144}{9} = 16 \text{ da. } \textit{Ans.}$$

22. What will be the cost of painting the walls of a room, at 1*s.* 7*d.* per square yard, the length being 19 feet $10\frac{1}{4}$ inches, the breadth 16 feet $1\frac{3}{4}$ inches, and the height 10 feet 3 inches? (Solve by decimals.)

19 ft. 10·25 in.
16 1·75
36 ft. 0 in. $\times$ 2 = 72 ft., the circuit of the room;
72 $\times$ 10·25 = 738 sq. ft. = 82 sq. yds. to be painted;
82 yds. at $19d. = 1558d. = 129{\cdot}83s. = £6{\cdot}4916.$ *Ans.*

23. A cistern has two pipes, by one of which it may be filled in 20 minutes, and by the other in 25 minutes; it has also a

discharging pipe, by which it might be emptied in 18. If all three were open together, in what time would the cistern be filled?

If the quantity supplied by the 1st pipe be called 1 measure, the content of the cistern will be 20 such measures.

Quantity received by the cistern per minute, when the three pipes run together, $= (\frac{1}{20} + \frac{1}{36} - \frac{1}{18})$ of 20 measures.

$$= 1 + \tfrac{4}{9} - \tfrac{10}{9} \text{ meas.} = 1 + \tfrac{26}{45} - \tfrac{50}{45} \text{ meas.}$$
$$= 1 - \tfrac{14}{45} \text{ meas.} = \tfrac{31}{45} \text{ meas.}$$

$\frac{31}{45}$ meas. in 1 min. $= 31$ meas. in 45 min. $= 1$ meas. in $\frac{45}{31}$ min.

$$= 20 \text{ meas. in } \frac{45 \times 20}{31} \text{ min. or in } 29\tfrac{1}{31} \text{ min. } \textit{Ans.}$$

24. Extract the square root of $514\frac{185}{256}$.

$$\sqrt{514\tfrac{185}{256}} = \sqrt{131769} \div \sqrt{256};$$

$\sqrt{131769} = 363$;

	9
66	417
6	396
723	2169
	2169

$\sqrt{256} = 16$;

	1
26	156
	156

Hence, $363 \div 16 = 22\frac{11}{16}$. *Ans.*

25. Extract the cube root of 228099131.

$\sqrt[3]{228099131} = 611$. *Ans.*

		216	
181	10800	12099	
	181		
	10981	10981	
	1		
1831	1116300	1118131	
	1831		
	1118131	1118131	

26. At what rate (simple interest) will £2063 15*s.* amount to £2249 9*s.* 9*d.* in $2\frac{1}{2}$ years?

Amt. of 41275*s.* in $2\frac{1}{2}$ yrs. $= 44989\frac{3}{4}s.$
do. of 1 in $2\frac{1}{2}$ yrs. $= \frac{179959}{165100}$.
do. of 100 in $2\frac{1}{2}$ yrs. $= \frac{179959}{1651} = 109$;
hence the rate $= 9$ per cent. in $2\frac{1}{2}$ years
$= 18$ per cent. in 5 years
$= 3\frac{3}{5}$ per cent. per annum. *Ans.*

27. A person invests £9075 in the 3 per Cents. at $90\frac{3}{4}$, and on heir rising to 91 transfers it to the $3\frac{1}{2}$ per Cents. at $97\frac{1}{2}$; what ıcrease does he make thereby in his annual income?

£9075 buys $\frac{9075}{90{\cdot}75} = 100$ cents. of 3 per Cent. stock.

100 cents. sold out at 91, to buy $3\frac{1}{2}$ per Cent. stock at $97\frac{1}{2}$, obtains $\frac{100 \times 91}{97\frac{1}{2}}$ cents. $= \frac{200 \times 91}{195} = \frac{40 \times 7}{3} = 93\frac{1}{3}$ cents. of $3\frac{1}{2}$ per Cent. stock; the income from which is $£3\frac{1}{2} \times 93\frac{1}{3}$ $= £\frac{7}{2} \times \frac{280}{3} = £326\ 13s.\ 4d.$: whereas the income from 3 per Cent. stock was $= 100$ cents. at £3 $= £300$; therefore the increase by the transfer is £26 13*s.* 4*d.* *Ans.*

28. A person buys teas at 3*s.* and 4*s.* the lb., and mixes them in ıe proportion of 4 to 7; what will he gain per cent. by selling the ıixture at 4*s.* 2*d.* per lb.?

4 lbs. at 3*s.* $= 12s.$
7 at 4*s.* $= 28s.$
$40s.$ is the prime cost of 11 lbs.
4*s.* 2*d.* $\times 11 = 45\frac{5}{6}s.$ the selling price of 11 lbs.

40 : 100 :: $5\frac{5}{6}$
or 24 : 10 :: 35 : $14\frac{7}{12}$ per cent. *Ans.*

APPENDIX (B).

TABLE OF ALL THE PRODUCTS UNDER 12000, THAT ARE COMPOSED OF PRIME FACTORS ABOVE 19.

[All integral numbers from 1 to 12000, except the following products of two prime factors, are either Primes, or else divisible by less than 20.]

Factor	Product	Factor	Product	Factor	Product	Factor	Product
23	529	31	1457	43	2021	47	2491
23	667	37	1517	23	47	41	2501
23	713	29	37	29	59	23	07
29	841	23	41	31	77	43	37
23	851	37	91	29	2117	31	73
29	899	23	1633	41	73	29	81
23	943	31	43	37	83	23	99
31	961	23	79	31	2201	43	2623
23	989	41	81	47	09	37	27
29	1073	29	1711	23	31	37	2701
23	81	37	39	37	57	41	47
31	1147	41	63	31	63	31	59
29	89	29	69	43	79	47	73
23	1219	23	1817	29	91	53	2809
29	47	31	29	23	2323	29	13
31	71	43	49	23	69	47	67
31	1333	31	91	29	2407	43	81
23	57	23	1909	41	19	41	2911
29	63	41	27	31	49	23	21
37	69	29	43	23	61	37	23
23	1403	37	61	37	79	29	29

Factor	Product	Factor	Product	Factor	Product	Factor	Product
29	2987	59	3481	37	4033	23	4531
41	93	31	3503	31	61	29	53
31	3007	53	51	61	87	47	59
23	13	43	69	23	4117	23	77
43	53	37	89	41	41	43	4601
37	71	59	99	23	63	31	19
29	3103	23	3611	43	71	41	33
53	27	41	49	37	81	59	61
31	31	29	83	47	83	31	81
43	39	47	3713	53	87	43	87
47	49	61	21	59	89	37	99
23	51	37	37	41	4223	53	4717
29	61	23	49	31	47	29	27
31	93	53	63	59	4307	47	47
23	97	29	99	31	09	67	57
53	3233	37	3811	29	21	61	4819
41	39	43	27	61	31	47	41
29	77	23	41	43	43	29	43
37	93	53	69	29	79	37	47
31	3317	47	3901	41	87	23	53
47	37	31	37	23	93	43	59
31	79	59	53	53	99	31	67
43	97	37	59	43	4429	67	91
41	3403	29	73	23	39	59	97
23	27	41	77	61	53	29	5017
47	31	23	79	41	69	47	29
23	73	29	4031	67	89	71	41

Factor	Product	Factor	Product	Factor	Product	Factor	Product
31	5053	29	5539	43	5977	23	6463
61	63	23	43	31	83	29	67
37	69	31	49	53	89	43	93
47	5123	67	61	37	6031	73	97
23	29	37	87	23	49	67	99
53	41	29	97	73	59	23	6509
37	43	71	5609	59	77	61	27
31	77	31	11	31	6107	47	33
71	83	41	17	41	09	31	41
29	91	43	33	29	19	79	57
41	5207	53	71	47	57	29	83
23	21	41	99	61	61	37	6623
29	49	29	5713	31	69	29	41
59	51	59	23	37	79	61	49
23	67	73	67	23	87	59	67
67	93	29	71	41	91	41	83
47	5311	23	73	23	6233	37	97
73	29	53	77	79	41	53	6731
53	53	37	5809	61	83	23	39
23	59	43	91	59	6313	43	51
31	63	71	93	71	19	29	57
41	71	23	5911	23	71	67	67
61	5429	61	17	37	6401	41	6847
53	59	31	21	43	07	71	87
43	61	59	59	59	31	83	89
23	97	67	63	41	37	61	93
37	5513	47	69	47	39	67	6901

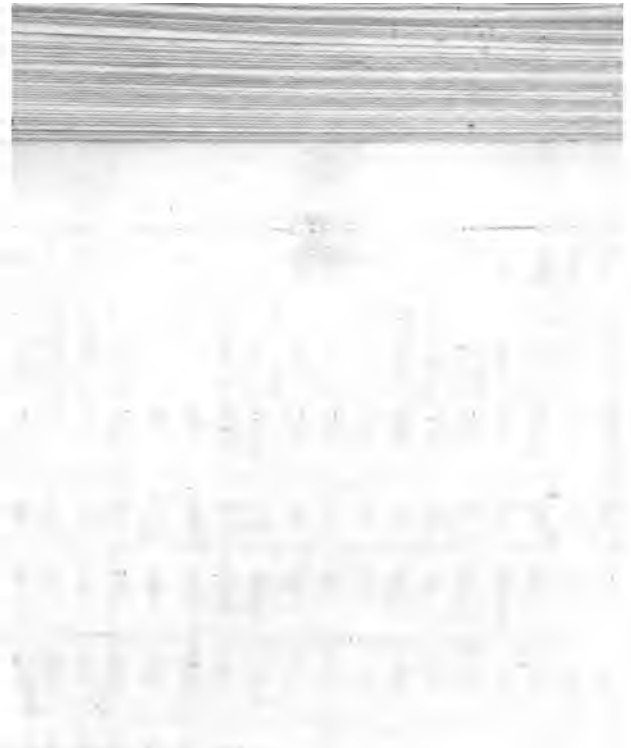
Factor	Product	Factor	Product	Factor	Product	Factor	Product
31	6913	41	7339	37	7807	29	8207
29	31	37	63	73	11	43	13
53	43	53	67	41	31	73	49
29	89	73	73	47	49	37	51
47	7003	47	79	29	59	23	57
43	09	83	87	53	97	43	99
79	31	31	7409	41	7913	53	8321
31	37	41	21	89	21	31	39
23	61	43	39	73	57	61	57
37	67	29	53	31	67	83	83
73	81	31	71	79	79	37	99
41	93	59	93	23	81	31	8401
47	97	73	7519	61	91	47	13
31	99	67	71	53	8003	23	41
37	7141	71	97	71	23	79	53
23	53	23	7613	23	27	43	71
67	69	29	27	29	33	37	73
71	71	47	61	83	51	61	79
43	81	79	63	41	77	29	97
23	99	43	97	59	83	47	8507
31	7223	59	7729	23	8119	67	09
53	61	71	39	47	31	83	49
29	79	61	47	79	37	43	57
37	89	23	51	29	49	23	79
23	91	31	81	31	53	31	87
67	7303	43	83	41	59	79	8611
71	13	29	7801	59	8201	37	21

Factor	Product	Factor	Product	Factor	Product	Factor	Product
31	5053	29	5539	43	5977	23	6463
61	63	23	43	31	83	29	67
37	69	31	49	53	89	43	93
47	5123	67	61	37	6031	73	97
23	29	37	87	23	49	67	99
53	41	29	97	73	59	23	6509
37	43	71	5609	59	77	61	27
31	77	31	11	31	6107	47	33
71	83	41	17	41	09	31	41
29	91	43	33	29	19	79	57
41	5207	53	71	47	57	29	83
23	21	41	99	61	61	37	6623
29	49	29	5713	31	69	29	41
59	51	59	23	37	79	61	49
23	67	73	67	23	87	59	67
67	93	29	71	41	91	41	83
47	5311	23	73	23	6233	37	97
73	29	53	77	79	41	53	6731
53	53	37	5809	61	83	23	39
23	59	43	91	59	6313	43	51
31	63	71	93	71	19	29	57
41	71	23	5911	23	71	67	67
61	5429	61	17	37	6401	41	6847
53	59	31	21	43	07	71	37
43	61	59	59	59	31	83	89
23	97	67	63	41	37	61	93
37	5513	47	69	47	39	67	6901

Factor	Product	Factor	Product	Factor	Product	Factor	Product
31	6913	41	7339	37	7807	29	8207
29	31	37	63	73	11	43	13
53	43	53	67	41	31	73	49
29	89	73	73	47	49	37	51
47	7003	47	79	29	59	23	57
43	09	83	87	53	97	43	99
79	31	31	7409	41	7913	53	8321
31	37	41	21	89	21	31	39
23	61	43	39	73	57	61	57
37	67	29	53	31	67	83	83
73	81	31	71	79	79	37	99
41	93	59	93	23	81	31	8401
47	97	73	7519	61	91	47	13
31	99	67	71	53	8003	23	41
37	7141	71	97	71	23	79	53
23	53	23	7613	23	27	43	71
67	69	29	27	29	33	37	73
71	71	47	61	83	51	61	79
43	81	79	63	41	77	29	97
23	99	43	97	59	83	47	8507
31	7223	59	7729	23	8119	67	09
53	61	71	39	47	31	83	49
29	79	61	47	79	37	43	57
37	89	23	51	29	49	23	79
23	91	31	81	31	53	31	87
67	7303	43	83	41	59	79	8611
71	13	29	7801	59	8201	37	21

Factor	Product	Factor	Product	Factor	Product	Factor	Product
89	8633	23	9131	43	9589	73	10001
53	39	41	43	53	93	43	19
41	51	89	67	29	99	37	27
31	8711	53	69	59	9617	79	33
23	17	67	79	23	37	89	57
31	73	29	93	31	41	29	63
67	77	61	9211	23	83	23	97
59	91	23	23	89	9701	67	10117
23	8809	47	59	31	03	29	21
37	43	59	63	71	27	53	23
53	51	73	71	37	31	73	47
83	81	37	87	43	61	61	87
29	8903	71	9301	29	73	23	89
59	09	41	07	97	97	101	10201
37	17	67	13	41	99	59	07
79	27	47	53	31	9827	53	29
23	47	83	79	43	47	29	37
47	77	41	89	59	53	37	49
89	89	23	9407	71	69	31	61
71	9017	97	09	41	81	43	77
29	19	53	87	23	9913	41	91
83	47	37	9509	47	17	23	10327
47	71	31	17	61	43	79	49
43	73	89	23	37	53	43	63
29	77	41	53	23	59	97	79
31	83	73	63	67	83	37	97
61	89	61	77	97	91	101	10403

Factor	Product	Factor	Product	Factor	Product	Factor	Product
29	10411	29	10817	103	11227	41	11603
53	41	31	19	47	33	59	23
31	47	79	23	59	69	29	29
37	71	37	41	29	81	103	39
47	81	83	73	23	93	61	51
23	10511	73	77	89	11303	43	53
67	19	61	10919	43	09	89	59
41	37	67	21	47	27	107	63
83	41	31	43	41	57	23	11707
53	47	47	51	37	59	37	29
61	53	97	61	83	71	59	41
59	61	79	81	31	77	31	49
97	73	29	91	59	87	79	71
71	79	101	11009	101	11413	61	73
23	10603	23	17	107	49	47	97
103	09	103	21	73	61	53	11819
29	43	73	23	23	77	71	57
23	49	41	29	37	11507	29	61
47	69	61	41	29	13	31	73
59	79	43	51	41	21	109	81
71	10721	29	11107	83	37	73	99
23	41	41	11	31	63	43	11911
31	57	31	29	43	67	79	29
47	63	71	47	23	69	23	83
41	83	53	83	71	73	67	93
43	93	67	89	37	81	41	12013
101	10807	23	11201	67	91	61	17



ANSWERS TO THE EXERCISES.

4.

(1) 653.
(2) 365.
(3) 405.
(4) 479.
(5) 613.
(6) 4720.
(7) 3928.
(8) 51391.
(9) 49015.
(10) 275382.
(11) 167761958.
(12) 1396026329.
(13) 5552.
(14) 25649.
(15) 93823.
(16) 28498.
(17) 3882169.
(18) 5301919.
(19) 875467742.

5.

(1) 3281757.
(2) 30930780.
(3) 998510330.
(4) 21718599.
(5) 6909991.
(6) 2103778097.
(7) 125970980.
(8) 376589973.
(9) 9647637.
(10) 112590982.
(11) 897028499.
(12) 193009609.
(13) 399098080.
(14) 109998684.

6.

(1) By 2 : *Ans.* 12778940570; by 3 : *Ans.* 19168410855.
5 : 31947351425; 8 : 51115762280.
10 : 63894702850; 12 : 76673643420.
(2) By 6 : *Ans.* 24598504098; by 7 : *Ans.* 28698254781.
4 : 16399002732; 9 : 36897756147.
11 : 45097257513; 12 : 49197008196.
(3) 24699753000; 37049629500; 555744442500.
(4) 9262407375; 10497395025; 41114888850.
(5) 18448004692; 27672007038; 41153241236.
(6) 5556519000; 65855040000; 322415300000.
(7) 17249767870. (8) 14120497024. (9) 27247417656.
(10) 37729047840. (11) 5734153152. (12) 1534115200.
(13) 262179249038. (14) 776909258400.
(15) 168115211972. (16) 158194175121.
(17) *97484053160340*. (18) 186286131500000.

7.

(1) By 2. *Ans.* 14826924; 57980171, rem. 1.
3. 9884616; 38653447, rem. 2.
5. 5930769, rem. 3; 23192068, rem. 3.
8. 3706731; 14495042, rem. 7.
10. 2965384, rem. 8; 11596034, rem. 3.
12. 2471154; 9663361, rem. 11.

(2) By 11. *Ans.* 9249, rem. 7; 20701, rem. 2; 77355, rem. 10.
12. 8478, rem. 10; 18976, rem. 1; 70909, rem. 7.

(3) By 7. *Ans.* 3353129001; by 8. *Ans.* 2933987875, rem. 7.
9. 2607989223; 12. 1955991917, rem. 3.

(4) 610395; 6103, rem. 950; 5086, rem. 750.

(5) 97799, rem. 1000; 15368, rem. 3900; 896499, rem. 20.

(6) 5926748; 4532219, rem. 1; 4055143, rem. 7.

(7) 847629; 791120, rem. 6; 659267.

(8) 725928, rem. 2; 575744; 451258, rem. 30.

(9) 282647, rem. 11; 206097; 157026, rem. 18.

(10) 135517; 115031, rem. 75; 101987, rem. 2.

(11) 897480; 930720; 52353. (12) 10406; 840, rem. 20.

(13) 275914. (14) 3050099. (15) 690058. (16) 135094, rem. 617.

(17) 2556, rem. 2278. (18) 90685. (19) 324, rem. 1423. (20) 2167.

8.

(1) 127891395; 144943581; 161995767.
(2) 115455877; 171775817; 199935787.
(3) 152820288; 133717752; 229230432.
(4) 55769, rem. 49; 47055, rem. 55; 61460, rem. 35.

9.

(1) $40\frac{1}{3}$. (2) $\frac{9}{35}$. (3) $\frac{4}{7}$. (4) $22\frac{11}{18}$.
(5) 3800. (6) $\frac{40}{81}$. (7) 5046.

10.

(1) £78; £236 19*s.*; £501 9*s.*
(2) £6 10*s.*; £19 14*s.* 11*d.*; £41 15*s.* 9*d.*
(3) £76 6*s.* 8*d.*; £70 13*s.* $4\frac{3}{4}d.$ (4) £11 7*s.* $1\frac{1}{2}d.$; £12 7*s.* $3\frac{1}{4}d.$
(5) 13*s.* $1\frac{1}{2}d.$; 17*s.* 11*d.*; 27*s.* $7\frac{1}{2}d.$ (6) 18427*d.*; 851*d.*; 233*d.*
(7) 619*f.*; 1814*f.*; 9124*f.* (8) 345289*f.*; 2880480*f.*
(9) 43175 *hf. d.*; 27370 hf. d.
(10) £20 6*s.* $4\frac{1}{2}d.$; £367 10*s.* $0\frac{1}{2}$d. (11) 1134000d.; 61441*d.*

(12) 217 *gu.*; 39 *gu.* (13) 1608 *fo.*; 4721 *fo.*
(14) 676 *thr.*; 6800 *thr.*; 112 *thr.* 2*d.* (15) 2257 *sixp.*; 73 *sixp.* $0\frac{1}{4}d.$
(16) 365 *gu.*; 409 *gu.* (17) 328680*d.*; 15570*d.*
(18) 396 *crs.* 8*d.*; 50 *crs.* 2*s.* 3*d.*
(19) 259 *hf. crs.* 1*s.* $11\frac{3}{4}d.$; 315 *hf. crs.*; 372 *hf. crs.*
(20) 74 *fl.* 1*s.* 1*d.*; 92 *fl.* 1*s.* 6*d.*; 3117 *fl.*
(21) 10 *lb.* 9 *oz.* 13 *dwt.* 20 *grs.*; 3 *lb.* 5 *oz.* 16 *grs.*
(22) 149208 *grs.*; 63669 *grs.*
(23) 5300 *grs.*; 1226899 *grs.*; 80976 *grs.*
(24) 22 *lb.* 4 *oz.* 15 *dwt.*; 56 *lb.* 10 *oz.*; 19 *lb.* 19 *dwt.*
(25) 7165 *lb.*; 32928 *lb.*; $1456\frac{1}{2}$ *lb.*
(26) 17 *cwt.* 2 *qr.* 14 *lb.*; 36 *cwt.* 1 *qr.* 3 *lb.* 12 *oz.*
(27) 88704 *oz.*; 16223 *oz.*; 1552 *oz.* (28) 130829 *oz.*; 85808 *oz.*
(29) 6 *t.* 10 *cwt.* 3 *qr.* 16 *lb.* 7 *oz.*; 15 *t.* 8 *cwt.* 2 *qr.* 15 *lb.*
(30) 17 *cwt.* 3 *qr.* 6 *lb.* 15 *oz.* 9 *drs.*; 227 *cwt.* 1 *qr.* 8 *lb.* 5 *oz.*
(31) 1726 *in.*; 51348 *in.* (32) 19326 *ft.*; 122411 *ft.*
(33) 591 *mi.* 2 *f.* 38 *yds.*; 120 *mi.* 4 *f.* 3 *yds.* 1 *ft.*
(34) 4232448 *sq. in.*; 35026660 *c. in.*
(35) 2391 *po.*; 42160 *po.*; $1137\frac{1}{4}$ *po.*
(36) 816 *ac.* 3 *ro.* 28 *po.*; 176000 *ac.* (37) 3824 *pts.*; 382 *pts.*
(38) 53 *bu.*; 28 *bu.*
(39) 503700 *min.*; 525960 *min.* (40) 252 *E. e.*; 275 *E. e.*
(41) 210 *yds.*; 262 *yds.* 1 *qr.* (42) 410 *gu.* 10*s.*; 262 *gu.* 8*s.*
(43) £59 17*s.*; £189. (44) 195 $\frac{5}{17}$ *lb. av.*; 72 *lb. av.*
(45) $58\frac{1}{3}$ *lb. tr.*; $89\frac{569}{3504}$ *lb. tr.*

11.

(1) £32 14*s.* 8*d.* (2) £269 15*s.* 10*d.* (3) £360 2*s.* $8\frac{3}{4}d.$
(4) £404 14*s.* 1*d.* (5) £440 15*s.* $6\frac{1}{4}d.$ (6) £428 3*s.* $9\frac{1}{2}d.$
(7) £334 19*s.* 4*d.* (8) £500 1*s.* $2\frac{1}{4}d.$ (9) £356 9*s.* $10d\frac{3}{4}$.
(10) £440 7*s.* 8*d.* (11) £526 8*s.* 4*d.* (12) £703 2*s.* $10\frac{1}{4}d.$
(13) £15822 15*s.* $4\frac{1}{2}d.$ (14) £14809 18*s.* 3*d.* (15) £7860 19*s.* 0*d.*
(16) 179 *cwt.* 2 *qr.* 18 *lb.* (17) 201 *lb.* 11 *oz.* 12 *dr.*
(18) 192 *t.* 12 *cwt.* 1 *qr.* 4 *lb.* 3 *oz.* (19) 140 *lb.* 7 *oz.* 8 *dwt.*
(20) 208 *yd.* 1 *ft.* 6 *in.* (21) 242 *mi.* 3 *f.* 120 *yd.* 11 *in.*
(22) 209 *ac.* 1 *ro.* 28 *po.* (23) 286 *sq. yd.* 5 *ft.* 120 *in.*
(24) 633 *c. yd.* 12 *ft.* 485 *in.* (25) 226 *gal.* 2 *qt.*
(26) 267 *qu.* 4 *bu.* 1 *pk.* (27) 1060 *da.* 13 *hr.* 41 *min.* 26 *sec.*
(28) £1325 2*s.* $11\frac{1}{4}d.$ (29) £12067 18*s.* 5*d.*
(30) £3145 13*s.* 6*d.*

12.

(1) £5 8s. 1¼d. (2) £15 16s. 10½d. (3) £48 8s. 5d.
(4) £49 19s. 6¾d. (5) £9 10s. 11½d. (6) £48 10s. 3d.
(7) £30 19s. 6½d. (8) £90 11s. 11½d. (9) £109 16s. 8¾d.
(10) £76 16s. 1¾d. (11) £70 0s. 11¼d. (12) £36 19s. 7¼d.
(13) 25 lb. 10 oz. 15 dwt. (14) 10 oz. 3 dwt. 23 gr.
(15) 15 cwt. 1 qr. 19 lb. (16) 29 yd. 2 ft. 6 in.
(17) 129 ac. 3 ro. 31 po. 13 yd. (18) 79 mi. 1468 yd. 1 ft.
(19) 286 da. 15 hr. 49 min. (20) £99 19s. 5½d., and £98 19s. 11¼d.
(21) £2091 1s. 9d. (22) £17 18s. 5d.

13.

(1) £92 17s. 7½d.; £139 6s. 5½d.; £185 15s. 3d.
(2) £285 19s. 4½d.; £381 5s. 10d.; £476 12s. 3½d.
(3) £175 13s. 10½d.; £527 1s. 7½d.; £702 15s. 6d.
(4) £302 9s. 2d.; £423 8s. 10d.; £544 8s. 6d.
(5) £454 1s. 9d.; £605 9s. 0d.; £681 2s. 7½d.
(6) £884 13s. 0¾d.; £2064 3s. 9¾d.; £2653 19s. 2½d.
(7) £2757 15s. 4½d.; £3217 7s. 11½d.; £3677 0s. 6d.
(8) £5833 18s. 6½d.; £6417 6s. 4¾d.; £7000 14s. 3d.
(9) £7967 13s. 11½d.; £8764 9s. 4½d.; £9561 4s. 9d.
(10) £3680 9s. 7d.; £4048 10s. 6½d.; £4416 11s. 6d.
(11) £534 15s. 4⅓d. (12) £1154 17s. 6d.
(13) £5863 2s. 6d. (14) £1339 5s. 4d.
(15) £7093 11s. 6d. (16) £18392 11s. 0d.
(17) £5028 2s. 7¾d. (18) £22138 1s. 11⅓d.
(19) £6761 11s. 3d. (20) £14807 8s. 2d.
(21) £25072 10s. 10d. (22) £13776 7s. 8¾d.
(23) £133393 1s. 0½d. (24) £70314 14s. 6d.
(25) 143 cwt. 2 qr. 11 lb. (26) 44 t. 4 cwt. 2 qr. 20 lb.
(27) 340 cwt. 1 qr. 5 lb. 9 oz. (28) 145 lb. 7 oz. 1 dwt. 16 grs.
(29) 1223 ac. 1 ro. 33 po. (30) 692 da. 15 hr. 59 min. 9 sec.

14.

(1) 15. (2) 70. (3) 91. (4) $6\frac{13}{17}$.
(5) 24. (6) $16\frac{1}{3}$. (7) 57. (8) $8\frac{66}{85}$.
(9) $4\frac{71}{946}$. (10) 28. (11) 31. (12) $32\frac{13113}{14420}$.
(13) 32. (14) $17\frac{3}{8}$. (15) $9300\frac{3}{17}$. (16) $198\frac{3}{5}$.

15.

(1) £29 18*s.* 10*d.*
(2) £36 15*s.* $9\frac{1}{2}$*d.*
(3) £88 16*s.* $5\frac{3}{4}$*d.*
(4) £0 17*s.* $10\frac{3}{8}$*d.*
(5) £57 13*s.* $10\frac{1}{2}$*d.*
(6) £9 10*s.* $3\frac{3}{4}$*d.*
(7) £9 10*s.* $3\frac{1}{4}$*d.*
(8) £14 6*s.* $2\frac{1}{14}$*d.*
(9) £3 13*s.* $11\frac{1}{2}$*d.*
(10) £25 11*s.* $11\frac{29}{35}$*d.*
(11) £9 4*s.* $6\frac{7}{11}$*d.*
(12) £3 0*s.* $11\frac{1}{8}$*d.*
(13) £4 14*s.* $0\frac{1}{4}$*d.*
(14) £9 14*s.* $11\frac{1}{8}$*d.*
(15) £47 8*s.* 1*d.*
(16) £3 9*s.* 7*d.*
(17) £0 0*s.* $7\frac{39}{56}$*d.*
(18) £3 3*s.* $10\frac{11}{44}$*d.*
(19) £0 6*s.* $0\frac{59}{68}$*d.*
(20) £11 8*s.* 9*d.*
(21) £2 10*s.* $7\frac{3}{4}$*d.*
(22) £1 3*s.* $9\frac{5}{7}$*d.*
(23) £0 3*s.* $6\frac{3}{4}$*d.*
(24) £3 12*s.* $10\frac{1}{2}$*d.*
(25) £0 11*s.* $6\frac{17}{100}$*d.*
(26) £5 16*s.* $11\frac{13}{40}$*d.*
(27) £1 15*s.* $2\frac{19}{130}$*d.*
(28) £0 13*s.* $0\frac{43}{152}$*d.*
(29) £2 4*s.* $7\frac{1}{5}$*d.*
(30) £2 13*s.* $11\frac{1}{2}$*d.*
(31) £0 19*s.* $11\frac{7}{48}$*d.*
(32) £1 0*s.* $11\frac{3}{4}$*d.*
(33) 7 *cwt.* 19 *lb.*
(34) 15 *cwt.* 1 *qr.* $23\frac{3}{8}$ *lb.*
(35) 3 *qr.* $2\frac{1}{8}$ *na.*
(36) 3 *da.* 12 *hr.* $15\frac{5}{51}$ *min.*
(37) 90 *ac.* 3 *ro.* $0\frac{1}{32}$ *po.*
(38) 6 *mi.* 4 *f.* $22\frac{1}{3}$ *po.*

16.

(1) £131 18*s.* $0\frac{3}{8}$*d.*
(2) £8 12*s.* $9\frac{3}{4}$*d.*
(3) 9*s.* $11\frac{1}{8}$*d.*
(4) £237 1*s.* $2\frac{3}{4}$*d.*
(5) £1376 10*s.* $6\frac{1}{4}$*d.*
(6) £3397 2*s.* $10\frac{1}{2}$*d.*
(7) £2 4*s.* $11\frac{1}{3}$*d.*
(8) £1528 14*s.* $7\frac{3}{40}$*d.*
(9) 2 *yd.* 1 *ft.* $0\frac{3}{17}$ *in.*
(10) 399 *cwt.* 2 *qr.* $18\frac{1}{7}$ *lb.*
(11) 472734 *in.*
(12) 23199372 *sq. in.*
(13) 555512 *sq. ft.*
(14) 32101272 *sq. in.*

17.

(1) £31 2*s.* $10\frac{3}{8}$*d.*
(2) £160 3*s.* 3*d.*
(3) £0 13*s.* $3\frac{5}{6}$*d.*
(4) £7 16*s.* $10\frac{1}{2}$*d.*
(5) £41 6*s.* $1\frac{1}{168}$*d.*
(6) £4 19*s.* $7\frac{1}{2}$*d.*
(7) £0 5*s.* $5\frac{5}{17}$*d.*
(8) £25 14*s.* $0\frac{111}{125}$*d.*
(9) 13 *lb.* 11 *oz.* $15\frac{1}{3}$ *dwt.*
(10) 1 *da.* 4 *hr.* $37\frac{5}{7}$ *min.*
(11) 9 *ac.* 3 *ro.* 22 *yd.*
(12) 1 *ac.* 16 *po.* 9 *yd.* 3 *ft.*
(13) 2 *ac.* 2 *ro.* 2 *po.* 23 *yd.* 7 *ft.* 51 *in.*
(14) 3 *yrs.* 300 *da.* 12 *hr.* 50 *min.*

18.

(1) 37392. (2) 4. (3) 9 *doz.* (4) £15 19*s.* 8½*d.*
(5) £49 15*s.* 4¾*d.* (6) 908 *times*; *rem.* 28*s.* 10*d.*
(7) 678 *pie.*; *rem.* 9*d.* (8) 180 *da.* (9) 250 *ac.* 18¾ *po.*

19.

(1) £1781 17*s.* (2) 100$\frac{8}{11}$. (3) £1 17*s.* 6*d.*
(4) £29 18*s.* 6*d.* (5) 8½ *gn.* (6) 62.
(7) 52 *yds.* (8) £1 9*s.* 4½*d.* (9) £28 11*s.* 0$\frac{3}{8}$*d.*

20.

(1) £59 1*s.* 3*d.* (2) 2*s.* 4½*d.* (3) 12 *boxes.*
(4) 14 *t.* 11 *cwt.* 48 *lb.*; £119. (5) 63 *yd.* 2 *ft.* 11 *in.*; 7128 of each.
(6) 22*s.* 8*d.* (7) 4¼ *gall.*
(8) Elder 7*s.* 7*d.*; younger 6*s.* 5*d.* (9) 8093 *sq. ft.* 36 *in.*

21.

(1) 13$\frac{13}{17}$ *yds.* (2) £650. (3) £8 18*s.* 6*d.* (4) 2920 *rup.*
(5) *W*, 22*s.* 6*d.*; *M*, 7*s.* 6*d.* (6) 5 *lb.* of each. (7) 400½, 166⅔.
(8) 25*s.* 1$\frac{2}{3}$*d.* (9) £428 18*s.* 3*d.*

22.

(1) 13*s.* 7½*d.* (2) £283 6*s.* 8*d.*
(3) *A*, £5 19*s.* 10½*d.*; *B*, £4 13*s.* 3½*d.* (4) 37 of each.
(5) £6. (6) *C*, 80 *ac.* 1 *ro.* 27 *po.*; *D*, 53 *ac.* 2 *ro.* 18 *po.*
(7) £2 16*s.* 10½*d.* (8) *A*, 46; *B*, 47; *C*, 54 *lb.*
(9) 14 *yd.* 7$\frac{3}{14}$ *in.*

23.

(1) 19. (2) £3 6*s.* 8*d.* (3) 7$\frac{17}{27}$. (4) 9*s.* 6*d.*
(5) 3$\frac{3}{5}$ *gall.* (6) 15 *ac.* 1 *ro.* 19 *po.* (7) 70 *lb.*
(8) *A*, 50; *B*, 55; *C*, 87; *D*, 90 *sal.*; £34 9*s.* 0$\frac{3}{4}$*d.*
(9) 18 *sq. po.* 14 *yd.* 1 *ft.* 100 *in.*

24.

(1) 3 *cwt.* 1 *qr.* 26 *lb.* (2) 28800 *times.* (3) 2436.
(4) 19*s.* (5) 1 *cwt.* 3 *qr.* 18$\frac{4}{7}$ *lb.* (6) 8*s.*
(7) *A*, £13 13*s.* 6*d.*; *B*, £15; *C*, £16 6*s.* 6*d.*
(8) 2$\frac{1}{3}$, 72$\frac{1}{3}$, 290 *da.* (9) 1 *ro.* 27 *po.* 21 *yd.* 7 *ft.*

25

(1) 20 *wks.* (2) Fore wh. 47520, Hind wh. 28160 *rev.*
(3) 21*s.* 6*d.* (4) 5*s.* 7*d.*, £1 2*s.* 4*d.*, £5 11*s.* 8*d.*
(5) *A*, £1 10*s.* 2*d.*; *B*, £2 0*s.* 2*d.*; *C*, £1 9*s.* 8*d.* (6) $1\frac{1}{8}$ *gall.*
(7) 2900 *foot. pass.*, 800 *carr.* (8) Each man 3*s.* 9*d.*, woman 1*s.* 3*d.*
(9) 166 *mi.* 1 *fur.*

26.

(1) 16 *ft.* $10\frac{1}{2}$ *in.* (2) 9 *gall.* (3) 44 *ells* 4 *qr.* (4) $17\frac{1}{2}$.
(5) A man 3*s.* 6*d.*, woman 1*s.* 2*d.*, youth 1*s.* 9*d.* (6) £36288.
(7) 24 *pieces.* (8) 27 *m.*, 54 *w.*, 81 *b.*
(9) *A*, 10 *ac.* 3 *ro.* 34 *po.* 19 *yd.*; *B*, 9 *ac.* 3 *ro.* 19 *po.* 5 *yd.*

27.

(1) 20. (2) 51. (3) 143. (4) $7\frac{3}{5}$.
(5) 40 *da.* (6) 1 *ft.* 8 *in.* (7) 120. (8) £51 12*s.*
(9) 5*s.* $7\frac{1}{2}$*d.* (10) 3 *lb.* 14 *dr.* (11) 4*s.* 6*d.* (12) 15*s.* $3\frac{3}{4}$*d.*
(13) 15 *cwt.* 3 *qr.* $0\frac{3}{5}$ *lb.* (14) 1750 *mi.* 6 *fur.* $24\frac{6}{11}$ *yd.*

28.

(1) 30*s.* (2) 36 *lb.* (3) £22 10*s.* (4) 2587 *rev.*
(5) 68. (6) 245 *cwt.* (7) 18 *da.* (8) 9 *sh.*
(9) 12 *da.* (10) 9*s.* $4\frac{1}{2}$*d.* (11) $17\frac{1}{2}$*d.* (12) 24 *ap.*
(13) $16\frac{1}{2}$ *qs.* (14) $15\frac{3}{4}$ *lb.* (15) $17\frac{1}{2}$ *da.* (16) 84 *min.*
(17) 26*s.* (18) 90*s.* (19) 15 *lb.* (20) 12 *hrs.*
(21) 37*s.* 1*d.* (22) £356. (23) £51 16*s.* 8*d.*
(24) 2 *lb.* 3 *oz.* 14 *dwt.* 16 *gr.* (25) 15 *oz.* (26) £5 11*s.* $1\frac{1}{2}$*d.*
(27) 4*s.* 8*d.* (28) £377 0*s.* $4\frac{1}{2}$*d.* (29) £164 15*s.* 6*d.*
(30) 6*s.* $4\frac{1}{24}$*d.* (31) $29\frac{1}{11}$ *da.* (32) 450 *m.*
(33) 19 *da.* (34) $21\frac{3}{5}$ *da.* (35) £30 7*s.* 9*d.*

29.

(1) 14 *m.* (2) 16 *ac.* (3) 7 *hrs.* (4) $3\frac{3}{4}$ *mo.*
(5) £7 3*s.* $1\frac{1}{2}$*d.* (6) £3 12*s.* $0\frac{1}{2}$*d.* (7) £555 2*s.* 1*d.*
(8) 49*s.* (9) 43 *lb.* 5 *oz.* (10) 4*d.*
(11) $9\frac{1}{2}$ *hrs.* (12) 410 *da.* (13) 34 *m.*
(14) £59 17*s.* (15) 11 *st.* (16) Average.
(17) 27 *m.* (18) $3\frac{1}{2}$ *ft.* (19) 40 *lb.*
(20) $58\frac{1}{3}$ *lb.* (21) $6\frac{3}{4}$ *cwt.*

30.

(1) $1\frac{7}{8}$; $2\frac{2}{3}$; 5; 1; $8\frac{2}{3}$; 8.
(2) $\frac{10}{10}$; $\frac{64}{5}$; $\frac{56}{7}$; $\frac{324}{18}$.
(3) $\frac{25}{5}$; $\frac{15}{1}$; $\frac{107}{13}$; $\frac{130}{7}$; $\frac{5217}{30}$.

31.

(1) 12. (2) 4. (3) 1. (4) 4.
(5) 16. (6) 7. (7) 13. (8) 21.
(9) 35. (10) 18. (11) 73. (12) 311.
(13) 7. (14) 107. (15) 205. (16) 52.
(17) 74. (18) 11.

32.

(1) 84. (2) 180. (3) 162. (4) 54.
(5) 24. (6) 528. (7) 168. (8) 480.
(9) 1260. (10) 1260. (11) 9900. (12) 31248.
(13) 101400. (14) 4284. (15) 7020. (16) 58968.
(17) 7560. (18) 110880.

33.

(1) $\frac{9}{10}$. (2) $\frac{7}{9}$. (3) $\frac{7}{8}$. (4) $\frac{16}{21}$.
(5) $\frac{13}{21}$. (6) $\frac{18}{28}$. (7) $\frac{6}{5}$. (8) $\frac{7}{12}$.
(9) $\frac{13}{17}$. (10) $\frac{71}{81}$. (11) $\frac{8}{3}$. (12) $\frac{113}{7}$.
(13) $\frac{127}{180}$. (14) $\frac{347}{1326}$. (15) $\frac{30}{43}$. (16) $\frac{5}{5}$.
(17) $\frac{67}{143}$. (18) $\frac{2246}{21}$. (19) $\frac{13}{111}$. (20) $\frac{1353}{122}$.

34.

(1) $\frac{45}{126}$, $\frac{77}{126}$, $\frac{60}{126}$.
(2) $\frac{7488}{15600}$, $\frac{9000}{15600}$, $\frac{195}{15600}$, $\frac{12400}{15600}$.
(3) $\frac{560}{3528}$, $\frac{1617}{3528}$, $\frac{864}{3528}$, $\frac{819}{3528}$.
(4) $\frac{53}{77}$, $\frac{3}{4}$, $\frac{101}{154}$. (5) $\frac{75}{256}$, $\frac{5}{17}$, $\frac{61}{20}$.
(6) $\frac{1771}{7007}$, $\frac{323}{1309}$, $\frac{28}{88}$. (7) 84 : 55. (8) 116 : 117.
(9) 23288 : 21615.
(10) $\frac{4}{225}$. (11) $3\frac{2}{5}$.
(12) $1\frac{5}{16}$. (13) 210. (14) $74\frac{3}{5}$. (15) $50\frac{2}{5}$.

35.

(1) $\frac{47}{72}$. (2) $\frac{9}{10}$. (3) $1\frac{17}{48}$. (4) $2\frac{239}{380}$.
(5) 1. (6) $2\frac{14}{15}$. (7) $7\frac{3}{4}$. (8) $2\frac{19}{18}$.
(9) $20\frac{19}{48}$. (10) $2\frac{13609}{22932}$. (11) $37\frac{37}{44}$. (12) $2\frac{263}{900}$.
(13) $240\frac{1}{240}$. (14) 2. (15) $1\frac{283}{528}$. (16) $2\frac{1678417}{2925600}$.
(17) £25 8*s.* $8\frac{85}{98}$*d.* (18) £31 16*s.* $6\frac{49}{140}$*d.*
(19) £21 10*s.* $3\frac{39}{140}$*d.* (20) £24 19*s.* 3*d.*

36.

(1) $\frac{11}{16}$. (2) $\frac{5}{48}$. (3) $\frac{5}{6}$. (4) $\frac{14}{195}$.
(5) $\frac{1}{8}$. (6) $\frac{56}{255}$. (7) $16\frac{17}{20}$. (8) $\frac{8}{19}$.
(9) $299\frac{9}{17}$. (10) $12\frac{7}{15}$. (11) $27\frac{3}{20}$. (12) $88\frac{3}{10}$.
(13) $17\frac{19}{24}$. (14) $19\frac{1}{3}$. (15) $\frac{101}{102}$. (16) $6\frac{17}{56}$.
(17) $10\frac{51}{65}$. (18) $99\frac{9}{25}$. (19) $977\frac{24}{25}$. (20) $80\frac{2170}{2171}$.
(21) $9\frac{3869}{3876}$. (22) £19 12*s.* $11\frac{1}{19}$*d.*
(23) 17 *cwt.* 3 *qrs.* $22\frac{7}{8}$ *lb.* (24) 6 *ac.* 2 *ro.* $0\frac{78}{111}$ *po.*

37.

(1) $\frac{20}{63}$. (2) $\frac{7}{18}$. (3) $\frac{10}{21}$. (4) $\frac{22}{45}$.
(5) $\frac{12}{25}$. (6) $9\frac{1}{3}$. (7) $33\frac{3}{4}$. (8) $\frac{22}{55}$.
(9) $1\frac{4}{5}$. (10) 14. (11) 20. (12) $7\frac{1}{2}$.
(13) $102\frac{2}{5}$. (14) $58\frac{1}{3}$. (15) $2\frac{11}{12}$. (16) $\frac{11}{108}$.
(17) $\frac{1}{3}$. (18) $2\frac{38}{45}$. (19) 1. (20) $16\frac{98}{165}$.
(21) $71\frac{1}{4}$. (22) $18\frac{3}{4}$. (23) $220581\frac{1}{2}$
(24) $13220\frac{44}{45}$. (25) $12184\frac{3}{8}$.

38.

(1) $1\frac{13}{33}$. (2) $\frac{9}{20}$. (3) $1\frac{1}{5}$. (4) $\frac{4}{27}$.
(5) $7\frac{1}{2}$. (6) $\frac{3}{5}$. (7) $1\frac{3}{5}$. (8) $19\frac{1}{2}$.
(9) $4\frac{78}{77}$. (10) $2\frac{1}{24}$. (11) $4\frac{4}{15}$. (12) $\frac{77}{240}$.
(13) $\frac{1}{14}$. (14) $\frac{5}{8}$. (15) $\frac{5}{14}$. (16) $\frac{3}{4}$.
(17) $\frac{20}{27}$. (18) $\frac{35}{64}$. (19) $\frac{1}{2}$. (20) $4\frac{29}{40}$.

39.

(1) $108\frac{5}{14}$. (2) $399\frac{8}{35}$. (3) $499\frac{12}{143}$. (4) $223\frac{16}{87}$.
(5) $39\frac{68}{119}$. (6) $319\frac{131}{384}$. (7) $214\frac{17}{80}$. (8) $47\frac{47}{48}$.
(9) $2\frac{31}{64}$. (10) $110482\frac{2}{9}$. (11) $13861\frac{5}{7}$. (12) $96461\frac{51}{65}$.
(13) $2186\frac{9}{16}$. (14) $12856\frac{52}{65}$. (15) $11192\frac{178}{180}$. (16) $248\frac{16}{87}$.
(17) $415\frac{40}{99}$. (18) $31\frac{1705}{1755}$. (19) $1\frac{47}{55}$. (20) $\frac{38}{81}$.
(21) $5\frac{376}{512}$. (22) $\frac{477}{2500}$. (23) $56\frac{56}{63}$. (24) $\frac{1}{27}$.

40.

(1) 8*s.* $10\frac{2}{3}$*d.*; 1 *ft.* $10\frac{1}{2}$ *in.*
(2) 5 *ac.* 1 *ro.* (3) £6 15*s.* $10\frac{2}{63}$*d.*
(4) £4 18*s.* $3\frac{5}{9}$*d.*
(5) 25 *mi.* 2 *fur.* $21\frac{5}{7}$ *po.*
(6) 815 *gns.* 1 *qt.* $0\frac{1}{10}$ *pt.*
(7) £297 8*s.* $7\frac{17}{56}$*d.*
(8) 139 *yrs.* 167 *da.* $1\frac{75}{113}$ *hr.*
(9) £12 16*s.* $11\frac{17}{30}$*d.*
(10) 8 *ac.* 1 *ro.* $18\frac{14}{33}$ *po.*
(11) £40 14*s.* $2\frac{1170}{3851}$*d.*
(12) 4 *lb.* 1 *oz.* $4\frac{80}{100}$ *dwt.*
(13) 2*s.* $11\frac{3}{4}$*d.*
(14) 50 *qu.* $2\frac{28}{37}$ *pks.*
(15) 129 *fur.* 34 *po.* 1 *yd.* $\frac{132}{176}$ *ft.*
(16) 5 *ac.* 24 *po.* 30 *sq. yds.* $1\frac{35}{64}$ *ft.*

41.

(1) $\frac{1}{20}s.$; $\frac{1}{240}£$. (2) $\frac{3}{1760}$ *mi.*; 2200 *sq. yds.*
(3) $\frac{1}{24}£$; $\frac{1}{100}$ *or.* (4) $\frac{13}{14}$ *guin.*; $\frac{7}{8}$ *kf. cr.* (5) $\frac{1}{280}$ *lb.*; $6\frac{3}{5}$ *d.*
(6) $\frac{25}{117}$ *moid.*; $\frac{7}{8}$ *kf. sov.* (7) $\frac{1}{2352}$ *cwt.*; 1020 *grs.*
(8) $\frac{35}{768}$ *lb. tr.*; $8\frac{101}{216}$ *oz. av.* (9) $\frac{20}{81}$; $\frac{5}{14}$. (10) $\frac{11}{145}$; $\frac{2}{5}$.
(11) $\frac{31}{155}$; $\frac{28}{55}$. (12) $\frac{27}{95}$; $\frac{9}{25}$. (13) $\frac{47}{85}$; $\frac{13}{11}$.
(14) $\frac{3}{8}$; $\frac{13}{51}$. (15) $\frac{317}{705600}$; $\frac{6}{175}$. (16) $\frac{3509}{31}$; $\frac{12679}{4165}$.

42.

(1) 3*s.* 9*d.*; $3\frac{1}{2}d.$; 4*s.* 6*d.*; 16*s.* 8*d.*
(2) 17*s.* $2\frac{1}{2}d.$; $15\frac{3}{4}d.$; 5*s.* $6\frac{6}{11}d.$; 16*s.* $8\frac{1}{2}d.$
(3) $24\frac{1}{2}$ *lb.*; $22\frac{1}{2}$ *gr.*; 2 *gall.* $1\frac{1}{2}$ *pt.*
(4) 16 *cwt.* $21\frac{1}{2}$ *lb.*; 20 *hrs.* 53 *min*, 20 *sec.*; 11 *sq. yd.* 5 *ft.* 90
(5) $16\frac{1}{2}$ *c. ft.*; 800 *yd.*; 3 *wk.* 6 *da.* 5 *hrs.* 20 *min.*
(6) 1 *yd.* $1\frac{2}{3}$ *na.*; 3 *sq. po.* 6 *yd.* $6\frac{1}{2}$ *ft.*; 237 *da.* 21 *hrs.* 36 *min*
(7) 15*s.* (8) £3 0*s.* $9\frac{7}{12}d.$ (9) £3 0*s.* $8\frac{1}{11}d.$
(10) £3 4*s.* $1\frac{11}{20}d.$ (11) 0.
(12) 1 *mi.* 4 *fur.* 12 *po.* $1\frac{7}{8}$ *yd.* (13) 134 *da.* 19 *hrs.* 1 *min.* $18\frac{1}{2}$
(14) 1 *ro.* 10 *po.* 17 *yd.* 2 *ft.* $127\frac{47}{700}$ *in.*

43.

(1) $\frac{7}{10}$, $\frac{209}{1000}$, $\frac{89}{100}$, $2\frac{51}{100}$; $15\frac{1}{5}$, $\frac{73}{200}$, $3\frac{13}{20}$, $\frac{9}{100}$; $\frac{19}{625}$, $11\frac{13}{4}$, $\frac{1}{4}$, $6\frac{1}{2}$; $\frac{1}{1000}$, $4\frac{2}{25}$, $20\frac{1}{200}$; $201\frac{2}{5}$, $\frac{3}{40000}$, $144\frac{1}{312500}$, $1\frac{733}{3125}$.
(2) ·8, ·24, ·3, 1·2, ·327; ·037, 15·6, 3·13, ·09185; ·221, 36, 1
·0000017.
(3) 23·456 and 2345·6; 4·75 and 47·5; 31081 and 3108100; 4
10; 3·575 and 357500.
(4) 607·02 and 6·0702; 6·0702 and ·060702; ·00215 and ·0(
365·4 and ·3654; ·00001 and ·000000001.

44.

(1) 7·0317. (2) 439·3284. (3) 156·32363. (4) 820.
(5) 9·895; 4·99. (6) 96·9945; ·290737.
(7) 75·0665. (8) 30·280965.

45.

(1) ·413; 1·0183. (2) 18·3924; ·69736.
(3) 2376·6; 388·89. (4) ·097305; ·0002336.
(5) 1430·715; ·037942884. (6) 1·860867; 67·51269.
(7) 36·9; 1·7005. (8) ·39434055325; 38.

46.

(1) 58·183; 3·7784. (2) ·5337; ·032615.
(3) 1·75; 100·65. (4) ·9375; ·03125; ·003125.
(5) 31·635; 405·6. (6) ·0195; ·00364.
(7) 37·5; 156·25; ·04296875. (8) ·0010494; ·0000625.
(9) 2951·25; 15000. (10) 25·09714$\frac{2}{7}$; ·07326$\frac{2}{3}$; ·00324$\frac{6}{11}$.
(11) 2321428$\frac{4}{7}$; 2·58 nearly.

47.

(1) ·5, ·25, ·75, ·125; ·1875, ·35, ·24, ·00875.
(2) ·16, 16·568, 3·39453125; 4·6875, 10·0208, 7·668.
(3) 11·171875; ·007. (4) ·0843$\frac{3}{13}$; ·3846$\frac{2}{13}$; ·0952$\frac{2}{21}$, 21·4259$\frac{23}{203}$.
(5) ·714286, ·272727, ·018333, 52·631579; ·004149, 11·615385, 1·620690, ·372671. (6) ·000013, ·4159.

48.

(1) ·4, ·3, ·38, ·416. (2) ·72, ·31707, ·245, ·24370.
(3) ·2871, ·063468, ·108801. (4) ·270322, ·21301180, ·0167364.

49.

(1) $\frac{2}{5}$, $\frac{6}{33}$, $\frac{11}{37}$, $\frac{10}{41}$. (2) $\frac{11749}{333503}$, $\frac{2524}{4845}$, $\frac{5}{7}$. (3) $\frac{21}{1025}$, $\frac{679}{1500000}$.
(4) $\frac{5}{28}$, $\frac{10}{73}$. (5) $\frac{11797}{98280}$, $\frac{1842153}{13732500}$. (6) $\frac{2602297}{5050000}$, $\frac{217}{504250}$.

50.

(1) 29·991310. (2) 11·549592. (3) ·100141; 2·899443.
(4) 2·098765432; ·96; 5·55867$\frac{1}{2}$ nearly; 2·246913458.
(5) ·003385 nearly; 11·1296 nearly. (6) 3·983$\frac{1}{2}$ nearly; ·49445.
(7) 6·336; 34·0426.

51.

(1) £·575; £·84375; £1·384375. (2) £·79583; £3·6572916; £·94259.
(3) ·90416 *cr.*; 1·389583. (4) ·02625; 1·1967592.
(5) ·978395$\frac{5}{81}$; ·1916. (6) ·53; ·7216492$\frac{7}{37}$.
(7) ·0375; 4·836. (8) £2·66875; 1·016.
(9) ·001875; 1·05. (10) ·79583 *lb.*; 1·68125 *oz.*
(11) ·14787946$\frac{3}{4}$ *cwt.*; ·045452$\frac{1}{113}$ *t.* (12) ·07714285 *cwt.*; 1·59505208$\frac{1}{3}$ *lb.tr.*
(13) ·047; 1·27232$\frac{1}{2}$. (14) ·031163194 *mi.*; 3·98.
(15) ·8022916 *da.*; ·839587 nearly.
(16) ·978125 *ac.*; ·81088083 nearly.
(17) 23·854938, &c. *sq. yd.*; ·728395061.
(18) 1·740625 *qu.*; 2·1986842$\frac{2}{19}$.

52.

(1) 7*s.* 6*d.*; 15*s.* $3\frac{3}{4}$*d.*; £1 16*s.* $2\frac{1}{2}$*d.*
(2) 1*s.* $10\frac{1}{2}$*d.*; $6\frac{3}{4}$*d.*; £9 13*s.* $9\frac{3}{8}$*d.* (3) 23*s.* $5\frac{3}{5}$*d.*; 42*s.* $3\frac{21}{100}$*d.*; 64*s.* $1\frac{1}{4}$*d.*
(4) $2\frac{11}{40}$*d.*; 8*s.* 6·156*d.*; 4*s.* $11\frac{251}{400}$*d.*
(5) 88*s.* 4*d.*; $\frac{2021}{4000}$*d.*; 86*s.* $8\frac{1}{4}$*d.* (6) 72*s.* $10\frac{261}{2000}$*d.*; 87*s.* $4\frac{1121}{2500}$*d.*
(7) 3 *qr.* 9 *lb.* 3·84 *oz.*; 1 *cwt.* 2 *qr.* 27·888 *lb.*; 5 *oz.* 3·84 *dr.*
(8) 7 *ac.* 1 *ro.* $13\frac{1}{2}$ *po.*; £10 18*s.* 4·224*d.*; 3 *yds.* 3·942 *in.*
(9) 1 *yd.* $8\frac{13}{40}$ *in.*; 7 *gal.* 3 *qt.* 0·3 *pt.*; 1 *ro.* 10·7104 *po.*
(10) 372·50048 *yd.*; 3·5 *guin.*
(11) 1 *cwt.* 41·2839 *lb.*; 4 *lb.* 2 *oz.* 1 *dwt.* 14·88 *gr.*
(12) 2*s.* $0\frac{88}{75}$*d.*; $5\frac{1}{4}$*d.*; 21*s.* $1\frac{2}{3}$*d.* (13) 5*s.* 10*d.*; £2 9*s.* $10\frac{1124}{2125}$*d.*
(14) 26*s.* 9·54*d.*; 2 *mi.* 2 *fur.* 70 *yd.* 2·2688 *ft.*

53.

(1) $\frac{97}{103}$; $\frac{97}{527}$. (2) 96. (3) 10·19368 nearly.
(4) $4\frac{101}{103}$; 4*s.* 3·765*d.* (5) $\frac{3}{7}$. (6) 720.
(7) $\frac{9}{11}$. (8) £10 18*s.* 4*d.* (9) $2\frac{5}{8}$*d.* (10) 355 very nearly.

54.

(1) $\frac{1467}{23000}$ *sq. po.* (2) $31\frac{5}{8}$. (3) $\frac{44}{111}$; 54·73. (4) $\frac{18}{49}$ *yd.*
(5) 75 : 128. (6) 75 nearly. (7) ·52.
(8) $12\frac{1}{2}$, $33\frac{3}{4}$, $46\frac{7}{8}$. (9) 38 *min.* $8\frac{1}{4}$ *sec.* (10) $\frac{22122}{2211955}$.

55.

(1) $\frac{17}{128}$. (2) 19300. (3) ·140625.
(4) 11 *ac.* 1 *rd.* 10·69 *po.* (5) $\frac{165}{224}$; $618\frac{3}{5}$. (6) $\frac{8}{9\frac{11}{21}}$, $\frac{8}{21\frac{1}{2}}$, $\frac{8}{26\frac{2}{3}}$.
(7) 32 *m.* (8) 23496 : 22025. (9) $5\frac{13}{16}$ *da.* (10) ·13552.

56.

(1) ·03759 nearly. (2) $425\frac{21}{51}$. (3) 531 : 4270.
(4) 5 *fur.* 3 *po.* 1·56024 *yd.* (5) ·60227. (6) 34 *lb.* $12\frac{4}{13}$ *oz.*
(7) $3\frac{9}{10}$ *oz.* (8) 14*s.* (9) $\frac{2645}{39}$. (10) 20.

57.

(1) 1·7287; ·015625. (2) 6*s.* 6·456*d.* (3) 5 *yd.* 1 *ft.* $1\frac{5}{8}$ *in.*
(4) 15*s.* (5) $29\frac{1}{2}$ of each. (6) 11 *wom.* (7) £265 5*s.* $9\frac{3}{5}$*d.*
(8) $34\frac{1}{2}$ *mi.* (9) $82\frac{1}{2}$ *tons.* (10) £35 12*s.* $9\frac{3}{5}$*d.*

58.

(1) $\frac{26}{33}$ *ell.*; ·43456 nearly. (2) 220·1 *gal.* nearly.
(3) $\frac{45}{56}$; 16*s.* $0\frac{9}{7}$*d.* (4) 1·2345 nearly.

(5) 908·70$\frac{3}{5}$ *fr.* (6) 19 *cwt.* 3 *qr.* 16 *lb.* 5·43 *oz.*
(7) £117 12*s.* (8) $\frac{5}{13}$. (9) 37$\frac{31}{45}$ *da.* (10) 12$\frac{1}{98}$.

59.

(1) £108 6*s.* 8*d.* (2) £228 2*s.* 6*d.* (3) £182 18*s.* 4*d.*
(4) £781 11*s.* 0*d.* (5) £2384 5*s.* 0*d.* (6) £512 0*s.* 0*d.*
(7) £976 10*s.* 0*d.* (8) £945 5*s.* 0*d.* (9) £1955 16*s.* 8*d.*
(10) £11735 2*s.* 0*d.* (11) £3769 14*s.* 10*d.* (12) £1594 5*s.* 4*d.*
(13) £1181 6*s.* 0$\frac{1}{2}$*d.* (14) £572 5*s.* 10$\frac{1}{2}$*d.* (15) £1529 4*s.* 0$\frac{3}{4}$*d.*
(16) £8617 17*s.* 6*d.* (17) £4414 6*s.* 3*d.* (18) £13790 10*s.* 3*d.*
(19) £664 8*s.* 4*d.* (20) £186 12*s.* 4$\frac{3}{5}$*d.* (21) £2711 8*s.* 8$\frac{7}{9}$*d.*
(22) £564 1*s.* 6$\frac{5}{6}$*d.* (23) £7978 19*s.* 0*d.* (24) £6290 4*s.* 3*d.*
(25) £1312 11*s.* 4*d.* (26) £7275 13*s.* 4*d.* (27) £1131 10*s.* 2$\frac{1}{3}$*d.*
(28) £470 19*s.* 1$\frac{1}{8}$*d.* (29) £1137 17*s.* 5$\frac{31}{80}$*d.* (30) £5267 1*s.* 5$\frac{13}{14}$*d.*
(31) £29 9*s.* 8·53$\frac{7}{8}$*d.* (32) £8 8*s.* 3$\frac{19}{20}$*d.* (33) £45 7*s.* 1$\frac{7}{32}$*d.*
(34) £41 11*s.* 10$\frac{5}{14}$*d.* (35) £36 19*s.* 10$\frac{1}{72}$*d.* (36) £317 6*s.* 8$\frac{7}{10}$*d.*
(37) £84 13*s.* 11$\frac{5}{9}$*d.* (38) £6 17*s.* 8$\frac{1}{12}$*d.* (39) £166 8*s.* 4$\frac{155}{156}$*d.*
(40) £4 18*s.* 4$\frac{893}{960}$*d.*

60.

(1) 43$\frac{3}{4}$ *p. c.* (2) 16$\frac{2}{3}$ *p. c.* (3) 2*s.* 6*d.* (4) 2$\frac{11}{13}$ *p. c.*
(5) 11*s.* (6) 7$\frac{1}{2}$*d.* (7) 1 *qr.* 12 *lb.*
(8) £9 8*s.* 3·4075*d.* (9) 61$\frac{7}{13}$ *p. c.* (10) 20 *p. c.*
(11) 1809 *pers.*; 326. (12) £1506 12*s.* (13) 7*d.*
(14) 95$\frac{5}{7}$ *p. c.* (15) 8$\frac{1}{3}$ *p. c.*
(16) 67 *p. c.*; 203$\frac{1}{33}$ *p. c.* (17) £8 2*s.* 6*d.* (18) 4$\frac{1}{6}$ *p. c.*
(19) 20. (20) 9$\frac{3}{5}$ *p. c.*
(21) 24*s. a gall.* (22) 8$\frac{3}{11}$ *p. c.*

61.

(1) £28. (2) £52 10*s.* (3) £203 12*s.* 6*d.*
(4) £90. (5) 355 18*s.* 4*d.* (6) £1520.
(7) 37$\frac{1}{2}$. (8) 37$\frac{1}{2}$. (9) £41 15*s.*
(10) 54*s.* (11) 1$\frac{1}{4}$ *p. c.* gain. (12) $\frac{5}{16}$ *p. c.* gain.
(13) 12033. (14) £29 5*s.* (15) *B*, $\frac{5}{37}$; *C*, $\frac{5}{23}$.
(16) 11 *yd.* (17) £1022 3*s.* 3$\frac{227}{767}$*d.* (18) £40 19*s.* 6$\frac{6}{41}$*d.*

62.

(1) 10*s.* (2) £39. (3) £12 5*s.*
(4) £26 6*s.* 6*d.* (5) £17 8*s.* 6$\frac{1}{2}$*d.* (6) £1166 13*s.* 4*d.*
(7) £880 15*s.* 7$\frac{1}{2}$*d.* (8) £66 0*s.* 3$\frac{101}{600}$*d.*
(9) *Amt.* £1265 13*s.* 8$\frac{4}{7}$*d.* (10) £180 4*s.* 6$\frac{3}{4}$*d.*

(11) £370 17*s.* $1\frac{7}{20}$*d.* (12) £60 17*s.* $8\frac{3}{10}$*d.* (13) £768 18*s.* $6\frac{1}{16}$*d.*
(14) £12190 12*s.* 6*d.* (15) £1097 9*s.* $6\frac{1}{2}$*d.* (16) 18*s.* $4\frac{68}{75}$*d.*
(17) £4 16*s.* 9·846*d.* (18) £15 18*s.* $6\frac{42}{125}$*d.* (19) £10 18*s.* 10·416*d.*
(20) £3 4*s.* $6\frac{75003}{125000}$*d.* (21) £196 16*s.* 11·04*d.* (22) £282 13*s.* $0\frac{281}{345}$*d.*
(23) £34 1*s.* 1·1976*d.* (24) £394 0*s.* $6\frac{254}{265}$*d.*

62.

(1) 5 *p. c.* (2) 3 *yrs.* (3) 7 *mnths.*
(4) $3\frac{3}{4}$ *p. c.* (5) £1650. (6) £735.
(7) 28 *da.* nearly. (8) £1226 13*s.* 4*d.* (9) £8 3*s.* 4*d.*
(10) £870 10*s.* (11) $44\frac{4}{9}$ *yrs.* (12) 24*s.* 6*d.* nearly.

64.

(1) £503 18*s.* $6\frac{3}{4}$*d.* (2) £1406 1*s.* $7\frac{1}{2}$*d.* (3) £826 10*s.* 10·62*d.*
(4) £7538 4*s.* $8\frac{1}{4}$*d.* (5) £98 5*s.* 0·82125*d.* (6) £1908 11*s.* 3·81*d.*
(7) £81 2*s.* $8\frac{7}{8}$*d.* nearly. (8) £192 15*s.* 1·55*d.* nearly.
(9) £52 15*s.* 6·778*d.* nearly. (10) £600 16*s.* 9·38*d.* nearly.

65.

(1) £82 4*s.* $6\frac{1}{2}$*d.* nearly. (2) £668 15*s.* 1·85*d.* nearly.
(3) £440 nearly. (4) £1232 5*s.* 5·305*d.* nearly. (5) £625.

66.

(1) £550. (2) $£20\frac{10}{17}$. (3) £7 19*s.* 3*d.*
(4) $£598\frac{33}{41}$. (5) $467\frac{29}{815}$. (6) £2 0*s.* $3\frac{6}{11}$*d.*
(7) $£8597\frac{1}{37}$. (8) $£33\frac{6}{23}$. (9) $£347\frac{27}{184}$.
(10) £114 1*s.* 3*d.* (11) 9 *mnths.* (12) $14\frac{3}{7}$ *p. c.* gain.
(13) 8 *p. c.* gain. (14) $10\frac{5}{13}$ *p. c.* (15) 8 *mnths.*

67.

(1) £407 8*s.* $7\frac{1}{2}$*d.* (2) £719 4*s.* $1\frac{1}{2}$*d.* (3) 12*s.* 9*d.*
(4) £1099 9*s.* 7*d.* (5) £1992 12*s.* $0\frac{3}{4}$*d.*; $6\frac{1}{2}$*d.*

68.

(1) 9, 45. (2) 12, 18, 24. (3) 46*s.* 8*d.*, 26*s.* 8*d.*, 18*s.* 8*d.*
(4) *A*, $12\frac{1}{2}$*d.*; *B*, $17\frac{1}{2}$*d.* (5) *A*, £100; *B*, 43 15*s.*; *C*, £116 5*s.*
(6) *A*, £13 6*s.* 8*d.*; *B*, £4 8*s.* $10\frac{2}{3}$*d.*; *C*, £22 4*s.* $5\frac{1}{3}$*d.* (7) *A*, 9; *B*, 12 *mi.*
(8) 78, 65, 52 *cwt.* (9) *A*, £192; *B*, £268 16*s.*
(10) *A*, 6*s.*; *B*, 3*s.*; *C*, 2*s.* 6*d.*; D, 8*s.* 6*d.*
(11) 300 *m.*, 280 *w.* 120 *ch.*

(12) *A*, £2100; *B*, £3570, *C*, £2380; *D*, £4550.
(13) £148 5*s*. $5\frac{1}{7}d$., £138 11*s*. $5\frac{1}{7}d$., £142 17*s*. $1\frac{5}{7}d$.
(14) $37\frac{1}{2}$, 15, $312\frac{1}{2}$. (15) 1 *sq. pole*.

69.

(1) *A man* 14*s*. $4\frac{1}{17}d$., *wom.* 8*s*. $9\frac{15}{17}d$., *ch.* 5*s*. $1\frac{13}{17}d$.
(2) £176 8*s*., £117 12*s*., £73 10*s*., £56.
(3) $£17\frac{23}{81}$, $£18\frac{4}{27}$, $£64\frac{16}{81}$. (4) *A*, $£18\frac{2}{3}$, *B*, $£15\frac{1}{3}$.
(5) *A*, $£55\frac{85}{113}$; *B*, $£84\frac{28}{113}$. (6) *A*, $£333\frac{513}{539}$; *B*, $£92\frac{412}{539}$; *C*, $£73\frac{153}{539}$.
(7) 27 *oz*. 1 *dwt*. 16 *grs*. (8) £1500.
(9) *A*, $£476\frac{2}{3}$; *B*, $£583\frac{1}{3}$; *C*, £825. (10) *A*, $2\frac{1}{2}$, *B*, $4\frac{1}{2}$; *C*, $5\frac{1}{2}$ *mths*.
(11) 144 *m*. 351 *w*. 480 *b*. (12) $15\frac{3}{16}s$., $10\frac{1}{8}s$., $12\frac{3}{4}s$.

70.

(1) £2350. (2) £2598 15*s*. (3) £1555 10*s*.
(4) £960; £36. (5) £3171 3*s*. (6) £311 2*s*. $2\frac{2}{3}d$.
(7) £5040; £201 12*s*.; $3\frac{37}{41}$ *p. c.* (8) $4\frac{13}{47}$ *p. c.*
(9) $£10277\frac{7}{9}$; £411 2*s*. $2\frac{2}{3}d$. (10) $3\frac{5}{9}$ *p. c.*
(11) $81\frac{1}{4}$. (12) 1*s*. 3*d*. more. (13) $70\frac{1}{2}$.
(14) 110. (15) £1 12*s*. $8\frac{8}{7}d$. more. (16) £298 1*s*. 3*d*.
(17) $78\frac{1}{8}$; £1920. (18) $79\frac{3}{4}$. (19) *B* gains £11 16*s* 3*d*.
(20) £5400. (21) £3 9*s*. 4*d*. more. (22) $101\frac{1}{4}$.
(23) $3\frac{1}{4}$ *p. c.* stock. (24) $87\frac{1}{5}$.

71.

(1) $18\frac{1}{8}$; $\frac{28}{45}$. (2) $6\frac{29}{30}$. (3) 4*s*. 2*d*. (4) 30*s*.
(5) 1*s*. 7*d*. (6) 12 *p. c.* (7) 8·66 *p. c.* nearly.
(8) 6 *lb*., 14 *lb*. (9) 9 : 5. (10) $11\frac{1}{2}$, $13\frac{1}{2}$. (11) 7 *oz*.
(12) 14. (13) 28, 72. (14) 18 *oz*. of each.
(15) 7 : 3. (16) £600, £225. (17) 10 *doz*. (18) £7200.
(19) 130 *lb*. (20) 5 : 4. (21) £120, £90.

72.

(1) $6\frac{1}{2}$ *mths*. (2) 31 *da*. (3) 9 *wks*. (4) $15\frac{1}{2}$ *mths*.
(5) 59 *da*. (6) 10 *mths*. (7) 18 *mths*.
(8) 1 *yr*. $279\frac{2}{17}$ *da*.; 1 *yr*. $280\frac{6}{37}$ *da*. (9) $8\frac{1008}{1402}$ *mths*.; $8\frac{2}{3}$ *mths*.

73.

(1) £40 5*s*. $9\frac{91}{211}d$. (2) 9072 *fr*. $14\frac{1}{2}$ *cts*. nearly.
(3) £246 13*s*. $6\frac{2262}{4465}d$. (4) £436. (5) £221 1*s*. $7\frac{1}{2}\frac{1}{5}\frac{1}{0}d$.
(6) 537·6 rup. (7) 25·32 fr. per £ nearly.
(8) 12 fl. $0\frac{1}{4}$ *cts. per* £ nearly. (9) 13 mks. 10 sch. per £.

(10) 36·5*d.* nearly. (11) 25·26$\frac{6}{15}$ *fr.* (12) 4098
(13) £1173 13*s.* 3$\frac{141}{221}$*d.* (14) £441 0*s.* 3*d.* (15) 25·5
(16) Indirect better by £10 11*s.* 3·081*d.*
(17) Direct better by 419·562 *ru.* (18) 46$\frac{1}{5}$ *fr.*
(19) About ·28, or nearly 3 *tenths per cent.* dearer in Paris.
(20) 25·31 *fr. per £* nearly. (21) About 2 *per mille* premi
(23) About the same price as in London. (24) 423·35

74.

(1) 393 *sq. ft.* 108 *in.* (2) 25 *sq. ft.* 27 *in.*
(3) 6004$\frac{1}{8}$ *sq. yds.* (4) $\frac{25}{96}$ *sq. yd.* (5) 54 *ac.*
(6) 955 *lks.* (7) 35 *sq. ft.* 1 *in.* (8) 7 *ft.* 8 *in.* (9) £22 4*s*
(10) 72 *planks.* (11) 9*s.* 2$\frac{55}{1152}$*d.* (12) 35$\frac{5}{7}$ *sq.*
(13) 64$\frac{1}{6}$ *yds.* (14) 21$\frac{3}{10}$ *yds.* (15) ·86673 *sq. yd.*
(16) 105$\frac{5}{14}$ *yds.* (17) 90 *sq. yds.* (18) 184$\frac{8}{9}$ *yds.* (19) 248$\frac{31}{33}$ *y*
(20) £21 1*s.* 3$\frac{5}{8}$*d.* (21) £5 12*s.* (22) £11 13
(23) 6$\frac{1}{2}$*d. a yd.* (24) 12 *ft.* 9 *in.*

75.

(1) 17$\frac{13}{16}$ *c. ft.* (2) 74 *c. ft.* 1628 *in.*
(3) 6 *cwt.* 2 *qr.* 7*lb.* 10$\frac{1}{2}$ *oz.* (4) 239$\frac{1}{2}$ *gall.* nearly.
(5) 6$\frac{1}{2}$ *in.* nearly. (6) 113288 *st.*
(7) 95 *c. yd.* 26 *ft.* 27 *in.*

76.

(1) 62 *sq. ft.* 2′ 9″ = 62 *sq. ft.* 33 *sq. in.*; 76 *sq. ft.* 1′ 4″ = 76 16 *sq. in.*
(2) 67 *sq. ft.* 2′ 2$\frac{1}{6}$″ = 67 *sq. ft.* 26$\frac{1}{6}$ *sq. in.*; 176 *sq. ft.* 11′ 4″ 1‴ 176 *sq. ft.* 136$\frac{9}{64}$ *sq. in.*
(3) 166 *sq. ft.* 9′ 6″ 3$\frac{11}{12}$‴ = 166 *sq. ft.* 114$\frac{47}{144}$ *sq. in.*; 3 *sq. ft.* 4′ 1 = 3 *sq. ft.* 59$\frac{1}{16}$ *sq. in.*
(4) 0 *sq. ft.* 3′ 1″ 7‴ = 37$\frac{7}{12}$ *sq. in.*; 6 *sq. ft.* 8′ 9″ 9‴ = 6 *sq. ft.* 105$\frac{3}{4}$
(5) 10 *sq. ft.* 2′ 11″ 11‴ = 10 *sq. ft.* 35$\frac{11}{12}$ *sq. in.*; 96 *sq. ft.* 7′ 8′ 96 *sq. ft.* 92$\frac{2}{3}$ *sq. in.*
(6) 37 *c. ft.* 4′ 10″ 6‴ = 37 *c. ft.* 702 *c. in.*
(7) 283 *c. ft.* 1′ 10$\frac{1}{2}$‴ = 283 *c ft.* 270 *c. in.*
(8) 88 *c. ft.* 3′ 5″ 4$\frac{5}{6}$‴ = 88 *c. ft.* 496$\frac{5}{6}$ *c. in.*
(9) 2 *ft.* 3′ 10″. (10) 9 units 10$\frac{3}{4}$ twelfths.
(11) 0 *ft.* 10′ 6″. (12) 2 ft. 6′.

77.

(1) 37; 79; 279. (2) 659; 860. (3) 904; 1195.
(4) 1866; 2307. (5) 4950; 60·17. (6) 98·989; 1·024.
(7) 59007; ·9807. (8) ·3198; 90·89. (9) ·0847; ·03126.
(10) 780900; ·099. (11) 4·968 nearly; 1·2345 nearly.
(12) 356·4008, &c.; ·548662, &c.
(13) 198·706 nearly; ·101508, &c.
(14) 2·4495 nearly; ·7746 nearly; 5·09902 nearly; ·14142, &c.
(15) ·63245, &c.; $\frac{2}{3}$; ·57735, &c.; 1·354, &c.
(16) 1·44337, &c.; $2\frac{1}{3}$; 7·42462, &c.; $5\frac{1}{2}$.
(17) 39·90405, &c.; 23·63; ·738549 nearly.
(18) 3·7052, &c.; ·26468, &c.; ·557055, &c.

78.

(1) 63; 44. (2) 36; 36. (3) 168; 15. (4) 280; $16\frac{1}{2}$.
(5) 390; $4\frac{1}{2}$. (6) $2\frac{1}{2}$; $8\frac{3}{4}$. (7) $9\sqrt{5}$; $44\sqrt{11}$.
(8) $\frac{5}{7}\sqrt{2}$. (9) $\frac{17}{156}$. (10) $\frac{19}{60}$. (11) $\frac{2}{5}\sqrt{5}$; $\frac{4}{9}\sqrt{6}$; $\frac{2}{5}$.
(12) $\frac{2}{3}\sqrt{5}$; $\frac{1}{60}\sqrt{5}$; 15.

79.

(1) $5\frac{1}{6}$ *ft.* (2) 365 *lks.* (3) 14 *ft.* (4) 53 *yd.*
(5) 60 *ft.* (6) 8 *sq. ft.* 108 *in.*
(7) 25·456 *yd.* nearly; 648 *sq. yd.* (8) 15 *ft.* $4\frac{2}{3}$ *in.*
(9) $4\frac{1}{2}$ *ft.*, $1\frac{1}{2}$ *ft.* (10) 58·4 *yd.* nearly.
(11) ·041 *of a mile.* (12) $1\frac{2}{3}$ *in.* nearly.

80.

(1) 57; 29. (2) 91; 640. (3) 215; 234. (4) 784; 109.
(5) 497. (6) 1905. (7) 2284. (8) 1399.
(9) 29·01. (10) ·135. (11) 4·1954; 3·6088, &c.
(12) 5·981, &c.; ·0464, &c. (13) ·5297 nearly; 3·124, &c.
(14) $97\frac{1}{3}$. (15) $\frac{3}{4}$; $\frac{3}{8}$; ·82207. (16) 8·4.

81.

(1) 4·41 *ft.* nearly. (2) 2·3506 *in.* (3) 1169·607 *sq. ft.* nearly.
(4) 54·352 *sq. ft.* (5) 721 : 500 nearly. (6) 8·543 *in.*

ANSWERS TO THE EXAMINATION PAPERS.

Exam. Paper (I).

(1) 2360398$\frac{1}{2}$ *ft.* (2) 102. (3) 307.
(4) £1434 9*s.* 9*d.*; 13*s.* 6$\frac{19}{112}$*d.* (5) 6$\frac{3}{4}$ *hr.* (6) 20 *qu.*
(7) £43 13*s.* 6$\frac{1}{2}$*d.* (8) £3046 18*s.* 3·2529*d.*
(9) £151$\frac{1}{2}$; 4·05 *p. c.* nearly; £4666$\frac{2}{3}$; £145 16*s.* 8*d.* (10) ·00882 *c. ft.*
(11) £1333$\frac{1}{3}$. (12) 29·0086; 9·441. (13) 8$\frac{3}{4}$.
(14) 7$\frac{7}{18}$ *da.* (15) 12 *gal.* 3 *pt.* (16) 3 *p. c.*
(17) Beginning of March. (18) 7$\frac{1}{2}$*d.*

Exam. Paper (II).

(1) 50 men; 4*s.* 2*d.* (2) 72. (3) 37$\frac{1}{3}$ *da.*
(4) £7 10*s.* 6·095*d.* (5) 144 times; *rem.* ·066.
(6) £11 18*s.* 6*d.* (7) 6 *doz.* (8) $\frac{149}{151}$. (9) $\frac{13}{18}$.
(10) $\frac{80}{81}$; $\frac{5413}{5550}$. (11) £38 8*s.* more. (12) £833 15*s.*
(13) 4$\frac{16}{21}$ *p. c.* (14) *A*, £5000, *B*, £5685. (15) 110$\frac{2}{5}$ *yds.*
(16) 182$\frac{181}{442}$ *st.* (17) 18$\frac{3}{8}$*d. p. gall.* (18) ·081.

Exam. Paper (III).

(1) £1277 12*s.* 1*d.* (2) 3$\frac{3}{5}$*d.* (3) 1$\frac{373}{459}$.
(4) £549 3*s.* 4*d.* (5) 1866. (6) £62 11*s.* 2$\frac{2}{5}$*d.* (7) 6$\frac{16}{35}$.
(8) 254 *sq. ft.* 7′ 5″ 11$\frac{3}{4}$‴ = 28 *sq. yd.* 2 *ft.* 89$\frac{47}{48}$ *in.* (9) 24*s.* 9*d.*
(10) 3$\frac{1}{2}$ *bo.* (11) *A*, £5 16*s.*; *B*, £4 7*s.*; *C*, £2 13*s.* (12) 8$\frac{1}{8}$ *a*
(13) £12 5*s.* nearly. (14) £4 9*s.* for 5 *gns.* (15) 35 *l*
(16) *A*, £37 19*s.* 2$\frac{10}{13}$*d.*; *B*, £36 3*s.* 0$\frac{12}{13}$*d.*; *C*, £40 7*s.* 8$\frac{4}{13}$*d.*
(17) 48 *gall.* (18) £1 5*s.* 2$\frac{1}{2}$*d.*

Exam. Paper (IV).

(1) £6 15*s.* (2) 28$\frac{89}{140}$ *yd.* (3) 12 *m.*
(4) £3295 17*s.* 9$\frac{3}{4}$*d.*; £13 1*s.* 5$\frac{55}{64}$*d.* (5) 62$\frac{1}{2}$ *yd.*
(6) *A*, £593 15*s.*; *B*, £504 13*s.* 9*d.*; *C*, £326 11*s.* 3*d.* (7) 7$\frac{1}{2}$ *days.*
(8) 4 *p. c. per ann.* (9) 46 *qr.* (10) 32 *hr.*
(11) 91. (12) *C*, 15$\frac{3}{4}$ *min.*; *D*, 2$\frac{1}{10}$ *hr.* (13) £26 0*s.* 10$\frac{3}{4}$
(14) 170 nearly. (15) 5 : 4. (16) 26$\frac{2}{3}$ *st.*
(17) 3$\frac{3}{4}$ *p. c.* (18) £2484.

Exam. Paper (V).

(1) 4802·8 *met.* nearly. (2) 93$\frac{169}{192}$ *cwt.* (3) 15*d.*
(4) 9$\frac{1}{2}$*d. per doz.* (5) 135 *da.* (6) 6 *da.* (7) ·0006.

(8) 52*s.* 6*d.* (9) 6*s.* 9*d.* (10) 35*s.* 9*d.* (11) 4 : 3.
(12) £14 14*s.* (13) 14 *da.* (14) 720, 400, 300 *ac.*
(15) 11 *da.* (16) £3967 10*s.*; £3954 3*s.* 11*d.*
(17) 75. (18) £1700, £300.

Exam. Paper (VI).

(1) 13 *cwt.* 2 *qr.* 24 *lb.* (2) £26253 4*s.* $10\frac{7}{8}$*d.*; £25 8*s.* $4\frac{25}{64}$*d.*
(3) 128 *ells.* (4) $18\frac{2}{7}$ *ft.* (5) $\frac{1}{3}$. (6) 6*s.* $11\frac{3}{4}$*d.*
(7) 189. (8) 5 *hr.* (9) £161 14*s.* 2*d.*; £327 5*s.* 6*d.*
(10) $5\frac{1}{2}$ *mi.* (11) $\frac{13}{63}$. (12) 3*s.* 4*d.*, 2*s.* 6*d.*
(13) 9*d.* (14) 3 *yrs.* ago. (15) $1\frac{1}{2}$ *hr.*
(16) 66*s.* 8*d.* (17) 2 *mths.* (18) 50 *ox.*

Exam. Paper (VII).

(1) 68 *oz.* $11\frac{3}{7}$ *dwt.* (2) $2509\frac{1}{2}$ *yds.* (3) $7\frac{1}{2}$ *yds.*
(4) $\frac{567}{136}$. (5) 24 *ft.* (6) $8\frac{1}{2}$*d.* (7) 8 *horses.*
(8) £8. (9) £55 10*s.* 8*d.* (10) 68 *da.*
(11) £33. (12) £532 13*s.* 4*d.*
(13) £480, £640, £880. (14) 4·3114 *p. c.*
(15) 15 *gall.*, 10 *gall.* (16) £21 17*s.* 6*d.*
(17) 3 *hr.* (18) Indirect better by £11 3*s.* $9\frac{3}{4}$*d.*

Exam. Paper (VIII).

(1) 15 *ac.* 2 *ro.* 30 *po.* 27 *yd.* 80 *in.* (2) 10·7.
(3) £78 6*s.* $11\frac{233}{320}$*d.* (4) $3\frac{1}{2}$ *per Cents.* (5) £1270 10*s.* 1*d.*
(6) £407 15*s.* (7) ·16016*s.* (8) $\frac{85}{153}$. (9) $32\frac{8}{11}$. *m. past* 6.
(10) 16 : 21. (11) 6 : 5. (12) £2035 2*s.* 6*d.* (13) 8*s.* 4*d.*
(14) $3\frac{1}{3}$ *mi.* (15) 2*s.* 11*d.* (16) $26\frac{26}{99}$ *p. c.*
(17) £2900, £2624. (18) 75 *da.*

Exam. Paper (IX).

(1) 20 *lb.* (2) $\frac{11}{12}$. (3) 3 *yr.* 235 *da.* 21 *hr.* $22\frac{1}{2}$ *min.*
(4) 625 *cwt.* 3 *qr.* 5 *lb.*; £1023 8*s.* 8·6675*d.*
(5) £4037 14*s.* $9\frac{1}{701}$*d.* (6) 9·75 *flor.* (7) £6 12*s.* $10\frac{5}{7}$*d.*
(8) $58\frac{19}{32}$ *min. past* 1. (9) 32*s.* 6*d.* (10) 1938 *ft.* nearly.
(11) 1 *ho.* $55\frac{5}{7}$ *m.*; $1\frac{3}{4}$ *ho.* (12) $2\frac{1}{4}$ *in.* nearly; 2·2795 *c. ft.*
(13) 57 *min.* (14) $22\frac{1}{2}$, 40. (15) 96 *min.* (16) 9, 6, 4.
(17) $12\frac{225}{227}$ *min.*; $13\frac{71}{83}$ *min. past* 5. (18) 28 *ox.*

Exam. Paper (X).

(1) 1068 *guin.* (2) 11·88 *ft.* nearly. (3) £2603370.
(4) ·62. (5) 14000. (6) 3*s.* 4*d.*; 24855 *m.*
(7) 7 *ac.* $25\frac{43}{103}$ *po.* (8) 75 *lb.* 1 *oz.* 19 *dwt.* nearly.
(9) 2046 *sov.* 8 *fl.* 7 *cts.* 5 *mils.* (10) $4\frac{88}{83}$ *p. c.*
(11) *B* owed *A* £1022 0*s.* 4*d.* (12) 77. (13) $1\frac{10}{11}$ *mi. an h*
(14) $3\frac{3}{4}$ *p. c.* (15) £250, £350. (16) 687 *da.*
(17) 8 *lb.* 2 *oz.* (18) $2\frac{1}{3}$ *doz. br.*; 3 *doz. rum.*

Exam. Paper (XI).

(1) 2 *ac* 39 *po.* 30 *yd.* 2 *ft.* 16 *in.* (2) $148\frac{4}{5}$ *lb.*
(3) £37 6*s.* $7\frac{7}{11}$*d.* (4) 3·5208 *p. c.* nearly.
(5) £3200. (6) 3·45 *oz.* (7) $\frac{41}{224}$. (8) £8 8*s.*
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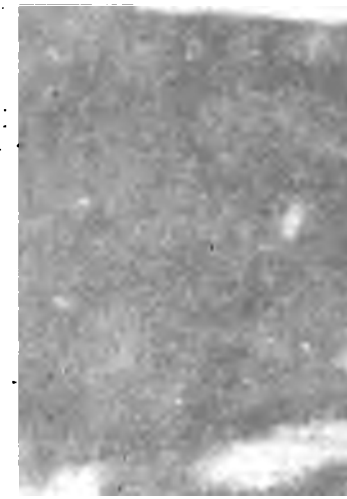

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